Concurrent Separation Logic

Peter O'Hearn, HCSS Tutorial, 21 May 2009

References:

- **O'Hearn**: Resources, concurrency, and local reasoning. Reynolds Festschrift. Theor. Comput. Sci. 375(1-3): 271-307, 2007. Prelim version in CONCUR'04.
- **Brookes**: A semantics for concurrent separation logic. Reynolds Festschrift. Theor. Comput. Sci. 375(1-3): 227-270, 2007. Prelim version in CONCUR'04.
- **Hoare**: Towards a theory of parallel programming. Operating Systems Techniques. 1971

Dijkstra's Principle

- *We have stipulated that processes should be loosely connected; by this we mean that apart from the (rare) moments of explicit intercommunication, the individual processes are to be regarded as completely independent of each other.*
  (*Cooperating Sequential Processes, EW Dijkstra, 1965*)

Disjoint Concurrency

$$\frac{\{P_1\}\, C_1\, \{Q_1\} \quad \{P_2\}\, C_2\, \{Q_2\}}{\{P_1 * P_2\}\, C_1 \parallel C_2 \{Q_1 * Q_2\}}$$

where

C_i *does not alter variables free in* P_j, Q_j *when* $i \neq j$.

Disjoint Concurrency

$$\frac{\{P_1\}\,C_1\,\{Q_1\} \quad \{P_2\}\,C_2\,\{Q_2\}}{\{P_1 * P_2\}\,C_1 \parallel C_2\,\{Q_1 * Q_2\}}$$

We can't prove racy programs like

$$\{10 \mapsto -\}$$
$$[10] := 42 \;\parallel\; [10] := 6$$
$$\{??\}$$

We cannot send 10 to both processes in their preconditions, since

$$(10 \mapsto -) * (10 \mapsto -)$$

is false. But...

Well specified programs don't go wrong

$\{(x \mapsto -) * P\}\ [x]{:=}7\ \{(x \mapsto 7) * P\}$

$\{\texttt{true}\}\ [x]{:=}7\ \{??\}$

$\{P * (x \mapsto -)\}\ \texttt{dispose}(x)\ \{P\}$

$\{\texttt{true}\}\ \texttt{dispose}(x)\ \{??\}$

$\{P\}\ x = \texttt{cons}(a, b)\ \{P * (x \mapsto a, b)\}\quad (x \notin \mathit{free}(P))$

If $\{P\}C\{Q\}$ *holds then* P *describes all the resources that* C *needs (at the beginning) in order to run.*

Disjoint Concurrency

$$\frac{\{P_1\}\,C_1\,\{Q_1\} \quad \{P_2\}\,C_2\,\{Q_2\}}{\{P_1 * P_2\}\,C_1 \parallel C_2\,\{Q_1 * Q_2\}}$$

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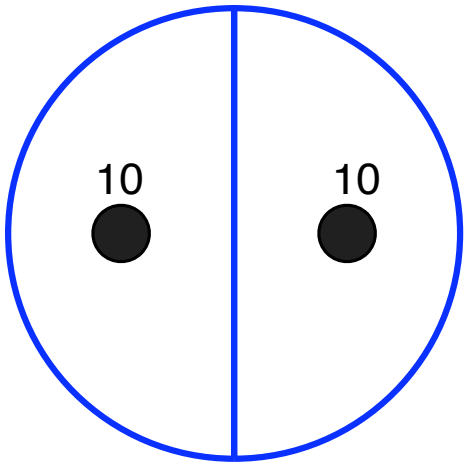
An inconsistency: trying to be two places at once

10|->3 * 10|->3



Disjoint Concurrency

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Preconditions can pick out race-free start-states, when they exist:

$$\begin{array}{ccc} & \{x \mapsto 3 * y \mapsto 3\} & \\ \{x \mapsto 3\} & & \{y \mapsto 3\} \\ [x] := 4 & \parallel & [y] := 5 \\ \{x \mapsto 4\} & & \{y \mapsto 5\} \\ & \{x \mapsto 4 * y \mapsto 5\} & \end{array}$$

Disjoint Concurrency

$$\frac{\{P_1\}\,C_1\,\{Q_1\} \quad \{P_2\}\,C_2\,\{Q_2\}}{\{P_1 * P_2\}\,C_1 \parallel C_2\,\{Q_1 * Q_2\}}$$

Preconditions can pick out race-free start-states, when they exist:

$$\begin{array}{c} \{x \mapsto 3 * y \mapsto 3\} \\ \{x \mapsto 3\} \qquad\qquad \{y \mapsto 3\} \\ [x]{:=}\,4 \qquad \parallel \qquad [y]{:=}\,5 \\ \{x \mapsto 4\} \qquad\qquad \{y \mapsto 5\} \\ \{x \mapsto 4 * y \mapsto 5\} \end{array}$$

That ‘proof figure” is an annotation form for

$$\frac{\{x \mapsto 3\}\,[x]{:=}\,4\,\{x \mapsto 4\} \qquad \{y \mapsto 3\}\,[y]{:=}\,5\,\{y \mapsto 5\}}{\{x \mapsto 3 * y \mapsto 3\}\,[x]{:=}\,4 \parallel [y]{:=}\,5\,\{x \mapsto 4 * y \mapsto 5\}}$$

Example: Parallel Mergesort

$\{array(a, i, j)\}$
procedure ms(a, i, j)
newvar $m := (i + j)/2$;
if $i < j$ then
$\big(\mathtt{ms}(a, i, m) \parallel \mathtt{ms}(a, m + 1, j)\big)$;
merge$(a, i, m + 1, j)$;
$\{sorted(a, i, j)\}$

Example: Parallel Mergesort

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$\{sorted(a, i, j)\}$

- Can't prove with disjoint concurrency rule

$$\frac{\{P\}C\{Q\} \quad \{P'\}C'\{Q'\}}{\{P \wedge P'\}C \parallel C'\{Q \wedge Q'\}}$$

where C does not modify any variables free in P', C', Q', and conversely. Because: Hoare logic treats an assignment to an array component as an assignment to the whole array.

Example: Parallel Mergesort

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$\quad \texttt{merge}(a, i, m + 1, j);$
$\{sorted(a, i, j)\}$

- To prove with invariants+preservation, you track many irrelevant interleavings
  - and... state complex recursion hypothesis

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$\quad \mathtt{merge}(a, i, m + 1, j);$
$\{sorted(a, i, j)\}$

- To prove with rely/guarantee, you complicate the spec (not just the reasoning)
  - Rely: no one else touches my segment
  - Guarantee: I only touch my own segment (frame axiom)

In Separation Logic[1]

- We just use the given pre/post spec.

$$\{array(a, i, m) * array(a, m+1, j)\}$$
$$\begin{array}{lcl} \{array(a, i, m)\} & & \{array(a, m+1, j)\} \\ \mathtt{ms}(a, i, m) & \| & \mathtt{ms}(a, m+1, j) \\ \{sorted(a, i, m)\} & & \{sorted(a, m+1, j)\} \end{array}$$
$$\{sorted(a, i, m) * sorted(a, m+1, j)\}$$

- Concurrency proof rule:

$$\frac{\{P_1\}C_1\{Q_1\} \quad \{P_2\}C_2\{Q_2\}}{\{P_1 * P_2\}C_1 \| C_2\{Q_1 * Q_2\}}$$

[1] $a[i]$ is sugar for $[a + i]$ in RAM model

Process Interaction:

Conditional Critical Regions (Hoare, 72)

Programs

init;
`resource` $r_1(\text{variable list}), \ldots, r_m(\text{variable list})$
$C_1 \parallel \cdots \parallel C_n$

CCRs

`with` r `when` B `do` C .

