Cryptol Tutorial
Part 2:
The Second Part

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HCSS 2011

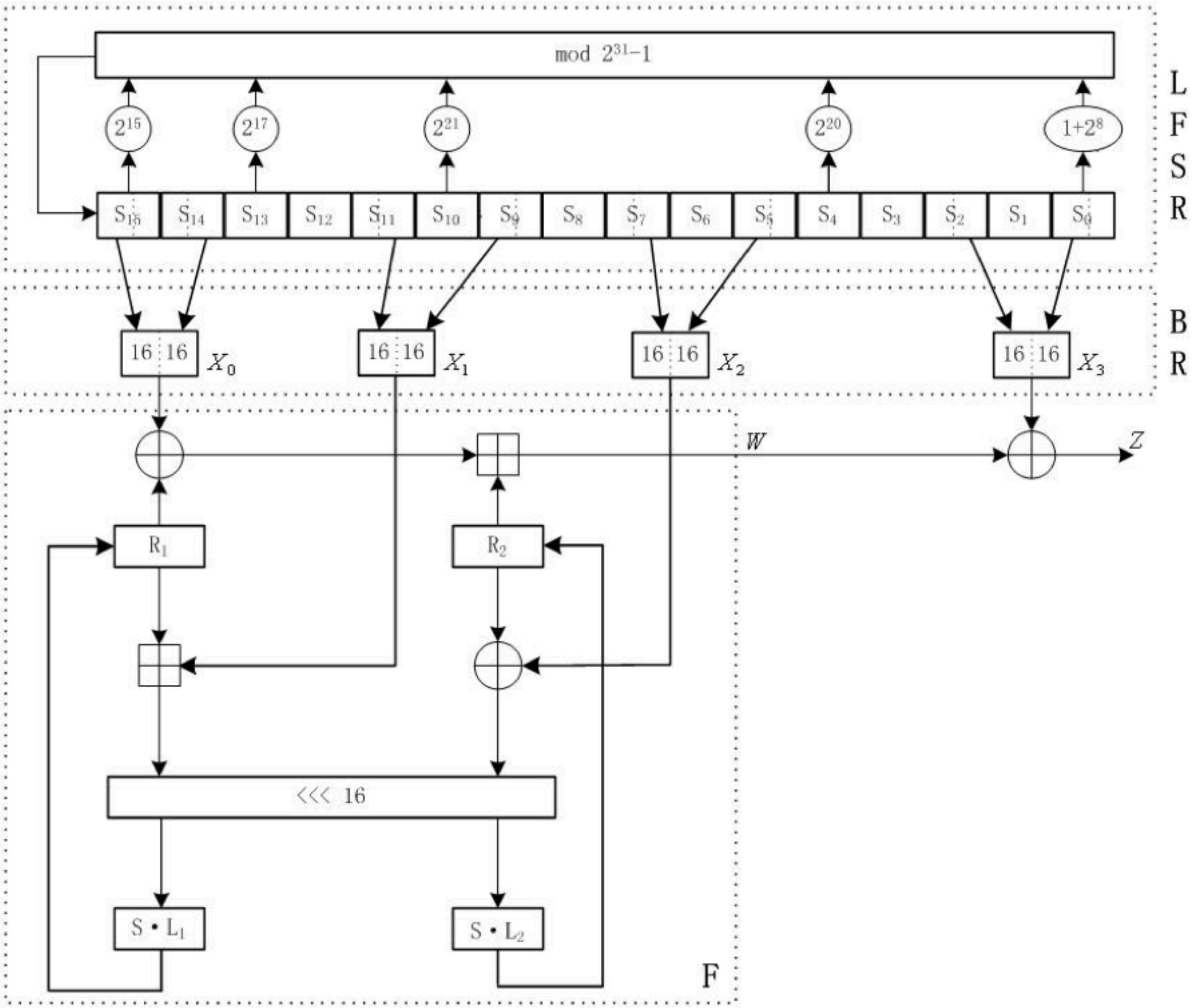
Outline

- Cryptol Demo
  - The ZUC stream cipher
- Verification of Inferior Language Source Code
  - Java AES
  - Java MD5

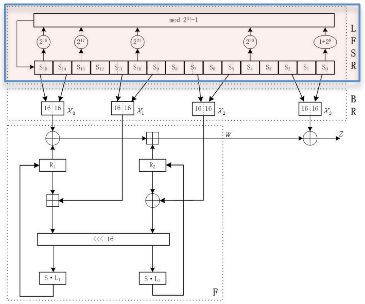
ZUC

- Latest initiative by 3GPP for securing mobile networks[1]
- Word-oriented stream cipher
  - 128-bit key
  - 128-bit initialization vector
  - Generates keystream of 32-bit words
- Forms the heart of the 3GPP confidentiality algorithm 128-EEA3 and the 3GPP integrity algorithm 128-EIA3[1]

