Applying Formal Methods to Prove Correctness of Surgical Robot Software

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Surgical Robotics (Cyber-Physical Systems)

da Vinci Surgical Robot

Figures courtesy of Intuitive Surgical, Inc.

da Vinci research opportunities

Augmented Reality

- Preoperative images
- \bullet Intraoperative data

Mechanical Assistance

- Virtual fixtures
- \bullet Motion primitives

Retinal Microsurgery System

NIH EB 007969

Credit: Russell Taylor

Surgical Assistant Workstation (SAW)

Joint development with Intuitive Surgical, Inc. NSF EEC 9731748, EEC 0646678, MRI 0722943

SAW Requirements

- •Support robot control and real-time image processing
- Concurrent execution within a process (multithreading) and between processes
- Plug & play of devices
- Safety for clinical testing

Hierarchical multi-rate robot control exactle and Real-time video stream

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SAW Design

- • Component-based software architecture
	- Within process and between processes
	- Uses *cisst* C++ libraries

ICE for data exchange between components in different processes

Efficient, lock-free data exchange between components in same process

- • State table (single writer, multiple readers)
- \bullet Mailboxes (single reader/writer FIFO)

Medical Device Safety

- Medical device safety: IEC 60601
- Medical device SW life cycle: IEC 62304
- Risk management: ISO 14971
	- Failure Modes Effects and Criticality Analysis (FMECA), IEC 60812

Focus on process, traceability, and testing

SAW Testing

Automated unit testing framework (uses CDash)

What about lock-free mechanisms for data exchange between concurrent threads?

Formal Methods to the Rescue!

- Use FM to validate a small subset of SAW (data exchange primitives)
	- Most difficult to validate by testing
	- Incremental introduction of FM

Goal

Model Driven Design: code generation from verified models

• (At least for a critical subset of code)

Design: State Vector Storage/Access

- \blacktriangleright Consider a system that has
	- \triangleright A single "writer" thread
	- \triangleright A single storage location for the state vector, with a version 'v' to distinguish between updates

 \blacktriangleright Many "reader" threads

- \triangleright Slow readers' data will be corrupted by a fast writer
- \blacktriangleright Readers cannot tell whether the vector has been updated/corrupted during the read

Design: Circular Buffer

- \triangleright SAW maintains a circular buffer of slots
- \blacktriangleright Each slot contains storage for the state vector and a version number
- \blacktriangleright Read and write indices indicate most recent slot completely updated, and slot currently being updated

Design: Starting State

Design: Advance Write Index

Design: Update Version

Design: Advance Read Index

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Design: Write State Vector

Design: Complete Write

Implementation: Write Cycle

- \triangleright Display one buffer slot as it changes state
- \blacktriangleright Time progresses from left to right

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Active-write portion of cycle

Implementation: Read Strategy

- \triangleright Check version before and after read to ensure no corruption of data
- \triangleright Reasoning: Writer updates version before writing, so any reader will notice different versions if the writer changed the data during the read

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Implementation: Detecting A Corrupted Read

 \triangleright This interleaving illustrates the detection of a corrupted read by observing a change in version numbers

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FM Approach: HLRG Program Logic

- \blacktriangleright HLRG predicates apply to traces, or sequences of system states representing the progression of the system over time
	- \blacktriangleright Temporal operators allow expression of statements connecting state in the present to states in the past
- \triangleright Rely/guarantee allows reasoning about concurrent threads
- \triangleright Separation logic allows local reasoning
- \triangleright Yale colleagues developed sound proof rules that worked in the presence of these traces and these operators¹²

 ${}^{1}X$. Feng. Local rely-guarantee reasoning, In Proc. 36th ACM Symp. on Principles of Prog. Lang., Jan. 2009

 $2M$. Fu, Y. Li, X. Feng, Z. Shao, and Y. Zhang. Reasoning about optimistic concurrency using a program logic for history. Yale [Tec](#page-22-0)[hni](#page-24-0)[c](#page-22-0)[al](#page-23-0) [R](#page-24-0)[epo](#page-12-0)[rt](#page-32-0) \longleftrightarrow \Rightarrow \Rightarrow \Diamond

FM Approach: Operators in HLRG

- \triangleright Some operators we might encounter:
	- \triangleright P \land Q is additive conjunction, where all of the context is used to satisfy P and Q
	- \triangleright P \ast Q multiplicative conjunction, where part of the context is used to satisfy P and another disjoint part satisfies Q
	- \triangleright P \triangleright Q means at some point in the past, P was true, and at some later time, Q became true
	- $P \triangleright Q$ means at some point in the past, P was true, and thereafer, Q held
	- $\triangleright \, \Leftrightarrow$ P means at some point in the past or in the present, P happened

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 \blacktriangleright \sqcap P means that P holds always

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FM Approach: Describing the Domain of Shared State

Thrray

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\stackrel{\text{def}}{=} \circledast_{i \in [0, \ldots, H-1]} \text{Ticks} + i \mapsto
$$
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$$
\text{Vector}(i)(j) \stackrel{\text{def}}{=} \text{Vec} + i \times N + j \mapsto
$$
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$$
\text{Vector}
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$$
\stackrel{\text{def}}{=} \circledast_{j \in [0, \ldots, N-1]} (\circledast_{i \in [0, \ldots, H-1]} \text{Vector}(i)(j))
$$
\n
$$
I \stackrel{\text{def}}{=} \exists X. Y. \text{Thrray} * \text{Vector} * \text{readindex} \mapsto X * \text{writeindex} \mapsto Y
$$

$$
\mathsf{Vector}(i) \mapsto D \quad \stackrel{\mathsf{def}}{=} \quad \circledast_{j \in [0, ..., N-1]} \mathsf{Vec} + i \times N + j \mapsto D(i, j)
$$

