Applying Formal Methods to Prove Correctness of Surgical Robot Software

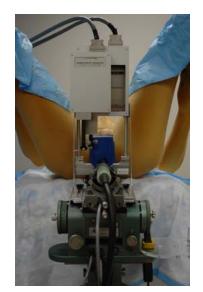
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HCSS, Annapolis MD, May 4, 2011

Surgical Robotics (Cyber-Physical Systems)

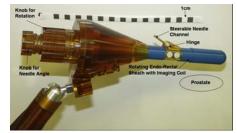








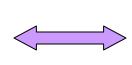






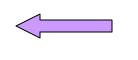
da Vinci Surgical Robot





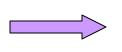














Figures courtesy of Intuitive Surgical, Inc.

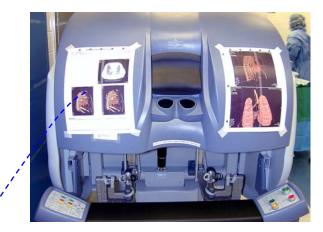
da Vinci research opportunities

Augmented Reality

- Preoperative images
- Intraoperative data

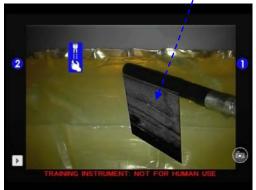
Mechanical Assistance

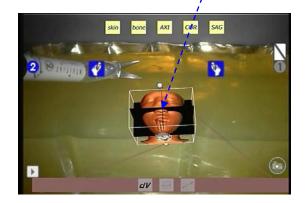
- Virtual fixtures
- Motion primitives



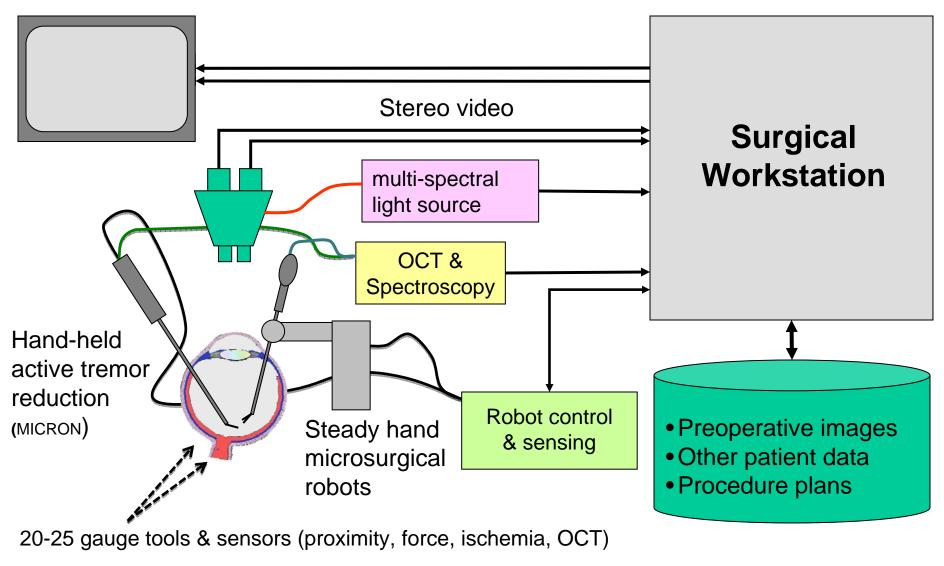








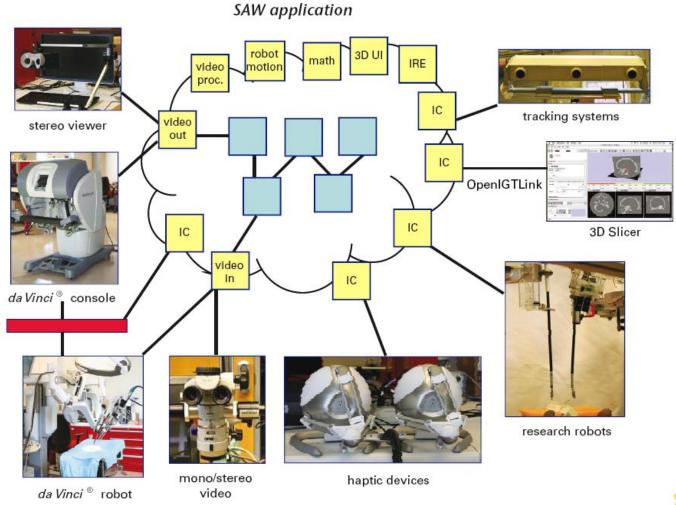
Retinal Microsurgery System



NIH EB 007969

Credit: Russell Taylor

Surgical Assistant Workstation (SAW)