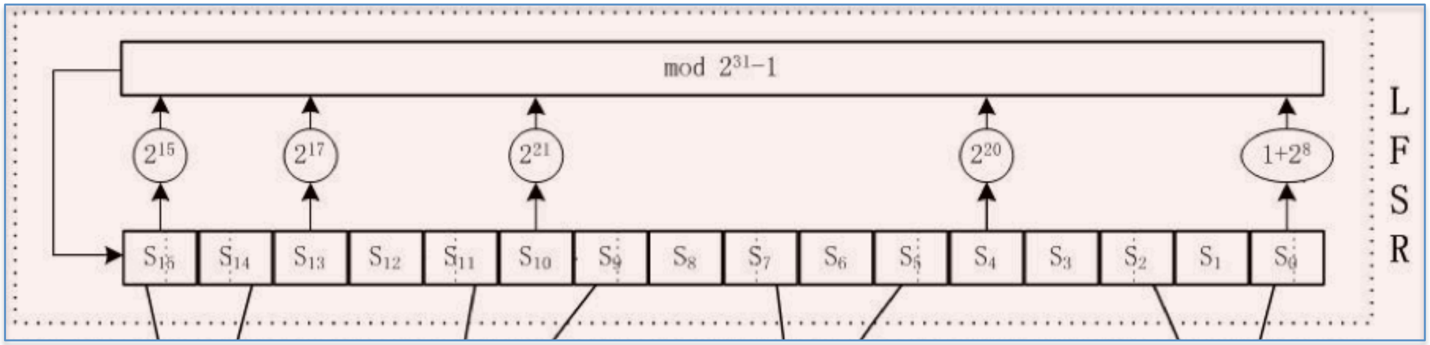
ZUC


mod $2^{31}-1$
2^{15}
2^{17}
2^{21}
2^{20}
$1+2^8$
S_{15} S_{14} S_{13} S_{12} S_{11} S_{10} S_9 S_8 S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0
LFSR
16 16 X_0
16 16 X_1
16 16 X_2
16 16 X_3
BR
W
Z
R_1
R_2
<<< 16
$S \cdot L_1$
$S \cdot L_2$
F

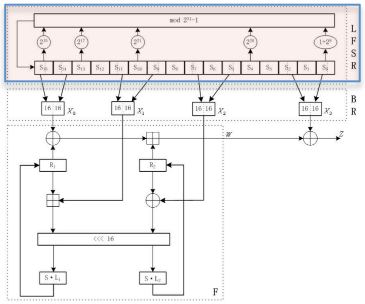
LFSR



- "The linear feedback shift register (LFSR) has 16 of 31-bit cells (s_0, s_1,..., s_{15})…" [2]
- "The LFSR has 2 modes of operations: the initialization mode and the working mode. In the initialization mode, the LFSR receives a 31-bit input word u…" [2]



LFSR



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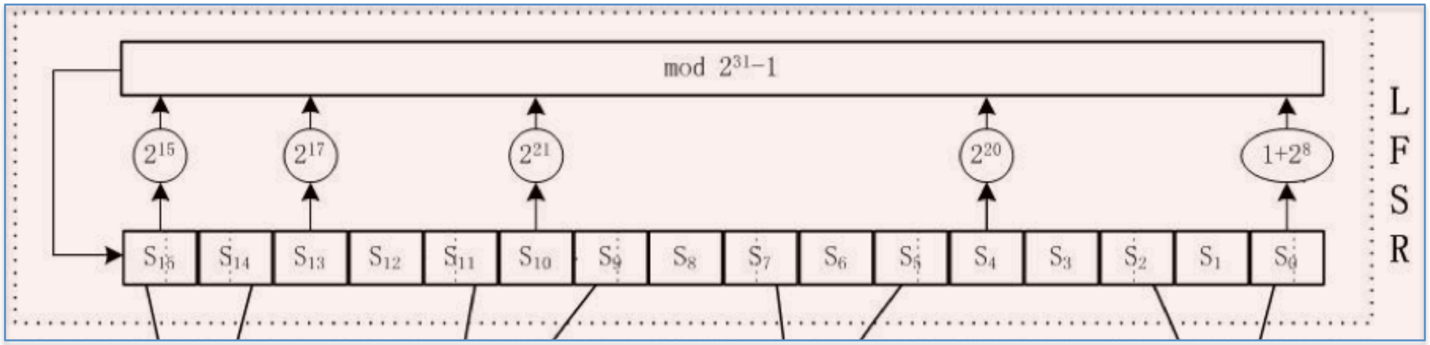
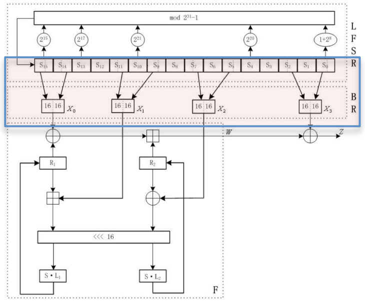
```
LFSRWithWorkMode : [16][31] -> [16][31];

LFSRWithInitialisationMode : ([31], [16][31]) -> [16][31];
```