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FM Approach: Atomic steps taken I UpdWrite $\stackrel{\mathsf{def}}{=} \mathsf{Id} \ast ((\mathsf{UpdData} \vartriangleright \mathsf{Id}) \wedge \exists X, X'.\mathtt{writeindex} \mapsto X \ltimes \mathsf{Id})$ writeindex $\mapsto X' \wedge X' = (X + 1) \text{mod} H$ I UpdVer $\stackrel{\mathsf{def}}{=} \mathsf{Id} \ast ((\mathsf{Update} \rhd \mathsf{Id}) \wedge \exists X, X', V, V'.\mathtt{writeindex} \mapsto X *$ $\operatorname{Ticks}+X'\mapsto V'\ltimes\bigl(\text{writeindex}\mapsto X*\text{Ticks}+X\mapsto V'+1*$ $\operatorname{Ticks}+X'\mapsto V')\wedge X=(X'+1){\operatorname{\mathsf{mod}}}{H})$ I UpdRead $\stackrel{\text{def}}{=}$ Id \ast ((UpdVer \triangleright Id) $\land \exists X, Y$ writeindex $\mapsto Y \ltimes$ readindex $\mapsto X * \text{writeindex} \mapsto Y$ $\wedge Y = (X + 1) \text{mod} H$ I $\mathsf{UpdData} \quad \stackrel{\mathsf{def}}{=} \quad ((\mathsf{UpdData} \lor \mathsf{UpdRead}) \rhd \mathsf{Id}) \land \bigvee_{j \in [0,...,N-1]} \exists X.(\mathsf{Vector}(X)(j) *$ writeindex $\mapsto X \ltimes \text{Vector}(X)(j) *$ writeindex $\mapsto X) *$ Id

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FM Approach: Program Description

 \triangleright We have transformed the program into a description of the possible atomic steps it may take that affect computer state I

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G \stackrel{\text{def}}{=} (Id \vee UpdData \vee UpdWrite \vee UpdVer \vee UpdRead) \wedge (I \ltimes I)
$$

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$$
R \stackrel{\text{def}}{=} \mathsf{Id} \wedge (I \ltimes I)
$$

$$
\mathcal{M} \stackrel{\text{def}}{=} \boxminus (R \vee G)
$$

Verification: First Attempt At Read Data Integrity

- \triangleright Now state the theorem we seek to prove, and show that our machine implies that theorem
- \blacktriangleright Prove key lemma:

$$
\mathcal{M} \Rightarrow \left(\frac{A}{\text{Ticks} + h \rightsquigarrow X} \blacktriangleright \frac{B}{\text{Vector}(h) \rightsquigarrow D \land} \right) \text{Vector}(h) \rightsquigarrow B \land \underbrace{\text{Vector}(h) \rightsquigarrow D'}_{C} \blacktriangleright \underbrace{\text{Ticks} + h \rightsquigarrow X}_{D} \Rightarrow (D = D') \right)
$$

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 \triangleright This is unprovable, therefore there is a flaw in our design

Verification: Example of Read Strategy Problem

- \blacktriangleright There is a brief span of time where the data becomes inconsistent without a change in the version number
- \triangleright An example interleaving that illustrates the problem with our read strategy
- \triangleright The reader cannot detect corruption in the read

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Verification: Improved Read Strategy

- \triangleright Check the position of the write index in between the first version check and the actual reading of data
- If the write index is pointing to the current slot (i.e. we are in the active write portion of the cycle, then assume that our data is corrupted
- If the version number has changed during the read, also assume the data is corrupted

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Verification: Data Read Integrity Theorem

- \triangleright Able to successfully complete proof of data read integrity
- \blacktriangleright Improved key lemma:

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\mathcal{M} \Rightarrow \left(\begin{array}{c} \mathcal{A} & \mathcal{B} & \mathcal{C} \\ \text{Ticks+h} \leadsto X & \text{writeindex} \leadsto h' & \text{Vector}(h) \leadsto D \land \\ \frac{\text{Vector}(h) \leadsto D'}{D} & \text{Ticks+h} \leadsto X \land (h \neq h') \Rightarrow (D = D') \\ \hline D & \mathcal{E} & \mathcal{E} \end{array} \right)
$$

 \triangleright When a read completes, the value that is returned accurately reflects what was stored in memory for that state vector element during the read; and that value was stable during the read, i.e. no writer was altering it or may have altered it during that time.

Conclusions

- \triangleright Towards practical application of formal methods in the design of medical systems
	- \triangleright Moving in the direction of incrementally introducing FM into development process, using the SAW as a test case
	- \triangleright Certify properties for critical pieces of reusable framework in which testing is inadequate
- \blacktriangleright Immediate benefit to the surgical assistant workstation
	- \triangleright Found and fixed a design flaw in the SAW software
	- \blacktriangleright Guaranteed that there are no more bugs in the state-table component that could unexpectedly impact the data integrity of the state vector
	- \triangleright Enumerated specific axioms on which this guarantee rests