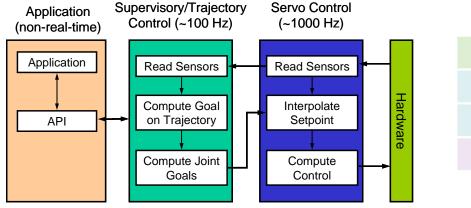


Joint development with Intuitive Surgical, Inc. NSF EEC 9731748, EEC 0646678, MRI 0722943

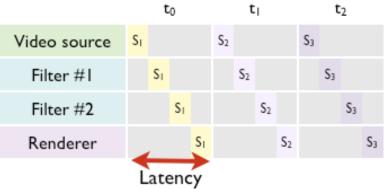


SAW Requirements

- Support robot control and real-time image processing
- Concurrent execution within a process (multithreading) and between processes
- Plug & play of devices
- Safety for clinical testing



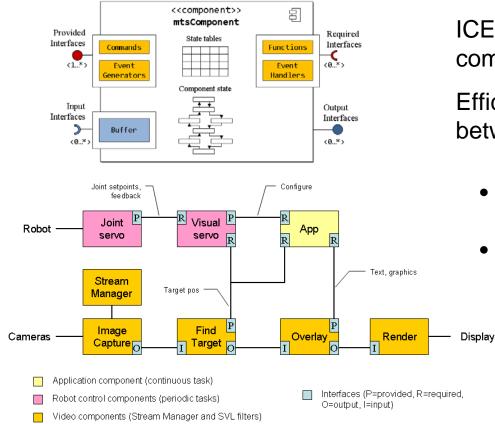
Hierarchical multi-rate robot control



Real-time video stream

SAW Design

- Component-based software architecture
 - Within process and between processes
 - Uses cisst C++ libraries



ICE for data exchange between components in different processes

Efficient, lock-free data exchange between components in same process

- State table (single writer, multiple readers)
- Mailboxes (single reader/writer FIFO)

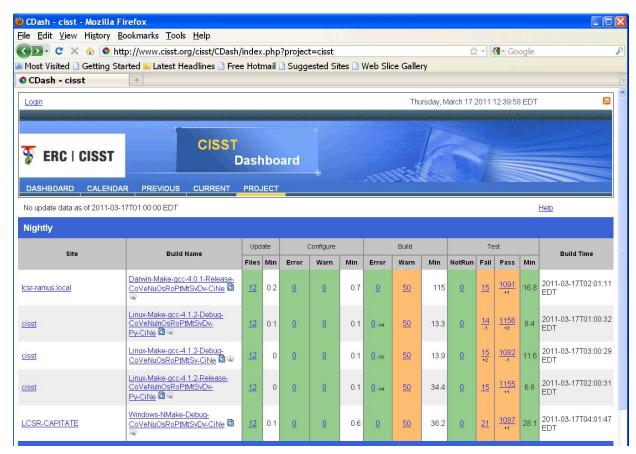
Medical Device Safety

- Medical device safety: IEC 60601
- Medical device SW life cycle: IEC 62304
- Risk management: ISO 14971
 - Failure Modes Effects and Criticality Analysis (FMECA), IEC 60812

Focus on process, traceability, and testing

SAW Testing

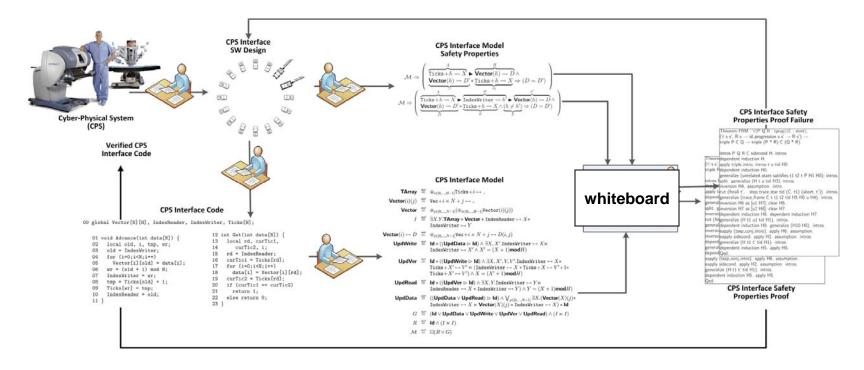
Automated unit testing framework (uses CDash)



What about lock-free mechanisms for data exchange between concurrent threads?

