A Rewriting-based Forwards Semantics for Maude-NPA

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Outline

Deriving Equivalent Logical Systems

- To prove properties of a program, we need make use of some logical sytem
- Different components, different aspects, different properties of a program may require different logical systems
  - This is especially the case in security, a many-faceted problem
- We need to show these different logics can work together, and what is proved in one system remains true in another
- In this talk, will show how applied this to a formal tool for cryptographic protocol analysis Maude-NPA

Problem Area: Symbolic Cryptographic Protocol Analysis

- Example: Diffie-Hellman Without Authentication
  1. $A \to B : g^{N_A}$
  2. $B \to A : g^{N_B}$
  3. A and B compute $g^{N_A * N_B} = g^{N_B * N_A}$

  Well-known attack
  1. $A \to I_B : g^{N_A}$
  2. $I_A \to B : g^{N_I}$
  3. $B \to I_A : g^{N_B}$
  4. $I_B \to A : g^{N_I}$
- A thinks she shares $g^{N_I * N_A}$ with B, but she shares it with I
- B thinks he shares $g^{N_I * N_A}$ with A, but he shares it with I
- Commutative properties of $*$ and fact that $(G^X)^Y = G^{X*Y}$ crucial to understanding both the protocol and the attack

Symbolic "Dolev-Yao" Model for Automated Cryptographic Protocol Analysis

- Start with a signature, giving a set of function symbols and variables
- For each role, give a program describing how a principal executing that role sends and receives messages
- Give a set of inference rules and equations the describing the deductions an intruder can make
  - E.g. if intruder knows K and $e(K, M)$, can deduce M, or;
  - $d(K, e(K, M)) = M$, where d is a decryption operator
- Assume that all messages go through intruder who can
  - Stop or redirect messages
  - Alter messages
  - Create new messages from already sent messages using inference rules

The Maude-NPA Tool

- A tool to find or prove the absence of attacks using backwards search
- Analyzes infinite state systems
  - Active intruder
  - No abstraction or approximation of nonces
    - If Maude-NPA finds path from initial state to insecure attack state, it is a genuine path
  - Unbounded number of sessions
    - If Maude-NPA terminates without finding path no such path exists
    - Problem is in general undecidable, so Maude-NPA may not terminate
    - Uses search-space pruning mechanisms making termination more likely
- Supports a number of equational theories, including: cancellation (e.g. encryption-decryption), AC, exclusive-or, Diffie-Hellman, bounded associativity, homormorphic encryption over various theories, various combinations, working on including more
- Executable semantics based on rewrite rules

Executable Formal Semantics

- Logical system that can also be executed
  - In our case, as state-exploration-based cryptographic protocol analysis tool, Maude-NPA
- By proving things about the logical system, we can prove things about results of the execution
- If we want to make modifications to the tool, we make modifications to the semantics
  - Prove new semantics sound and/or complete to the old
  - Have applied this approach to extend the capabilities of Maude-NPA and prove that these extensions are sound and complete

What Happens When the Process Breaks?

- Require major changes to semantics in order to achieve the functionality we want
  - In our case, we needed to reverse the direction of the execution
  - In this talk, we show how we handled this problem

Outline

Important Tools Used by Maude-NPA: Equational Unification

- Given a signature Σ and an equational theory E, and two terms s and t built from Σ:
- A unifier of $s =_E ?t$ is a substitution σ to the variables in s and t s.t. σs can be transformed into σt by applying equations from E to σs and its subterms
- Example: $\Sigma = \{d/2, e/2, m/0, k/0\}, E = \{d(K, e(K, X)) = X\}$. The substitution $\sigma = \{Z \mapsto e(T, Y)\}$ is a unifier of $d(T, Z)$ and Y.
- The set of most general unifiers of $s = ?t$ is the set Γ s.t. any unifier σ is of the form $\rho\tau$ for some ρ, and some τ in Γ.
- Example: $\{Z \mapsto e(T, Y), Y \mapsto d(T, Z)\}$ mgu's of $d(T, Z)$ and Y.
- Given the theory, can have:
  - at most one mgu (empty theory)
  - a finite number (AC)
  - an infinite number (associativity)
- Problem can also be undecidable