–if a variable belongs to a resource, it cannot appear in a parallel process except in a critical section for that resource

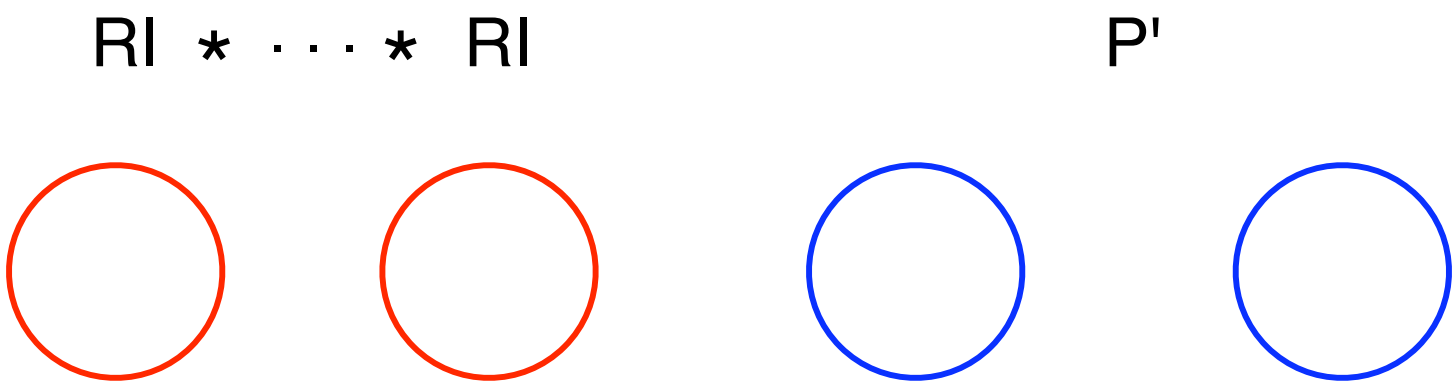
–if a variable is changed in one process, it cannot appear in another unless it belongs to a resource.

The Sequential Processes

$$
\begin{array}{rcl}
C & ::= & x{:=}\,E \mid x{:=}\,[E] \mid [E]{:=}\,F \\
& \mid & x{:=}\,\texttt{cons}(E_1, \ldots, E_n) \mid \texttt{dispose}(E) \\
& \mid & \texttt{skip} \mid C; C \mid \texttt{if } B \texttt{ then } C \texttt{ else } C \\
& \mid & \texttt{while } B \texttt{ do } C \\
& \mid & \texttt{with } r \texttt{ when } B \texttt{ do } C
\end{array}
$$

Some sugar: $x.i{:=}\,E$ for $[x + i - 1]{:=}\,E$

$$\frac{\{P\}\,init\,\{RI_{r_1} * \cdots * RI_{r_m} * P'\} \qquad \{P'\}\,C_1 \parallel \cdots \parallel C_n\,\{Q\}}{\begin{array}{l}\{P\}\\ init;\\ \texttt{resource}\ r_1(\text{variable list}), \ldots, r_m(\text{variable list})\\ C_1 \parallel \cdots \parallel C_n\\ \{RI_{r_1} * \cdots * RI_{r_m} * Q\}\end{array}}$$



$$\frac{\{P\}\,init\,\{RI_{r_1} * \cdots * RI_{r_m} * P'\} \qquad \{P'\}\,C_1 \parallel \cdots \parallel C_n\{Q\}}{}$$

$\{P\}$

init;

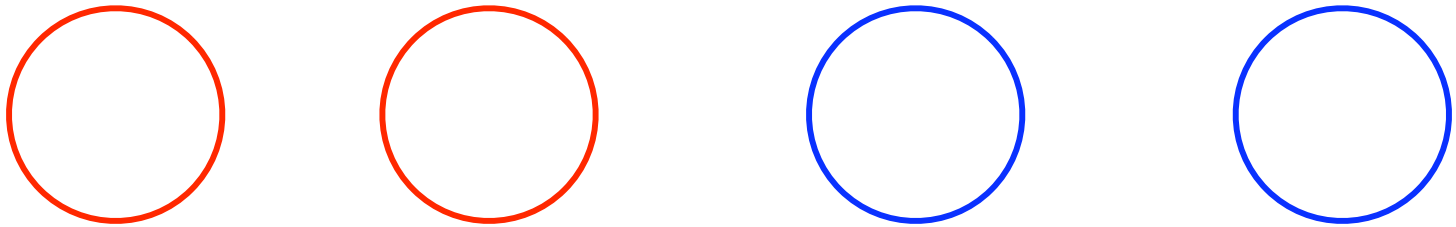
`resource` r_1(variable list), ..., r_m(variable list)

$C_1 \parallel \cdots \parallel C_n$

$\{RI_{r_1} * \cdots * RI_{r_m} * Q\}$

RI * · · · * RI

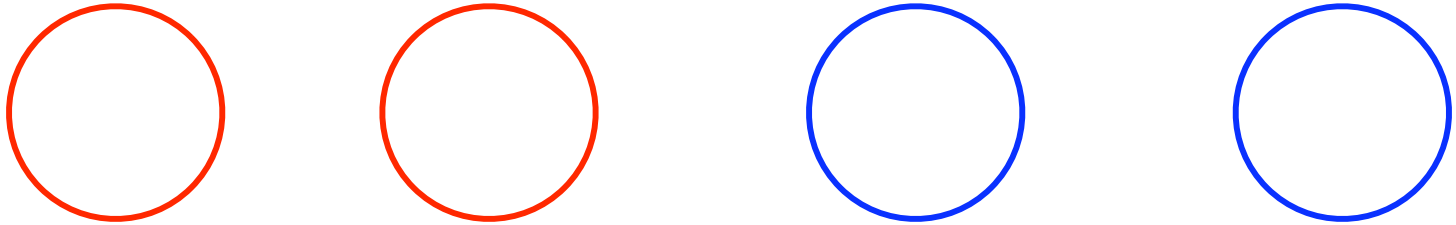
P1 * · · · * Pn



$$\frac{\{P_1\}\, C_1\, \{Q_1\} \;\cdots\; \{P_n\}\, C_n\, \{Q_n\}}{\{P_1 * \cdots * P_n\}\, C_1 \parallel \cdots \parallel C_n\, \{Q_1 * \cdots * Q_n\}}$$