Formal Methods to the Rescue!

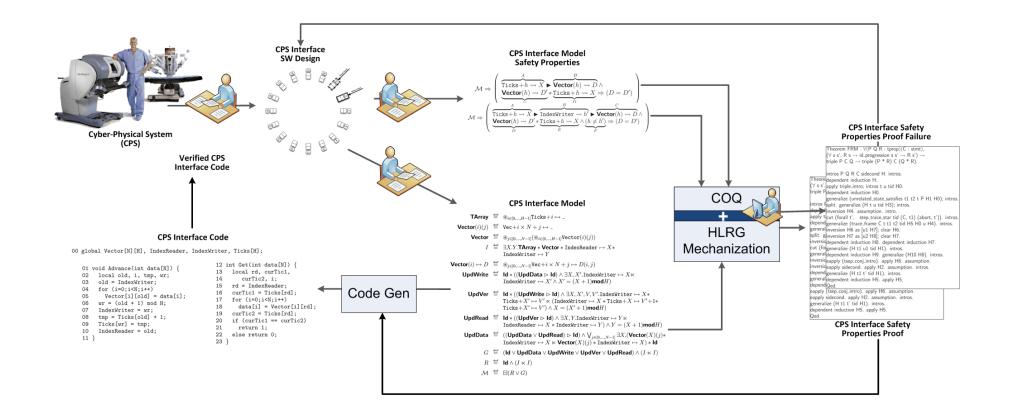
- Use FM to validate a small subset of SAW (data exchange primitives)
 - Most difficult to validate by testing
 - Incremental introduction of FM



Goal

Model Driven Design: code generation from verified models

• (At least for a critical subset of code)



Design: State Vector Storage/Access

- Consider a system that has
 - A single "writer" thread
 - A single storage location for the state vector, with a version 'v' to distinguish between updates

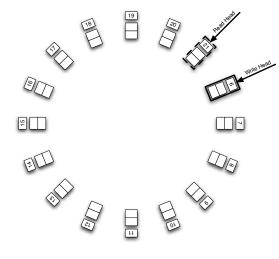
Many "reader" threads



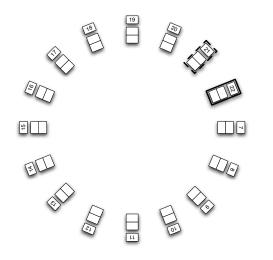
- Slow readers' data will be corrupted by a fast writer
- Readers cannot tell whether the vector has been updated/corrupted during the read

Design: Circular Buffer

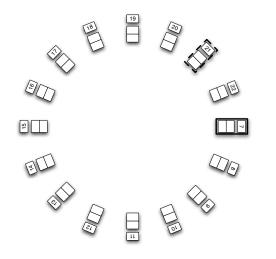
- SAW maintains a circular buffer of slots
- Each slot contains storage for the state vector and a version number
- Read and write indices indicate most recent slot completely updated, and slot currently being updated



Design: Starting State

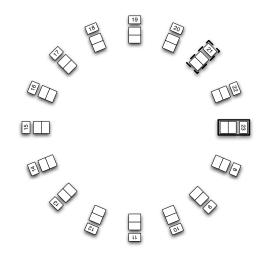


Design: Advance Write Index

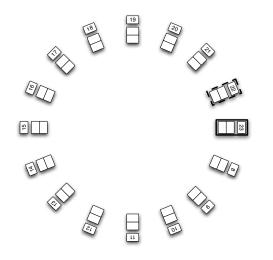


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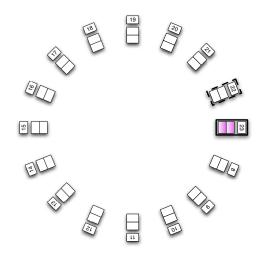
Design: Update Version