Important Tools Used by Maude-NPA: Rewrite Rules and Narrowing

- A rewrite theory $\mathcal{R}$ is a triple $\mathcal{R} = (\Sigma, E, R)$, with:
  - Σ a signature
  - (Σ, R) a set of rewrite rules of the form $t \to s$
    e.g. $e(K_A, N_A; X) \to e(K_B, X)$
  - E a set of equations of the form $t = s$
- Rewriting: If t is a ground term (no variables), $t \to_{\sigma,R,E} s$ if there are
  - a non-variable position $p \in Pos(t)$;
  - a rule $l \to r \in R$;
  - a substitution σ (modulo E) such that $t\theta =_E l$ and $s = \theta(t[r]_p)$
- Narrowing: If t is a symbolic term (may have variables) $t \rightsquigarrow_{\sigma,R,E} s$ if there are
  - a non-variable position $p \in Pos(t)$;
  - a rule $l \to r \in R$;
  - a unifier σ (modulo E) of $t|p =_E ?l$ such that $s = \sigma(t[r]_p)$.

Comparison of Rewriting and Narrowing

- In favor of narrowing
  - Narrowing wrt symbolic terms means you can handle a possibly infinite number of terms in one narrowing step
  - For that reason, good for reasoning about infinite state systems
- In favor of rewriting
  - Rewriting simpler and faster than narrowing
  - Software support for rewriting (in particular, Maude itself!)
- Conclusion: Use narrowing when it can most benefit you, rewriting otherwise

Protocols Specified Using Strand Spaces

- Maude-NPA uses concept of strand spaces due to Thayer, Herzog, and Gutmann (2001)
- A strand is a sequence of messages representing the actions of a principal executing a role, or of an intruder making a computation
  - A negative term represents a message received by a principal
  - A positive term represents a message sent by a principal
- Example: Initiator's strand in DH

```
:: r, r' ::  [nil , +(A ; B ; exp(g,n(A,r))),  -(A ; B ; XE),
              +(e(exp(XE,n(A,r)),sec(A,r'))), nil]
```

- Example: Attacker exponentiation strand in DH

```
:: nil :: [ nil | -(GE), -(NS), +(exp(GE,NS)), nil ]
```

- Note: Capital letters stand for logical variables, terms inside "::" are special variables used to construct nonces

States in Maude-NPA

- A state is a set of strands plus the intruder knowledge (i.e., a set of terms)
  1. Each strand is divided into past and future
     $[\ m_1{}^{\pm},\ \ldots,\ m_i{}^{\pm} \mid m_{i+1}{}^{\pm},\ \ldots,\ m_k{}^{\pm}\ ]$
  2. Initial strand $[\ nil \mid m_1^{\pm},\ \ldots,\ m_k^{\pm}\ ]$, final strand $[\ m_1^{\pm},\ \ldots,\ m_k^{\pm}\ \mid nil\ ]$
  3. The intruder knowledge contains terms $m{\notin}\mathcal{I}$ and $m{\in}\mathcal{I}$
     $\{\ t_1{\notin}\mathcal{I}, \ldots, t_n{\notin}\mathcal{I},\ s_1{\in}\mathcal{I}, \ldots, s_m{\in}\mathcal{I}\ \}$
  4. Initial intruder knowledge $\{\ t_1{\notin}\mathcal{I}, \ldots, t_n{\notin}\mathcal{I}\ \}$,
     final intruder knowledge $\{\ s_1{\in}\mathcal{I}, \ldots, s_m{\in}\mathcal{I}\ \}$

Example

- State in which initiator has sent first message, attacker has learned that message, and attacker will learn secret value in future

```
SS & :: r, r' ::  [nil , +(a; b ; exp(g,n(a,r))) |
              -(a ; b ; XE),
              +(e(exp(XE,n(a,r)),sec(a,r'))), nil] &
          {exp(g,n(a,r) inI ,
              sec(a,r') notinI , K}
```

- Note that it is possible (and expected) for states to contain variables
- Since XE hasn't been received yet, we don't know what it is

Maude-NPA Backwards Semantics

- Expressed in terms of forwards executing rewrite rules
- Rewrite rule: a rule of the form $\ell \to r$ meaning "replace expression ℓ with expression r
  1. $SS \;\&\; [\, L \mid M^-, L' \,] \;\&\; \{M{\in}\mathcal{I}, K\} \to SS \;\&\; [\, L, M^- \mid L' \,] \;\&\; \{M{\in}\mathcal{I}, K\}$
     Moves input messages into the past
  2. $SS \;\&\; [\, L \mid M^+, L' \,] \;\&\; \{K\} \to SS \;\&\; [\, L, M^+ \mid L' \,] \;\&\; \{K\}$
     Moves output message that are not read into the past
  3. $SS \;\&\; [\, L \mid M^+, L' \,] \;\&\; \{M{\notin}\mathcal{I}, K\} \to SS \;\&\; [\, L, M^+ \mid L' \,] \;\&\; \{M{\in}\mathcal{I}, K\}$
     Joins output message with term in intruder knowledge.
  4. $SS \;\&\; [\, l_1 \mid u^+] \;\&\; SS \;\&\; \{u{\notin}\mathcal{I}, K\} \to \{u{\in}\mathcal{I}, K\}$ where $[\, l_1 \mid u^+]$ is a prefix of a strand in the protocol specification
     Introduces new strand or prefix of strand, and joins output message with term in intruder knowledge.
- To obtain backwards semantics, just reverse the arrows!