RI $*$ $\cdots$ $*$ RI

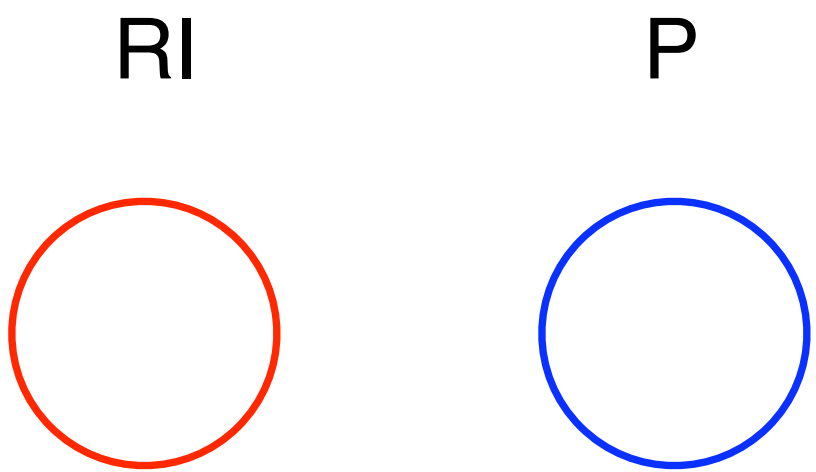
P1 $*$ $\cdots$ $*$ Pn



– no variable free in P_i or Q_i is changed in C_j when $j \neq i$.

– any modified variable free in RI_r must occur only within a critical region for r.

$$\frac{\{(P * RI_r) \wedge B\}\ C\ \{Q * RI_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \{Q\}}$$



- No other process modifies variables in P or Q

Example: Pointer Transfer

$$\texttt{resource}\ \mathit{buf}(c, \mathit{full} := \texttt{false});$$

$$\begin{array}{l} x := \texttt{cons}(-); \\ \texttt{with}\ \mathit{buf}\ \texttt{when}\ \neg \mathit{full}\ \texttt{do} \\ \quad (c := x;\ \mathit{full} := \texttt{true};) \end{array} \quad \| \quad \begin{array}{l} \texttt{with}\ \mathit{buf}\ \texttt{when}\ \mathit{full}\ \texttt{do} \\ \quad (y := c;\ \mathit{full} := \texttt{false};) \\ \texttt{dispose}(y); \end{array}$$

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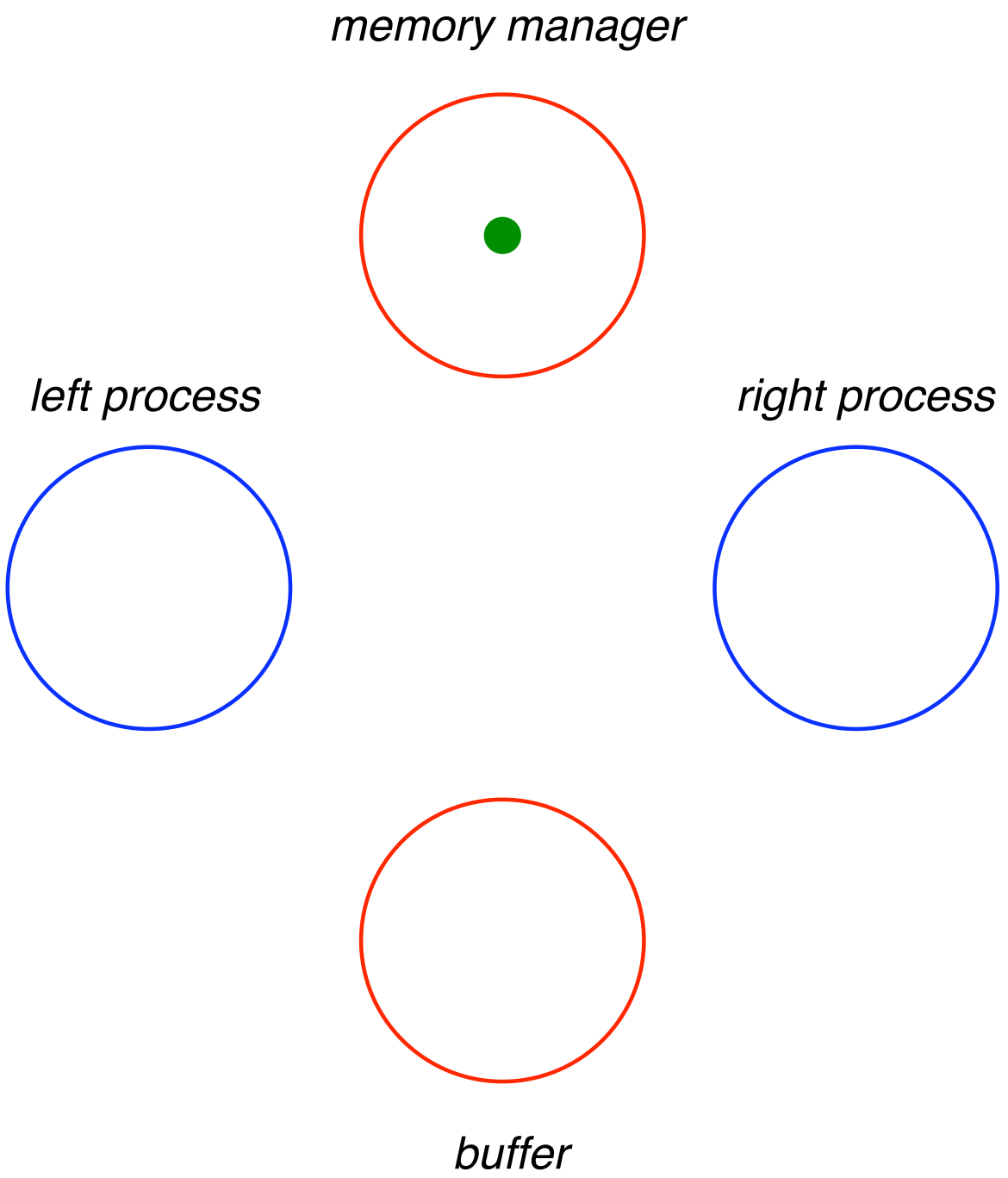
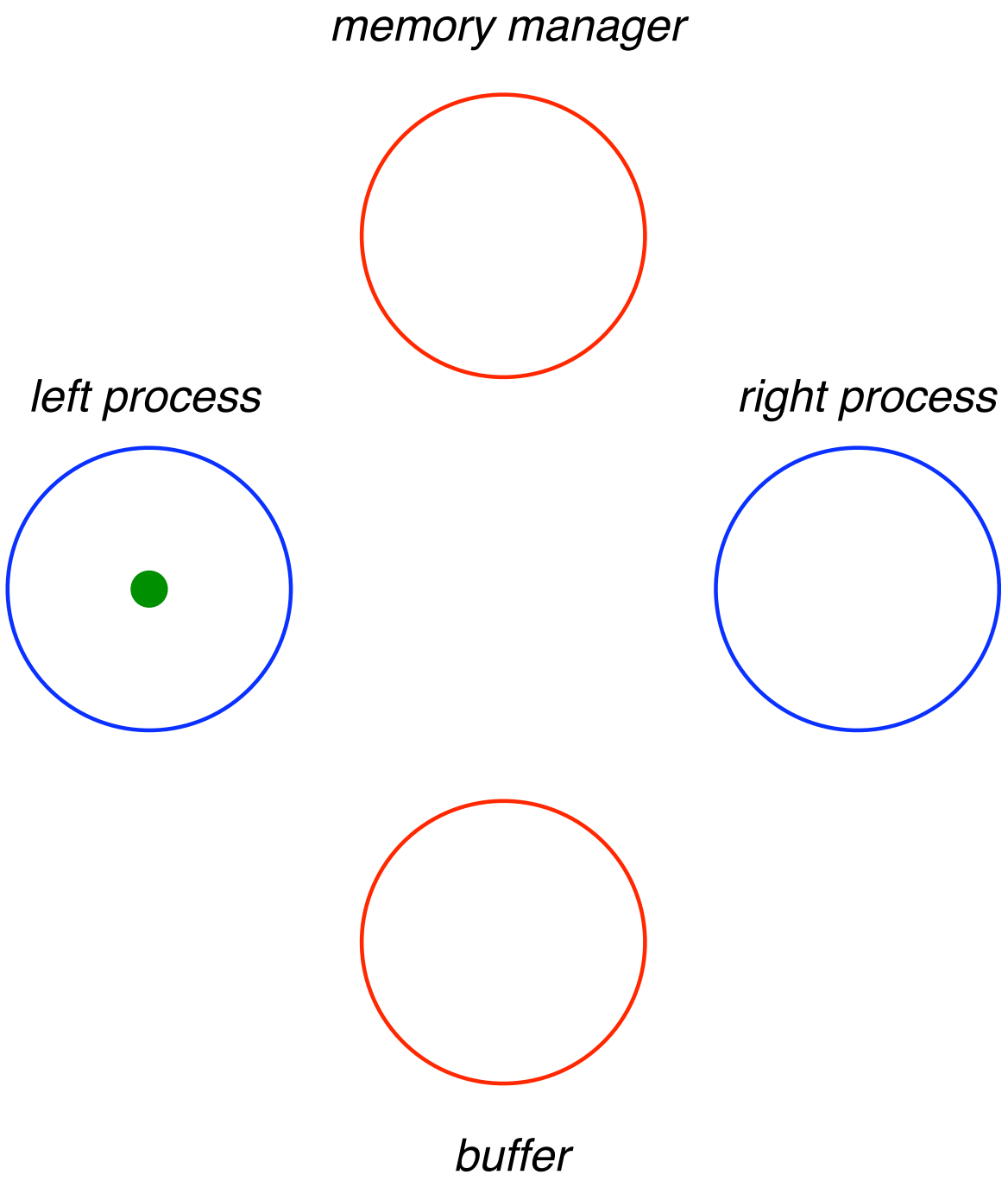