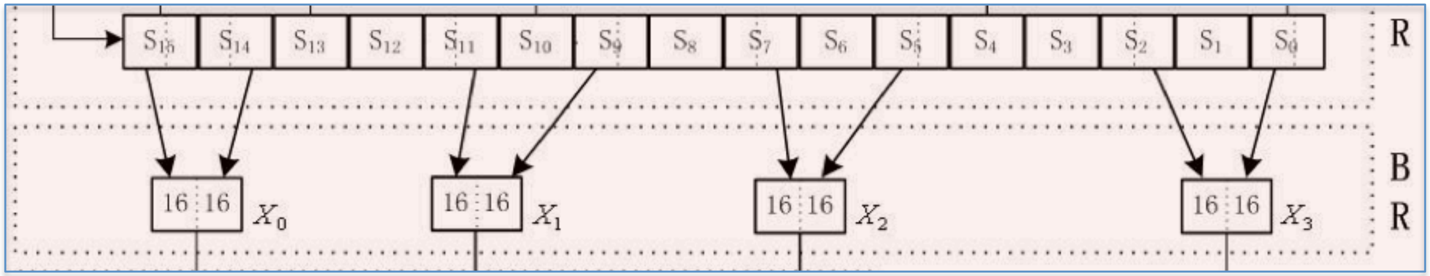


Bit Reorganization

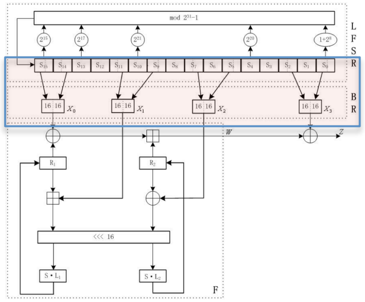


- “The middle layer of the algorithm is the bit-reorganization. It extracts 128 bits from the cells of the LFSR and forms 4 of 32-bit words…” [2]

```
Bitreorganization()
{
    1. $X_0 = s_{15H} || s_{14L}$
    2. $X_1 = s_{11L} || s_{9H}$
    3. $X_2 = s_{7L} || s_{5H}$
    4. $X_3 = s_{2L} || s_{0H}$
}
```



H and L

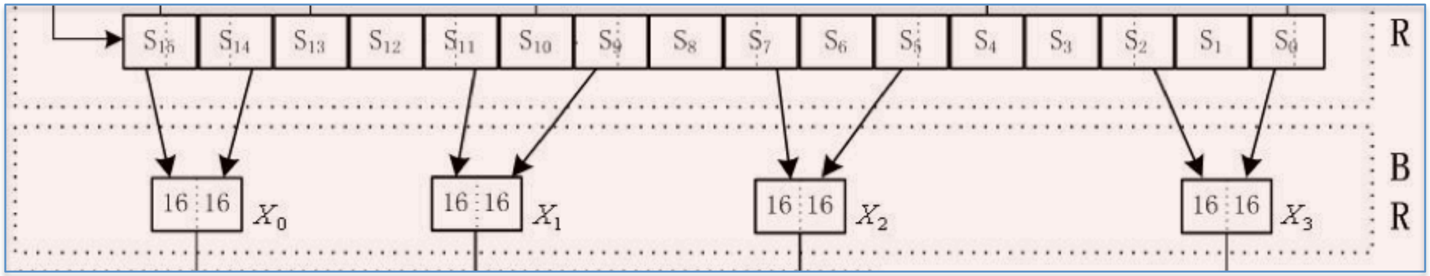


- “a_H: The leftmost 16 bits of integer a.” [2]

```
H : {b} (fin b, b >= 16) => [b] -> [16];
H(a) = drop(width(a)-16, a);
```

- “a_L: The rightmost 16 bits of integer a.” [2]

```
L : {b} (b >= 16) => [b] -> [16];
L(a) = take(16, a);
```



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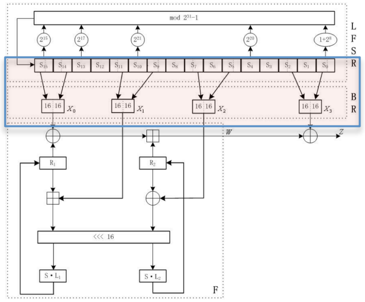

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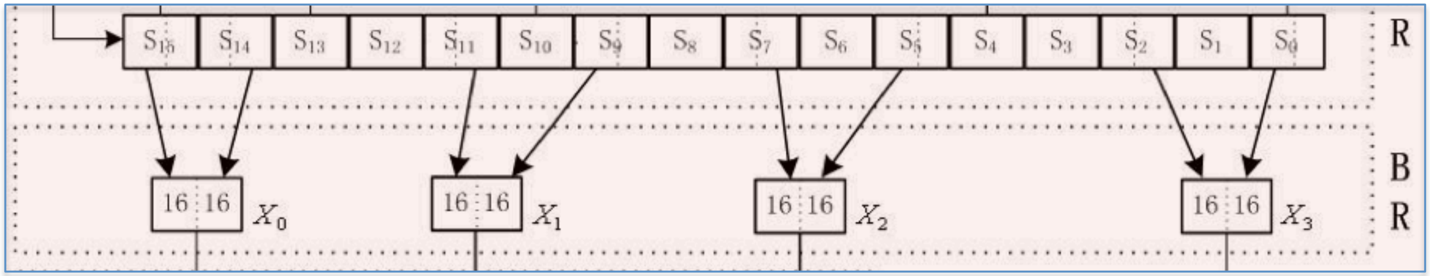