Executing the Backwards Semantics

- Begin by specifying an attack state pattern
  - An attack state pattern describes an insecure state and may contain variables
  - Example : Attack state in which responder B has finished execution of protocol, apparently with initiator A, but attacker knows the secret

```
:: r :: [nil, -(a ; b ; XE), +(a ; b ; exp(g,n(b,r))),
            -(e(exp(XE,n(b,r)),sec(a,r'))) | nil]
      || sec(a,r') inI
```

- Use backward narrowing via the rewrite rules, to determine if an initial state can be reached
- If you reach an initial state, you will have constructed a path to an instance of the attack pattern

Outline

When We May Need Forward Execution

- Practical Reasons
  - Narrowing is powerful, but computationally expensive
  - If you execute forwards instead of backwards, states will contain no variables, and you can use rewriting instead of narrowing
  - Example: Suppose that you want to simulate protocol to see if it can reach a final state in absence of attackers
    - Narrowing is overkill
- Theoretical Reasons
  - In many cases, it is more natural to reason about forward rather than backwards execution
  - We found this when developing a theory of indistinguishability for Maude-NPA

Important: Forwards semantics must be sound and complete with respect to backwards semantics

- Allows us to switch between forwards and backwards semantics
- We use simulation to verify protocol specified correctly using forwards semantics, but verify security using backwards semantics
- We use forwards semantics to formulate our indistinguishability framework, but prove indistinguishability using backwards semantics

Why Can't We Just Execute the Backwards Semantics Forwards?

- Maude-NPA already has a forwards semantics, obtained by reversing the backwards semantics
  - Why can't we just use that and save ourselves a lot of work?
- Backwards semantics contains too much information about the future!
  - Initial state contains all strands and intruder knowledge used to reach the final state
  - Part of the strand after the bar may need to contain variables
    - This is problematic for rewriting

How We Represent States in the Forwards Semantics

- No variables allowed in state
- Only information about the past allowed, not the future
  - Terms $t \notin \mathcal{I}$ can't appear, since they represent future knowledge of the intruder
  - Information after the bar in a strand can't appear, since it represents future execution

Some Rules in the Forwards Semantics

- Adding a positive term the intruder doesn't know already to a strand

$$\left\{\begin{array}{l} \forall\ [u_1^{\pm}, \ldots, u_{j-1}^{\pm}, u_j^{+}, u_{j+1}^{\pm}, \ldots, u_n^{\pm}] \in \mathcal{P} \wedge j > 1 : \\ \{SS \,\&\, \{IK\} \,\&\, [u_1^{\pm}, \ldots, u_{j-1}^{\pm}] \,\&\, \langle N \rangle\} \\ \rightarrow \\ \{SS \,\&\, \{u_j{\uparrow}_N^M{\in}\mathcal{I}, IK\} \,\&\, [u_1^{\pm}, \ldots, u_{j-1}^{\pm}, (u_j{\uparrow}_N^M)^{+}] \,\&\, \langle M \rangle\} \\ \qquad\qquad \text{IF } (u_j{\uparrow}_N^M{\in}\mathcal{I}) \notin IK \end{array}\right\} \quad (1)$$

- Adding a strand that begins with a positive term the intruder doesn't know already

$$\left\{\begin{array}{l} \forall\ [u_1^{+}, \ldots, u_n^{\pm}] \in \mathcal{P} : \\ \{SS \,\&\, \{IK\} \,\&\, \langle N \rangle\} \rightarrow \{SS \,\&\, [(u_1{\uparrow}_N^M)^{+}] \,\&\, \{IK\} \,\&\, \langle M \rangle\} \end{array}\right\} \quad (2)$$

Outline

Lifting Relation

Definition (Lifting relation)

Given a symbolic $\mathcal{P}$-state S and a ground state s we say that s *lifts* to S, or that S *instantiates* to s with a *grounding* substitution $\theta : (Var(S) - \{SS, IK\}) \rightarrow \mathcal{T}_{\Sigma}$, writen $S >^{\theta} s$ iff