memory manager
left process
right process
buffer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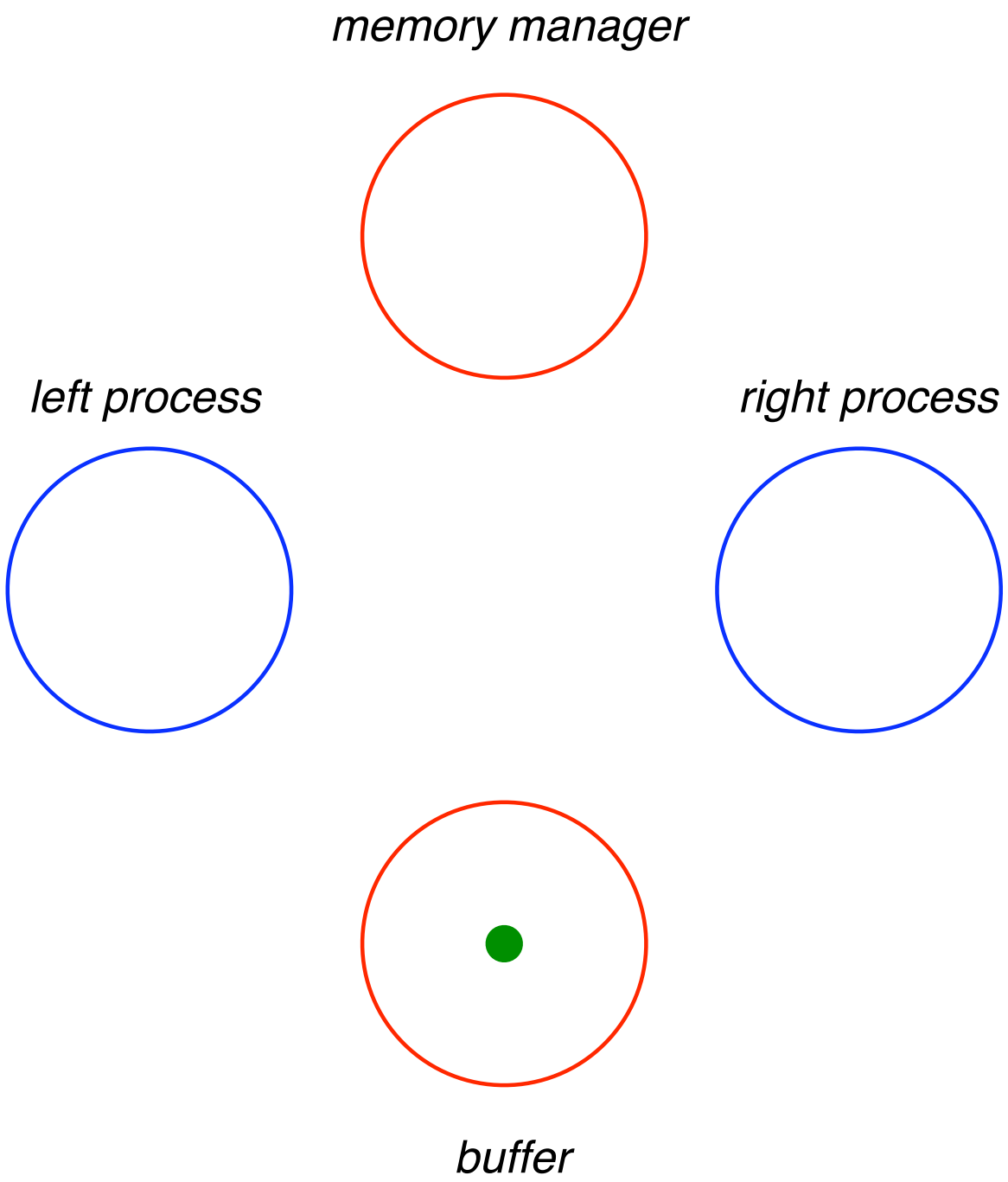
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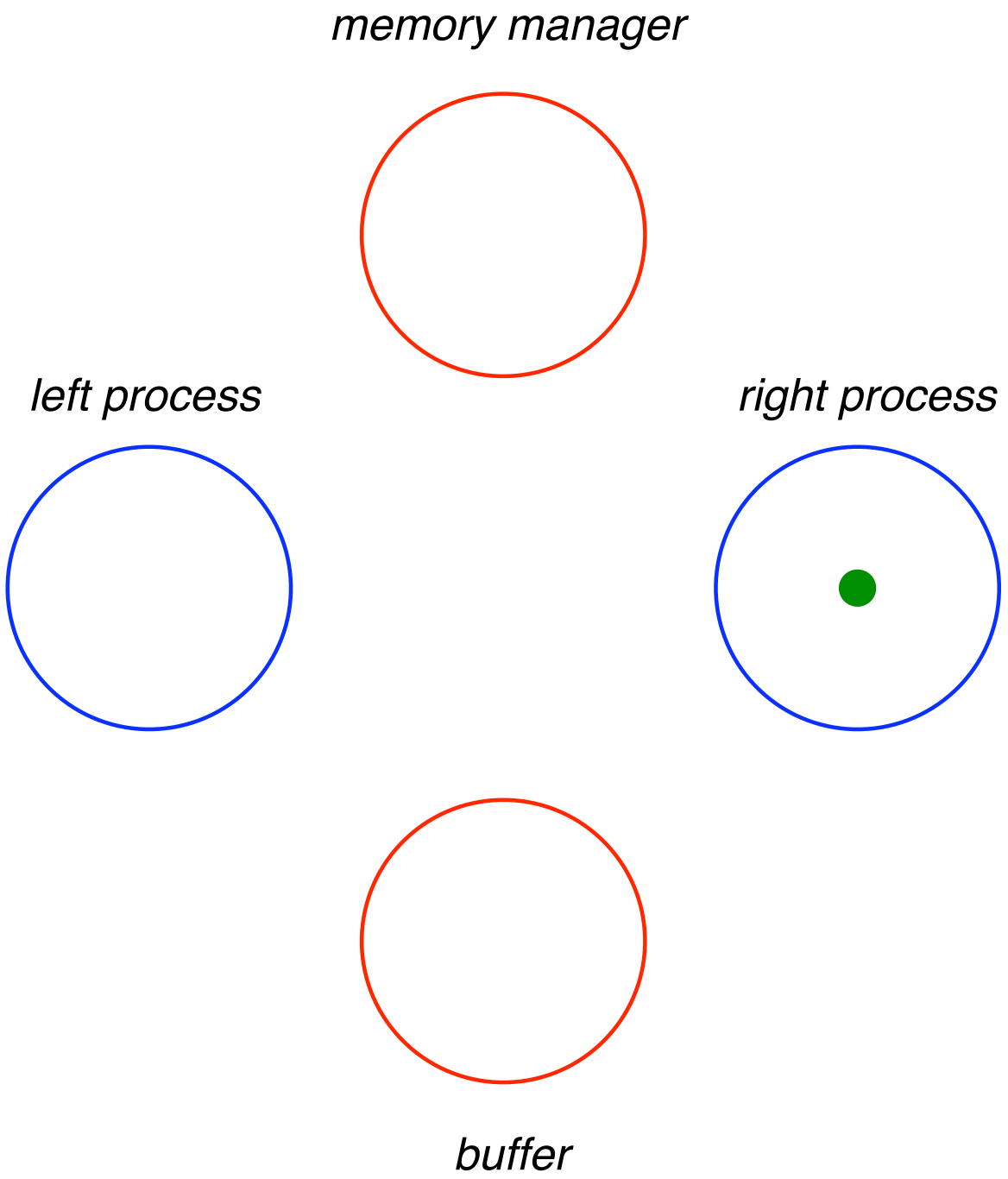