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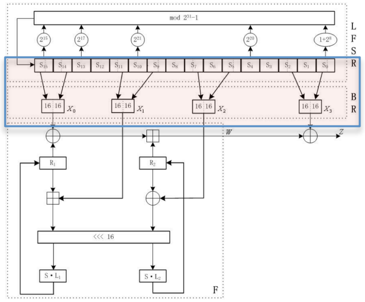

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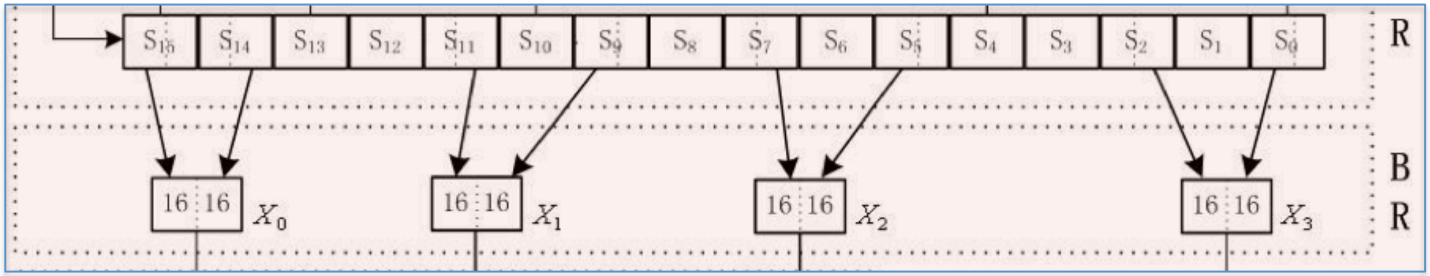
}
```

```
Bitreorganization : [16][31] -> [4][32];
Bitreorganization (s) = [X0 X1 X2 X3]
  where {
    X0 = L(s@14) # H(s@15);
    X1 = H(s@9)  # L(s@11);
    X2 = H(s@5)  # L(s@7);
    X3 = H(s@0)  # L(s@2);
  };
```



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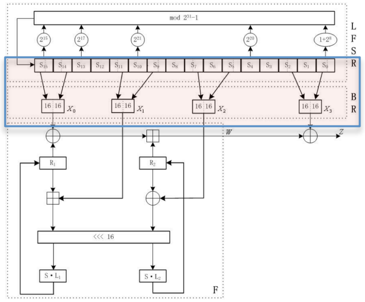

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    X2 = H(s@5)  # L(s@7);
    X3 = H(s@0)  # L(s@2);
```
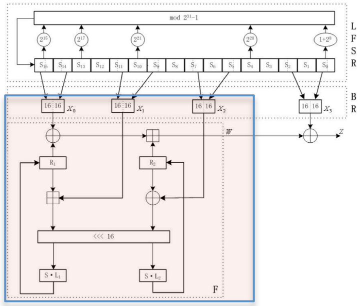
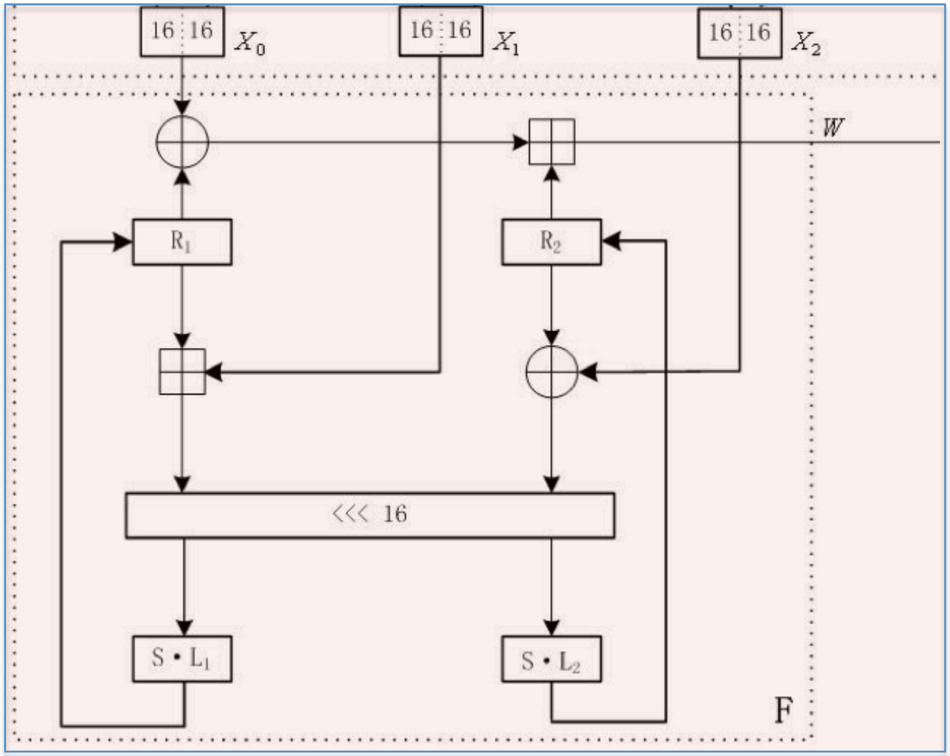

```
void Bitreorganization() {
  BRC_X0 = ((LFSR_S15 & 0x7FFF8000) << 1) | (LFSR_S14 & 0xFFFF);
  BRC_X1 = ((LFSR_S11 & 0xFFFF) << 16)    | (LFSR_S9 >> 15);
  BRC_X2 = ((LFSR_S7 & 0xFFFF) << 16)     | (LFSR_S5 >> 15);
  BRC_X3 = ((LFSR_S2 & 0xFFFF) << 16)     | (LFSR_S0 >> 15);
}
```