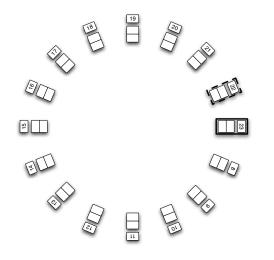
Design: Advance Read Index



Design: Write State Vector

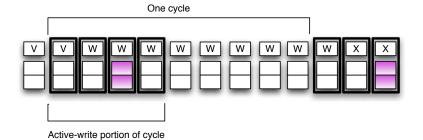


Design: Complete Write



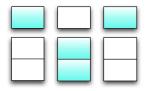
Implementation: Write Cycle

- Display one buffer slot as it changes state
- Time progresses from left to right



Implementation: Read Strategy

- Check version before and after read to ensure no corruption of data
- Reasoning: Writer updates version before writing, so any reader will notice different versions if the writer changed the data during the read



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Implementation: Detecting A Corrupted Read

 This interleaving illustrates the detection of a corrupted read by observing a change in version numbers



FM Approach: HLRG Program Logic

- HLRG predicates apply to traces, or sequences of system states representing the progression of the system over time
 - Temporal operators allow expression of statements connecting state in the present to states in the past
- Rely/guarantee allows reasoning about concurrent threads
- Separation logic allows local reasoning
- Yale colleagues developed sound proof rules that worked in the presence of these traces and these operators¹²

¹X. Feng. Local rely-guarantee reasoning, In Proc. 36th ACM Symp. on Principles of Prog. Lang., Jan. 2009

²M. Fu, Y. Li, X. Feng, Z. Shao, and Y. Zhang. Reasoning about optimistic concurrency using a program logic for history. Yale Technical Report (2) (2) (3)

FM Approach: Operators in HLRG

- Some operators we might encounter:
 - $P \land Q$ is additive conjunction, where all of the context is used to satisfy P and Q
 - P * Q multiplicative conjunction, where part of the context is used to satisfy P and another disjoint part satisfies Q
 - P ► Q means at some point in the past, P was true, and at some later time, Q became true
 - P ▷ Q means at some point in the past, P was true, and thereafer, Q held
 - ► ⇔ P means at some point in the past or in the present, P happened
 - $\Box P$ means that *P* holds always

►

FM Approach: Describing the Domain of Shared State

$$\begin{array}{rcl} \mathsf{TArray} & \stackrel{\mathrm{def}}{=} & \circledast_{i \in [0, \dots, H-1]} \mathsf{Ticks} + i \mapsto _ \\ \mathsf{Vector}(i)(j) & \stackrel{\mathrm{def}}{=} & \mathsf{Vec} + i \times N + j \mapsto _ \\ \mathsf{Vector} & \stackrel{\mathrm{def}}{=} & \circledast_{j \in [0, \dots, N-1]} (\circledast_{i \in [0, \dots, H-1]} \mathsf{Vector}(i)(j)) \\ I & \stackrel{\mathrm{def}}{=} & \exists X. Y. \mathsf{TArray} * \mathsf{Vector} * \mathsf{readindex} \mapsto X * \\ & \texttt{writeindex} \mapsto Y \end{array}$$

$$\mathsf{Vector}(i) \mapsto D \stackrel{\mathsf{def}}{=} \circledast_{j \in [0, \dots, N-1]} \mathsf{Vec} + i \times N + j \mapsto D(i, j)$$

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FM Approach: Atomic steps taken UpdWrite $\stackrel{\text{def}}{=}$ Id * ((UpdData \triangleright Id) $\land \exists X, X'$.writeindex $\mapsto X \ltimes$ writeindex $\mapsto X' \wedge X' = (X+1) \mod H$ $\stackrel{\mathsf{def}}{=} \mathsf{Id} * ((\mathsf{UpdWrite} \rhd \mathsf{Id}) \land \exists X, X', V, V'. \mathsf{writeindex} \mapsto X *$ UpdVer $\mathrm{Ticks} + X' \mapsto V' \ltimes (\mathrm{writeindex} \mapsto X * \mathrm{Ticks} + X \mapsto V' + 1 *$ $\texttt{Ticks} + X' \mapsto V') \land X = (X' + 1) \texttt{mod} H$ def UpdRead $\mathsf{Id} * ((\mathsf{UpdVer} \rhd \mathsf{Id}) \land \exists X, Y. writeindex \mapsto Y \ltimes$ readindex $\mapsto X * \text{writeindex} \mapsto Y) \land Y = (X + 1) \mod H$ $\mathsf{UpdData} \quad \stackrel{\mathsf{def}}{=} \quad ((\mathsf{UpdData} \lor \mathsf{UpdRead}) \rhd \mathsf{Id}) \land \bigvee_{j \in [0, \dots, N-1]} \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \ast \mathsf{Id}) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \land \forall j \in [0, \dots, N-1] \exists X.(\mathsf{Vector}(X)(j) \land \forall j \in [0, \dots, N-1] \lor \forall i \in [0, \dots, N-1]) \land \forall$