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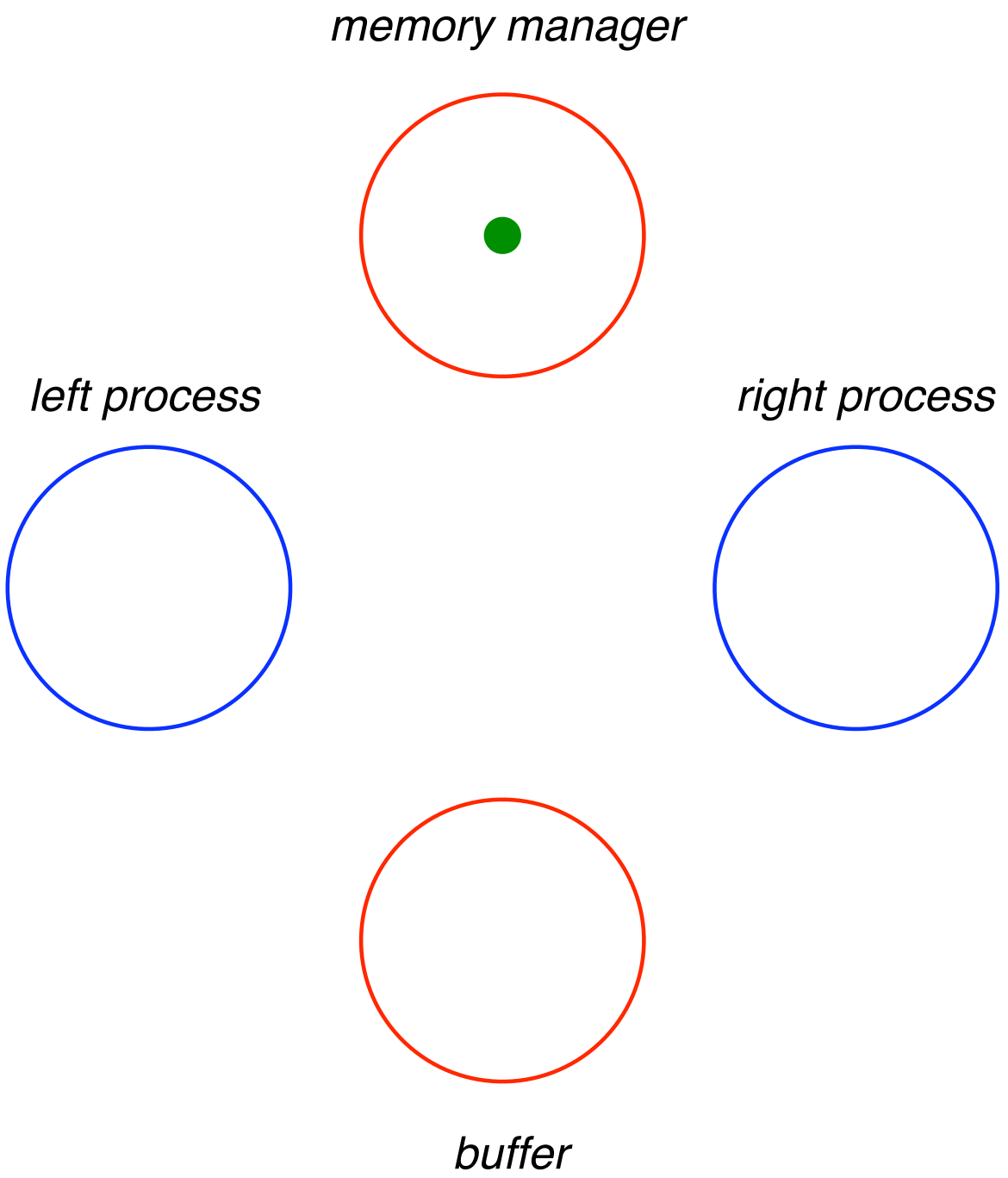
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$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\texttt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