F

- "The nonlinear function F has 2 of 32-bit memory cells R1 and R2. Let the inputs to F be X0, X1 and X2, which come from the outputs of the bit-reorganization (see section 3.3), then the function F outputs a 32-bit word W." [2]





```
F ($X_0$, $X_1$, $X_2$)
{
  1. $W = (X_0 \oplus R_1) \boxplus R_2$
  2. $W_1 = R_1 \boxplus X_1$
  3. $W_2 = R_2 \oplus X_2$
  4. $R_1 = S(L_1(W_{1L} || W_{2H}))$
  5. $R_2 = S(L_2(W_{2L} || W_{1H}))$
}
```

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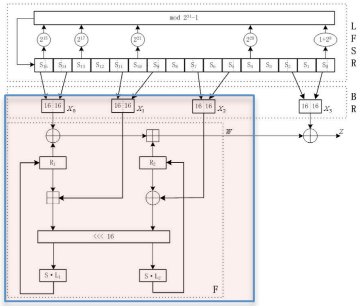

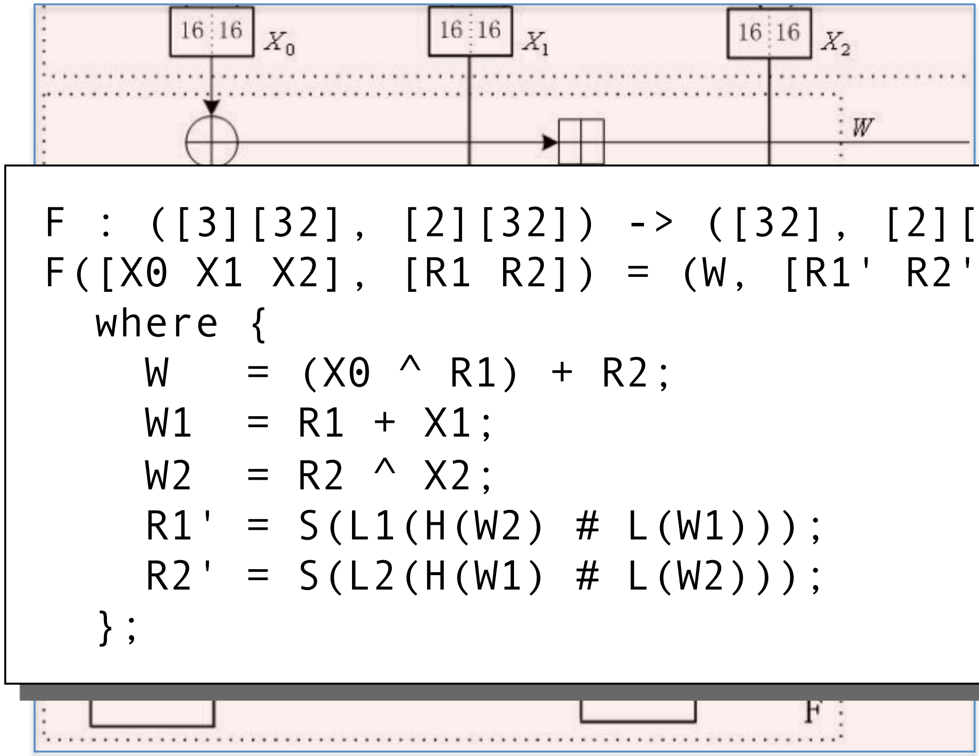

```
F : ([3][32], [2][32]) -> ([32], [2][32]);
F([X0 X1 X2], [R1 R2]) = (W, [R1' R2'])
  where {
    W   = (X0 ^ R1) + R2;
    W1  = R1 + X1;
    W2  = R2 ^ X2;
    R1' = S(L1(H(W2) # L(W1)));
    R2' = S(L2(H(W1) # L(W2)));
  };
```

```
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  4. $R_1 = S(L_1(W_{1L} || W_{2H}))$
  5. $R_2 = S(L_2(W_{2L} || W_{1H}))$
}
```
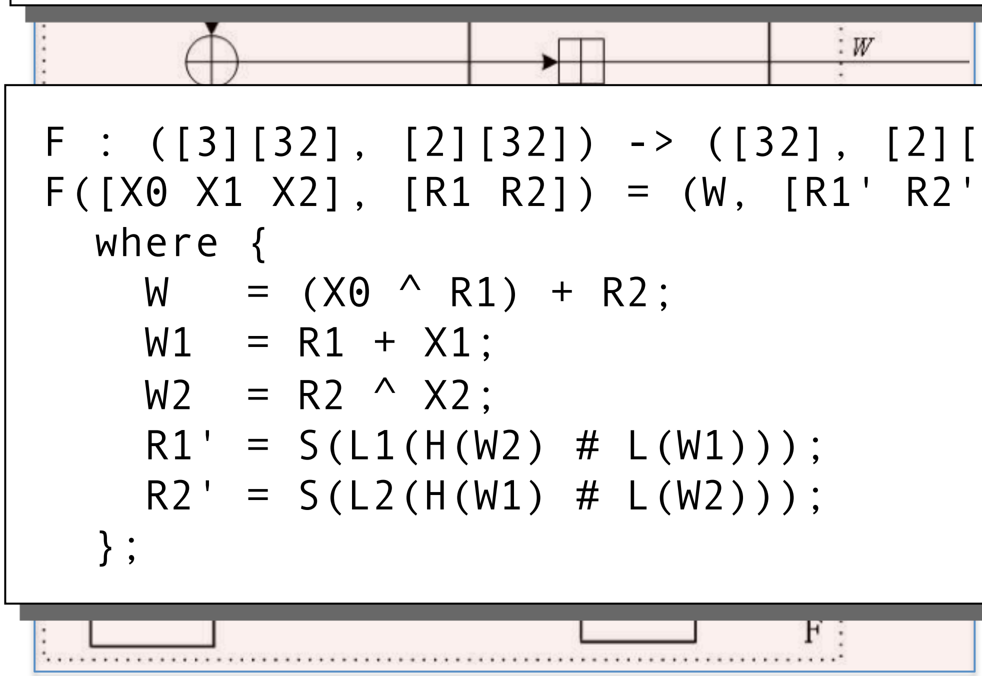

```
u32 F() {
  u32 W, W1, W2, u, v;
  W = (BRC_X0 ^ F_R1) + F_R2;
  W1 = F_R1 + BRC_X1;
  W2 = F_R2 ^ BRC_X2;
  u = L1((W1 << 16) | (W2 >> 16));
  v = L2((W2 << 16) | (W1 >> 16));
  F_R1 = MAKEU32(S0[u >> 24], S1[(u >> 16), S0[(u >> 8) & 0xFF], S1[u &
  F_R2 = MAKEU32(S0[v >> 24], S1[(v >> 16), S0[(v >> 8) & 0xFF], S1[v &
  return W;
}
```