writeindex $\mapsto X \ltimes \operatorname{Vector}(X)(j) \ast \operatorname{writeindex} \mapsto X) \ast \operatorname{Id}$

FM Approach: Program Description

 We have transformed the program into a description of the possible atomic steps it may take that affect computer state

$$G \stackrel{ ext{def}}{=} (\mathsf{Id} \lor \mathsf{UpdData} \lor \mathsf{UpdWrite} \lor \mathsf{UpdVer} \lor \mathsf{UpdRead}) \land (I \ltimes I)$$

$$R \stackrel{\text{def}}{=} \operatorname{Id} \wedge (I \ltimes I)$$

 $\mathcal{M} \stackrel{\mathsf{def}}{=} \boxminus (R \lor G)$

Verification: First Attempt At Read Data Integrity

- Now state the theorem we seek to prove, and show that our machine implies that theorem
- Prove key lemma:

$$\mathcal{M} \Rightarrow \left(\underbrace{\begin{array}{c} \underbrace{\mathsf{Ticks} + h \rightsquigarrow X}_{\mathsf{Vector}(h) \rightsquigarrow D} \land \\ \underbrace{\mathsf{Vector}(h) \rightsquigarrow D'}_{\mathsf{C}} \ast \underbrace{\mathtt{Ticks} + h \rightsquigarrow X}_{\mathsf{D}} \Rightarrow (\mathsf{D} = \mathsf{D}') \end{array} \right)$$

This is unprovable, therefore there is a flaw in our design

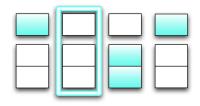
Verification: Example of Read Strategy Problem

- There is a brief span of time where the data becomes inconsistent without a change in the version number
- An example interleaving that illustrates the problem with our read strategy
- The reader cannot detect corruption in the read



Verification: Improved Read Strategy

- Check the position of the write index in between the first version check and the actual reading of data
- If the write index is pointing to the current slot (i.e. we are in the active write portion of the cycle, then assume that our data is corrupted
- If the version number has changed during the read, also assume the data is corrupted



Verification: Data Read Integrity Theorem

- Able to successfully complete proof of data read integrity
- Improved key lemma:

$$\mathcal{M} \Rightarrow \left(\underbrace{\underbrace{\mathsf{Ticks}}_{\mathsf{D}} + h \rightsquigarrow X}_{D} \bullet \underbrace{\underbrace{\mathsf{Writeindex}}_{F} \rightarrow h' \bullet \underbrace{\mathsf{Vector}(h) \rightsquigarrow D}_{F} \land \underbrace{\mathsf{Vector}(h) \rightsquigarrow D'}_{F} \ast \underbrace{\underbrace{\mathsf{Ticks}}_{F} + h \rightsquigarrow X}_{E} \land \underbrace{(h \neq h')}_{F} \Rightarrow (D = D') \right)$$

When a read completes, the value that is returned accurately reflects what was stored in memory for that state vector element during the read; and that value was stable during the read, i.e. no writer was altering it or may have altered it during that time.

Conclusions

- Towards practical application of formal methods in the design of medical systems
 - Moving in the direction of incrementally introducing FM into development process, using the SAW as a test case
 - Certify properties for critical pieces of reusable framework in which testing is inadequate
- Immediate benefit to the surgical assistant workstation
 - Found and fixed a design flaw in the SAW software
 - Guaranteed that there are no more bugs in the state-table component that could unexpectedly impact the data integrity of the state vector
 - Enumerated specific axioms on which this guarantee rests