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Warning

- There are weird resource invariants, like

$$\texttt{true} \qquad\qquad 10 \mapsto - \vee 42 \mapsto \vee$$

  which are unclear about which storage is "owned"
- Soundness broken by this weirdness (cf, Reynolds counterexample)

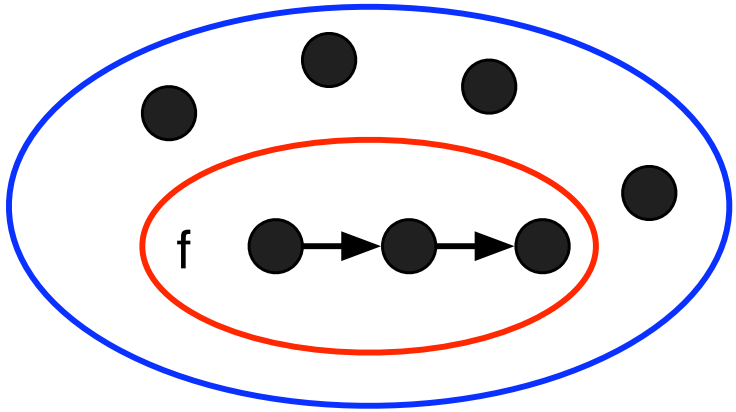
$$\frac{\{(P * \mathit{weird}) \wedge B\}\ C\ \{Q * \mathit{weird}\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \texttt{endwith}\ \{Q\}}$$

- A class of precise predicates, rule out weirdness.
- **Brookes's Soundness Theorem**: The proof rules are sound if we restrict the proof rule to be

$$\frac{\{(P * \mathit{precise}) \wedge B\}\ C\ \{Q * \mathit{precise}\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \texttt{endwith}\ \{Q\}}$$

Precise Predicates

- A predicate is **precise** if, for every state, there is at most one substate that satisfies it.



- For all s, h, exists at most one $h' \sqsubseteq h$ where $s, h' \models P$.

Further Work

- For more concurrency today, see the talks of **Zhong Shao** and **Alexey Gotsman** this afternoon.

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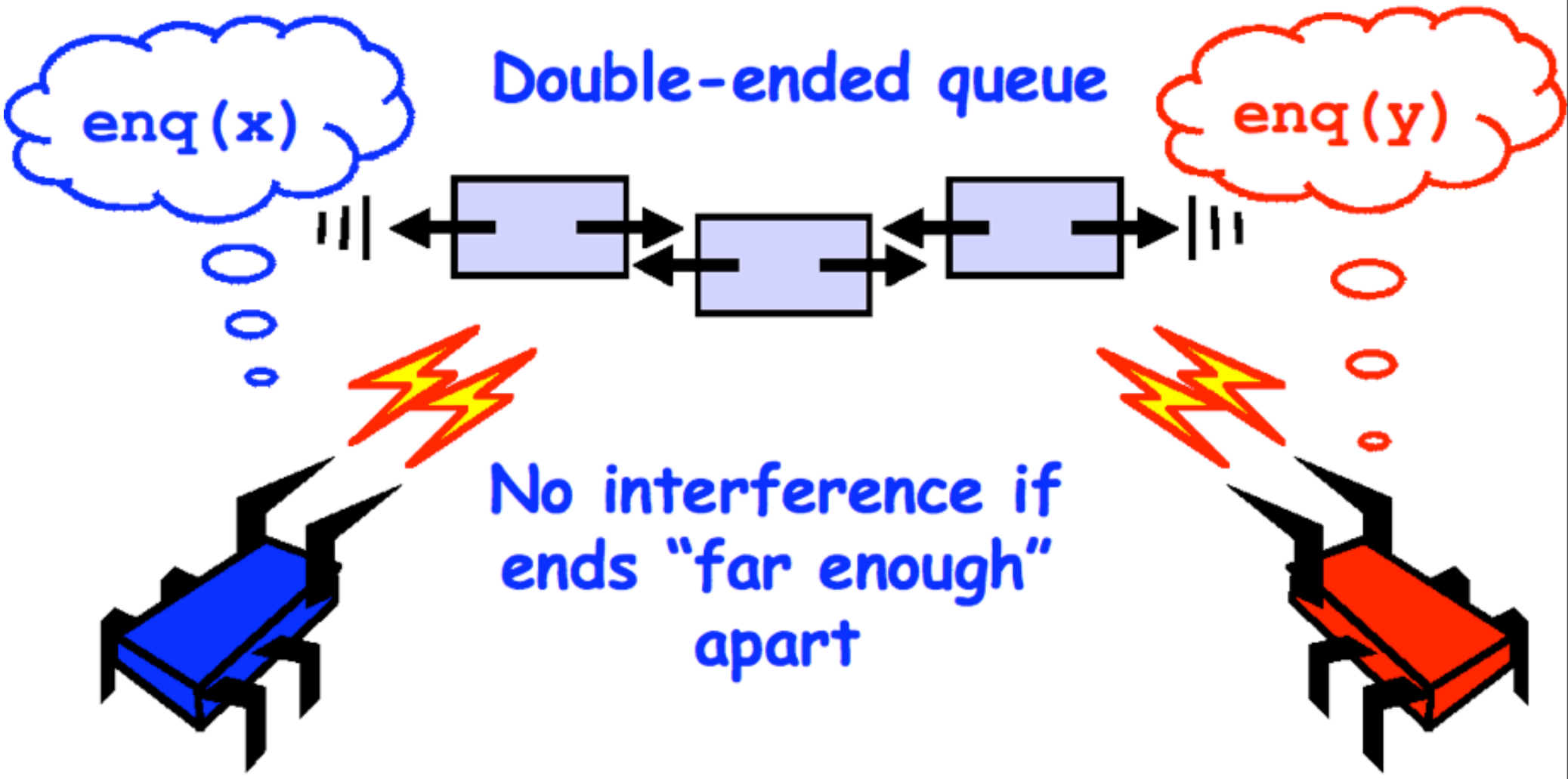
- For more concurrency today, see the talks of **Zhong Shao** and **Alexey Gotsman** this afternoon.
- There is also much further work on **fine-grained concurrency** and **separation logic meets rely guarantee**.
  - Xinyu Feng, Rodrigo Ferreira, Zhong Shao: On the Relationship Between Concurrent Separation Logic and Assume-Guarantee Reasoning. ESOP 2007: 173-188
  - Viktor Vafeiadis, Matthew J. Parkinson: A Marriage of Rely/Guarantee and Separation Logic. CONCUR 2007: 256-271
  - Cristiano Calcagno, Matthew J. Parkinson, Viktor Vafeiadis: Modular Safety Checking for Fine-Grained Concurrency. SAS 2007: 233-248
  - Xinyu Feng: Local rely-guarantee reasoning. POPL 2009: 315-327
  - Viktor Vafeiadis: Shape-Value Abstraction for Verifying Linearizability. VMCAI 2009: 335-34
  - Mike Dodds, Xinyu Feng, Matthew J. Parkinson, Viktor Vafeiadis: Deny-Guarantee Reasoning. ESOP 2009: 363-377