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```
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  4. $R_1 = S(L_1(W_{1L} || W_{2H}))$
  5. $R_2 = S(L_2(W_{2L} || W_{1H}))$
}
```
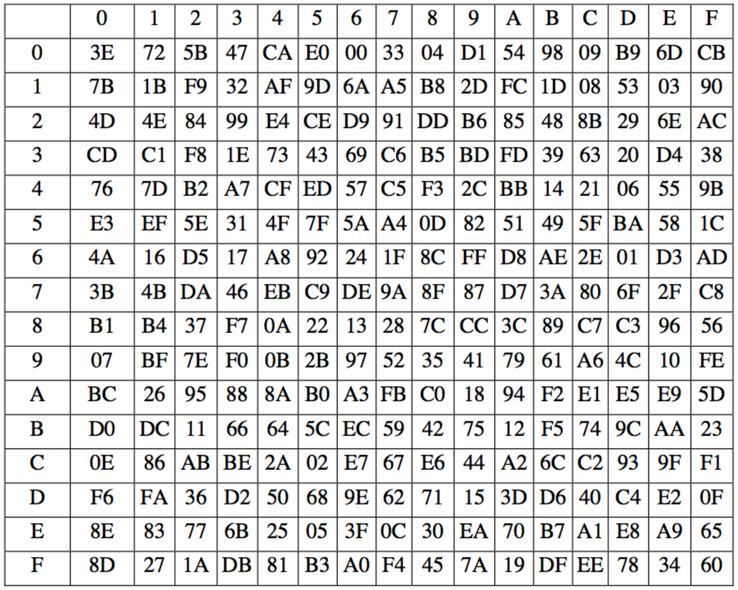

S-boxes

- “The 32x32 S-box S is composed of 4 juxtaposed 8x8 S-boxes, i.e., S=(S_0,S_1,S_2,S_3), where S_0=S_2, S_1=S_3” [2]

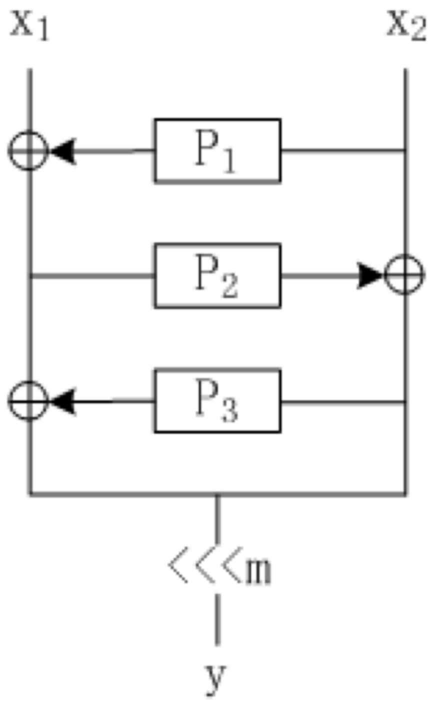


	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	3E	72	5B	47	CA	E0	00	33	04	D1	54	98	09	B9	6D	CB
1	7B	1B	F9	32	AF	9D	6A	A5	B8	2D	FC	1D	08	53	03	90
2	4D	4E	84	99	E4	CE	D9	91	DD	B6	85	48	8B	29	6E	AC
3	CD	C1	F8	1E	73	43	69	C6	B5	BD	FD	39	63	20	D4	38
4	76	7D	B2	A7	CF	ED	57	C5	F3	2C	BB	14	21	06	55	9B
5	E3	EF	5E	31	4F	7F	5A	A4	0D	82	51	49	5F	BA	58	1C
6	4A	16	D5	17	A8	92	24	1F	8C	FF	D8	AE	2E	01	D3	AD
7	3B	4B	DA	46	EB	C9	DE	9A	8F	87	D7	3A	80	6F	2F	C8
8	B1	B4	37	F7	0A	22	13	28	7C	CC	3C	89	C7	C3	96	56
9	07	BF	7E	F0	0B	2B	97	52	35	41	79	61	A6	4C	10	FE
A	BC	26	95	88	8A	B0	A3	FB	C0	18	94	F2	E1	E5	E9	5D
B	D0	DC	11	66	64	5C	EC	59	42	75	12	F5	74	9C	AA	23
C	0E	86	AB	BE	2A	02	E7	67	E6	44	A2	6C	C2	93	9F	F1
D	F6	FA	36	D2	50	68	9E	62	71	15	3D	D6	40	C4	E2	0F
E	8E	83	77	6B	25	05	3F	0C	30	EA	70	B7	A1	E8	A9	65
F	8D	27	1A	DB	81	B3	A0	F4	45	7A	19	DF	EE	78	34	60