- for each strand $:: r_1, \ldots, r_m :: [u_1^{\pm}, \ldots u_{i-1}^{\pm} \mid u_i^{\pm}, \ldots, u_n^{\pm}]$ in S, there exists a strand $[v_1^{\pm}, \ldots v_{i-1}^{\pm}]$ in s such that $\forall 1 \leqslant j \leqslant i-1$, $v_j =_{E_{\mathcal{P}}} u_j\theta$.
- for each positive intruder fact $w {\in} \mathcal{I}$ in S, there exists a positive intruder fact $w' {\in} \mathcal{I}$ in s such that $w' =_{E_{\mathcal{P}}} w\theta$, and
- for each negative intruder fact $w {\notin} \mathcal{I}$ in S, there is no positive intruder fact $w' {\in} \mathcal{I}$ in s such that $w' =_{E_{\mathcal{P}}} w\theta$.

Example of Lifting Relation

- Symbolic state

```
SS & :: r, r' ::  [nil , +(a; b ; exp(g,n(a,r))) |
                  -(a ; b ; XE),
                  +(e(exp(XE,n(a,r)),sec(a,r'))), nil] &
              {exp(g,n(a,r)) inI ,
                  sec(a,r') notinI , K}
```

- Ground State

```
 [ +(a; b ; exp(g,n(a,1))) ] &
          {exp(g,n(a,1)) inI,
          a inI,
          b inI,
          a; b ; exp(g,n(a,1)) inI}
```

- Lifting via $\theta = \{r \to 1\}$

Soundness and Completeness Theorems

Theorem (Completeness)

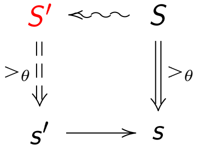
Given a protocol $\mathcal{P}$*,*
two ground states s, s_0*, a symbolic* $\mathcal{P}$*-state* S*, a substitution* θ *s.t. (i)* s_0 *is an initial state, (ii)* $s_0 \rightarrow^n s$*, and (iii)* $S >^\theta s$ *then there exist a symbolic initial* $\mathcal{P}$*-state* S_0*, two substitutions* μ *and* θ'*, and* $k \leqslant n$*, s.t.* $S_0 \overset{k}{\leftsquigarrow}_\mu S$*, and* $S_0 >^{\theta'} s_0$*.*

Theorem (Soundness)

Given a protocol $\mathcal{P}$*, two*
symbolic $\mathcal{P}$*-states* S_0, S'*, an initial ground state* s_0 *and a substitution* θ *s.t. (i)* S_0 *is a symbolic initial state, and (ii)* $S_0 \overset{*}{\leftsquigarrow} S'$ *, and (iii)* $S_0 >^\theta s_0$ *then there exist a ground state* s' *and a substitution* θ'*, s.t. (i)* $s_0 \rightarrow^* s'$*, and (ii)* $S' >^{\theta'} s'$*.*

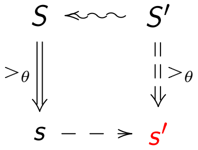
Proof of Soundness and Completeness

- (Lifting Lemma) Given rewriting step $s' \to s$ and lifting relation S $>_\theta$ s we can complete the diagram with S' as follows:



[Soundness]: Given a forward rewriting sequence iterate lifting lemma to get corresponding backwards narrowing sequence

- (Grounding Lemma) Given narrowing step $S \rightsquigarrow S'$ and lifting relation S $>_\theta$ s we can complete the diagram with an s' as follows:



[Completeness]: Given a backwards narrowing sequence iterate grounding lemma to get corresponding forwards rewriting sequence

Outline

Implementation of Forwards Semantics in Maude

- Implemented rewriting-based forward semantics in Maude
- Maude's support for rewriting made it possible to do this very quickly
- Implemented some heuristic state space reduction techniques to reduce state space explosion
  - Plan to investigate these further in the future, in particular adapting Maude-NPA's state space reduction techniques to a forwards setting
  - Expect soundness and completeness result to help us here
- Applied it two various protocols in the literature, tool was able to reproduce attacks found by Maude-NPA

Outline

Conclusion

- We started out wanting a theoretical tool to help us reason about indistinguishability, but we wound up with
  - A novel executable semantics for model-checking cryptographic protocols
  - A new logical foundation for Maude-NPA, designed for model-checking
  - The beginnings of a new crypto protocol model-checker
- And we got a new theoretical tool to help us reason about indistinguishability!