Sadistic Homework Assignment



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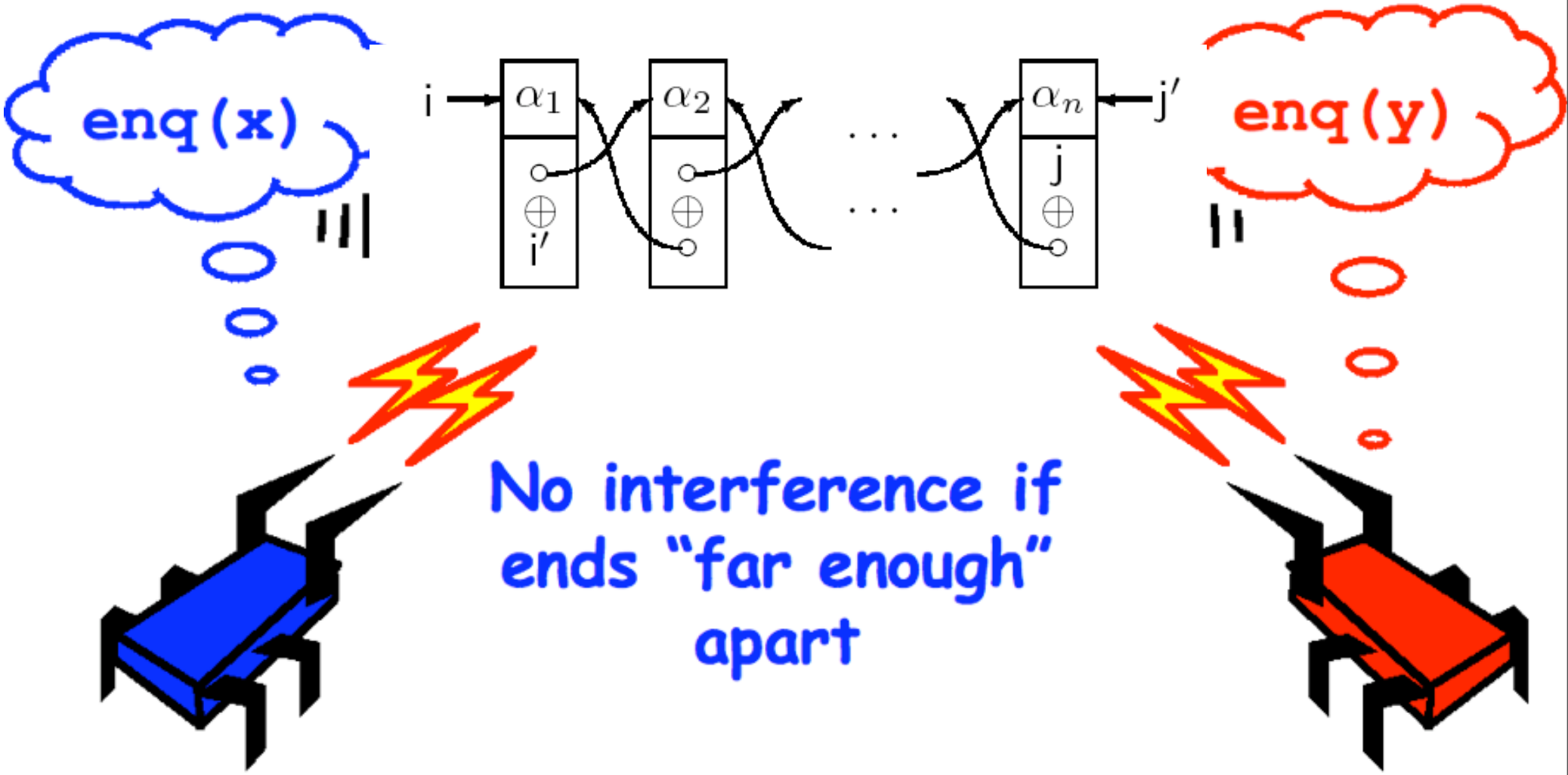

No interference if ends "far enough" apart

Additional Slides

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} \wedge \neg \mathit{full}\}$

$x := \mathtt{cons}(-);$		$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$
$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$	$\|\|$	$y := c;\ \mathit{full} := \mathtt{false}$
$c := x;\ \mathit{full} := \mathtt{true};$		$\mathtt{endwith};$
		$\mathtt{dispose}(y);$
$\mathtt{endwith};$		

$$\frac{\{(P * RI_r) \wedge B\}\ C\ \{Q * RI_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$RI \quad \triangleq \quad (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

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$\{\mathtt{emp} * \mathtt{emp} * (\mathtt{emp} \wedge \neg \mathit{full})\}$

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$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

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$\quad c := x;\ \mathit{full} := \mathtt{true};$

$\mathtt{endwith};$

$\|$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

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$\{\mathtt{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\mathtt{endwith};$

$\|$

$\{\mathtt{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$

$\mathtt{endwith};$

$\|$

$\{\mathtt{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$

$\{(c \mapsto - \wedge \mathit{full}\}$

$\mathtt{endwith};$

$\|$

$\{\mathtt{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$

$\{\mathit{RI}\}$

$\mathtt{endwith};$

$\|$

$\{\mathtt{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$

$\{\mathtt{emp} * \mathit{RI}\}$

$\mathtt{endwith};$

$\|$

$\{\mathtt{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$

$\{\mathtt{emp} * \mathit{RI}\}$

$\mathtt{endwith};$

$\{\mathtt{emp}\}$

$\|$

$\{\mathtt{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$
$x := \mathtt{cons}(-);$
$\{x \mapsto -\}$
$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$
$\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$
$c := x;\ \mathit{full} := \mathtt{true};$
$\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$
$\{\mathtt{emp} * \mathit{RI}\}$
$\mathtt{endwith};$
$\{\mathtt{emp}\}$

$\|$

$\{\mathtt{emp}\}$
$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$
$\{\mathtt{emp} * ((c \mapsto -) \wedge \mathit{full})\}$
$y := c;\ \mathit{full} := \mathtt{false}$
$\{(y \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$
$\mathtt{endwith};$

$\mathtt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathsf{emp}\}$

$\texttt{resource}\ \mathit{buf}(c, \mathit{full} := \texttt{false});$

$\{\mathsf{emp} * \mathsf{emp} * \mathit{RI}\}$

$\{\mathsf{emp}\}$

$x := \texttt{cons}(-);$

$\{x \mapsto -\}$

$\texttt{with}\ \mathit{buf}\ \texttt{when}\ \neg \mathit{full}\ \texttt{do}$

$\{(x \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \texttt{true};$

$\{(c \mapsto -) * (\mathsf{emp} \wedge \mathit{full})\}$

$\{\mathsf{emp} * \mathit{RI}\}$

$\texttt{endwith};$

$\{\mathsf{emp}\}$

$\|$

$\{\mathsf{emp}\}$

$\texttt{with}\ \mathit{buf}\ \texttt{when}\ \mathit{full}\ \texttt{do}$

$\{\mathsf{emp} * ((c \mapsto -) \wedge \mathit{full})\}$

$y := c;\ \mathit{full} := \texttt{false}$

$\{(y \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$

$\texttt{endwith};$

$\{y \mapsto -\}$

$\texttt{dispose}(y);$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathsf{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathsf{emp}\}$