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	55	C2	63	71	3B	C8	47	86	9F	3C	DA	5B	29	AA	FD	77
1	8C	C5	94	0C	A6	1A	13	00	E3	A8	16	72	40	F9	F8	42
2	44	26	68	96	81	D9	45	3E	10	76	C6	A7	8B	39	43	E1
3	3A	B5	56	2A	C0	6D	B3	05	22	66	BF	DC	0B	FA	62	48
4	DD	20	11	06	36	C9	C1	CF	F6	27	52	BB	69	F5	D4	87
5	7F	84	4C	D2	9C	57	A4	BC	4F	9A	DF	FE	D6	8D	7A	EB
6	2B	53	D8	5C	A1	14	17	FB	23	D5	7D	30	67	73	08	09
7	EE	B7	70	3F	61	B2	19	8E	4E	E5	4B	93	8F	5D	DB	A9
8	AD	F1	AE	2E	CB	0D	FC	F4	2D	46	6E	1D	97	E8	D1	E9
9	4D	37	A5	75	5E	83	9E	AB	82	9D	B9	1C	E0	CD	49	89
A	01	B6	BD	58	24	A2	5F	38	78	99	15	90	50	B8	95	E4
B	D0	91	C7	CE	ED	0F	B4	6F	A0	CC	F0	02	4A	79	C3	DE
C	A3	EF	EA	51	E6	6B	18	EC	1B	2C	80	F7	74	E7	FF	21
D	5A	6A	54	1E	41	31	92	35	C4	33	07	0A	BA	7E	0E	34

S_0

- "Both x_1 and x_2 are 4-bit strings, m=5, and P_1, P_2, P_3 are transforms over GF(16), which are defined as:" [2]



P_1

Input	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Output	9	15	0	14	15	15	2	10	0	4	0	12	7	5	3	9

P_2

Input	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Output	8	13	6	5	7	0	12	4	11	1	14	10	15	3	9	2

P_3

Input	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Output	2	6	10	6	0	13	10	15	3	3	13	5	0	9	12	13

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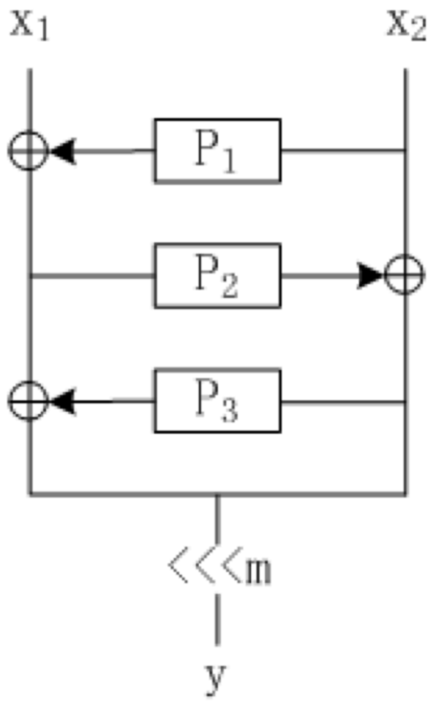
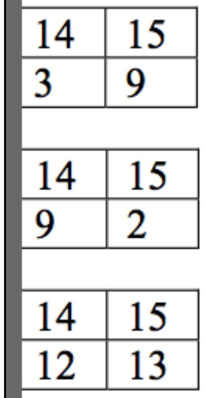