$\texttt{resource}\ \mathit{buf}(c, \mathit{full} := \texttt{false});$

$\{\mathsf{emp} * \mathsf{emp} * \mathit{RI}\}$

$\{\mathsf{emp}\}$	$\{\mathsf{emp}\}$
$x := \texttt{cons}(-);$	$\texttt{with}\ \mathit{buf}\ \texttt{when}\ \mathit{full}\ \texttt{do}$
$\{x \mapsto -\}$	$\quad \{\mathsf{emp} * ((c \mapsto -) \wedge \mathit{full})\}$
$\texttt{with}\ \mathit{buf}\ \texttt{when}\ \neg \mathit{full}\ \texttt{do}$	$\quad y := c;\ \mathit{full} := \texttt{false}$
$\quad \{(x \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$	$\quad \{(y \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$
$\quad c := x;\ \mathit{full} := \texttt{true};$	$\texttt{endwith};$
$\quad \{(c \mapsto -) * (\mathsf{emp} \wedge \mathit{full})\}$	$\{y \mapsto -\}$
$\quad \{\mathsf{emp} * \mathit{RI}\}$	$\texttt{dispose}(y);$
$\texttt{endwith};$	$\{\mathsf{emp}\}$
$\{\mathsf{emp}\}$	

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathsf{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathsf{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathsf{emp} * \mathsf{emp} * \mathit{RI}\}$

$\{\mathsf{emp}\}$

$x := \mathtt{cons}(-);$

$\{x \mapsto -\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$

$\{(x \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \mathtt{true};$

$\{(c \mapsto -) * (\mathsf{emp} \wedge \mathit{full})\}$

$\{\mathsf{emp} * \mathit{RI}\}$

$\mathtt{endwith};$

$\{\mathsf{emp}\}$

$\mathtt{dispose}(x);$

$\|$

$\{\mathsf{emp}\}$

$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$

$\{\mathsf{emp} * ((c \mapsto -) \wedge \mathit{full})\}$

$y := c;\ \mathit{full} := \mathtt{false}$

$\{(y \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$

$\mathtt{endwith};$

$\{y \mapsto -\}$

$\mathtt{dispose}(y);$

$\{\mathsf{emp}\}$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathsf{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathsf{emp}\}$

$\texttt{resource}\ \mathit{buf}(c, \mathit{full} := \texttt{false});$

$\{\mathsf{emp} * \mathsf{emp} * \mathit{RI}\}$

$\{\mathsf{emp}\}$

$x := \texttt{cons}(-);$

$\{x \mapsto -\}$

$\texttt{with}\ \mathit{buf}\ \texttt{when}\ \neg \mathit{full}\ \texttt{do}$

$\{(x \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$

$c := x;\ \mathit{full} := \texttt{true};$

$\{(c \mapsto -) * (\mathsf{emp} \wedge \mathit{full})\}$

$\{\mathsf{emp} * \mathit{RI}\}$

$\texttt{endwith};$

$\{\mathsf{emp}\}$

$\texttt{dispose}(x);$

$\{???\}$

$\|$

$\{\mathsf{emp}\}$

$\texttt{with}\ \mathit{buf}\ \texttt{when}\ \mathit{full}\ \texttt{do}$

$\{\mathsf{emp} * ((c \mapsto -) \wedge \mathit{full})\}$

$y := c;\ \mathit{full} := \texttt{false}$

$\{(y \mapsto -) * (\mathsf{emp} \wedge \neg \mathit{full})\}$

$\texttt{endwith};$

$\{y \mapsto -\}$

$\texttt{dispose}(y);$

$\{\mathsf{emp}\}$

$$\frac{\{(P * \mathit{RI}_r) \wedge B\}\ C\ \{Q * \mathit{RI}_r\}}{\{P\}\ \texttt{with}\ r\ \texttt{when}\ B\ \texttt{do}\ C\ \{Q\}}$$

$$\mathit{RI} \triangleq (\mathsf{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

$\{\mathtt{emp}\}$

$\mathtt{resource}\ \mathit{buf}(c, \mathit{full} := \mathtt{false});$

$\{\mathtt{emp} * \mathtt{emp} * \mathit{RI}\}$

$\{\mathtt{emp}\}$	$\{\mathtt{emp}\}$
$x := \mathtt{cons}(-);$	$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \mathit{full}\ \mathtt{do}$
$\{x \mapsto -\}$	$\quad\{\mathtt{emp} * ((c \mapsto -) \wedge \mathit{full})\}$
$\mathtt{with}\ \mathit{buf}\ \mathtt{when}\ \neg \mathit{full}\ \mathtt{do}$	$\quad y := c;\ \mathit{full} := \mathtt{false}$
$\quad\{(x \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$	$\quad\{(y \mapsto -) * (\mathtt{emp} \wedge \neg \mathit{full})\}$
$\quad c := x;\ \mathit{full} := \mathtt{true};$	$\mathtt{endwith};$
$\quad\{(c \mapsto -) * (\mathtt{emp} \wedge \mathit{full})\}$	$\{y \mapsto -\}$
$\quad\{\mathtt{emp} * \mathit{RI}\}$	$\mathtt{dispose}(y);$
$\mathtt{endwith};$	$\{\mathtt{emp}\}$
$\{\mathtt{emp}\}$	

$\{RI\}$

$$\frac{\{(P * RI_r) \wedge B\}\ C\ \{Q * RI_r\}}{\{P\}\ \mathtt{with}\ r\ \mathtt{when}\ B\ \mathtt{do}\ C\ \{Q\}}$$

$$RI \quad \triangleq \quad (\mathtt{emp} \wedge \neg \mathit{full}) \vee (c \mapsto - \wedge \mathit{full})$$

The Reynolds Counterexample

$\texttt{resource}\ r()$ $\quad$ *invariant* $RI_r = \texttt{true}$

From

$$\dfrac{\dfrac{\{\texttt{true}\}\texttt{skip}\{\texttt{true}\}}{\{(\texttt{emp} \vee \texttt{one}) * \texttt{true}\}\texttt{skip}\{\texttt{emp} * \texttt{true}\}}}{\{\texttt{emp} \vee \texttt{one}\}\texttt{with}\ r\ \texttt{when true do skip}\ \{\texttt{emp}\}}$$

We obtain an inconsistency

$$\dfrac{\dfrac{\dfrac{\dfrac{\{\texttt{emp} \vee \texttt{one}\}\texttt{with} \cdots\ \{\texttt{emp}\}}{\{\texttt{emp}\}\texttt{with} \cdots\ \{\texttt{emp}\}}}{\{\texttt{emp} * \texttt{one}\}\texttt{with} \cdots\ \{\texttt{emp} * \texttt{one}\}}}{\{\texttt{one}\}\texttt{with} \cdots \{\texttt{one}\}} \qquad \dfrac{\{\texttt{emp} \vee \texttt{one}\}\texttt{with} \cdots\ \{\texttt{emp}\}}{\{\texttt{one}\}\texttt{with} \cdots \{\texttt{emp}\}}}{\{\texttt{one} \wedge \texttt{one}\}\texttt{with}\ r\ \texttt{when true do skip}\ \{\texttt{emp} \wedge \texttt{one}\}}}{\{\texttt{one}\}\texttt{with}\ r\ \texttt{when true do skip}\ \{\texttt{false}\}}$$