```
S0_byte : ([4], [4]) -> [8];
S0_byte(x1, x2) = y
  where {
    x1'  = x1 ^ P1@x2;
    x2'  = P2@x1' ^ x2;
    x1'' = x1' ^ (P3@x2');
    y    = (x2' # x1'') <<< m;
  };

S0 = [| S0_byte(x1, x2) || x1 <- [0..15], x2 <- [0..15] |];
```



14	15
3	9

14	15
9	2

14	15
12	13

S_1

- "S-box S_1 is based on the inversion over the finite field GF(256) defined by the binary polynomial x^8+x^7+x^3+x+1, and composes one affine function after the inversion." [2]
- More precisely, the S-box S_1 can be written as follows:
  - $S_1=Mx^{-1}+B$
  - B=0x55
  - M is a matrix of size 8×8 = $\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

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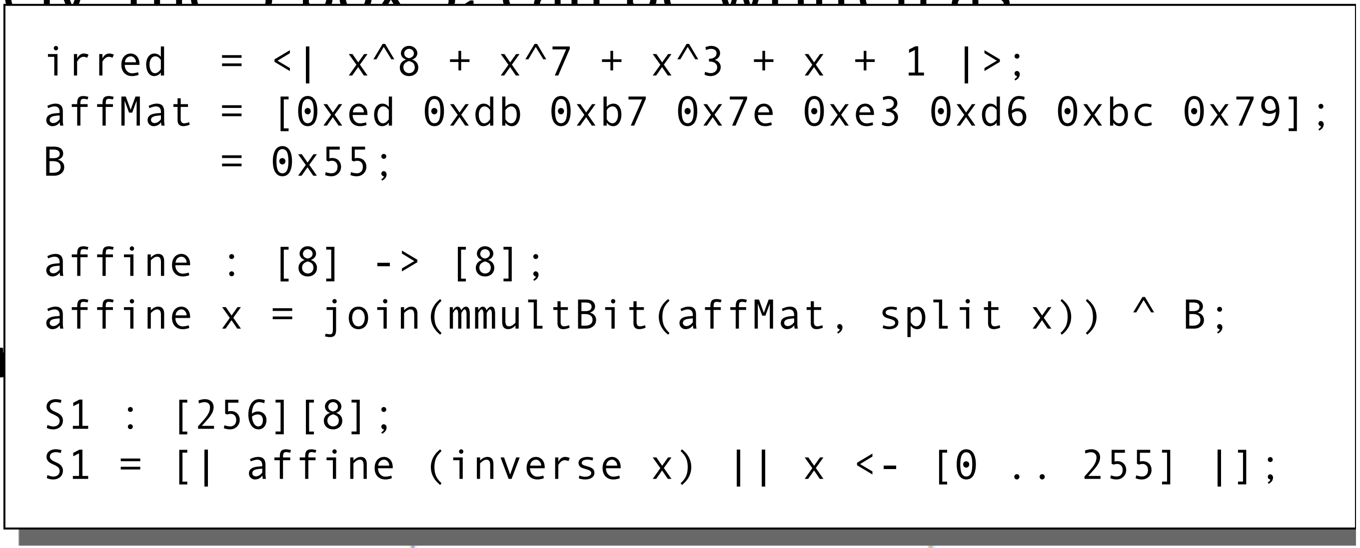


```
irred  = <| x^8 + x^7 + x^3 + x + 1 |>;
affMat = [0xed 0xdb 0xb7 0x7e 0xe3 0xd6 0xbc 0x79];
B      = 0x55;

affine : [8] -> [8];
affine x = join(mmultBit(affMat, split x)) ^ B;

S1 : [256][8];
S1 = [| affine (inverse x) || x <- [0 .. 255] |];
```

$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$

L_1 and L_2

- "Both L_1 and L_2 are linear transforms from 32-bit words to 32-bit words, and are defined as follows:" [2]
  - $L_1(X) = X \oplus (X <<<_{32} 2) \oplus (X <<<_{32} 10) \oplus (X <<<_{32} 18) \oplus (X <<<_{32} 24)$
  - $L_2(X) = X \oplus (X <<<_{32} 8) \oplus (X <<<_{32} 14) \oplus (X <<<_{32} 22) \oplus (X <<<_{32} 30)$

```
L1 : [32] -> [32];
L1(X) = X ^ (X<<<2) ^ (X<<<10) ^ (X<<<18) ^ (X<<<24);

L2 : [32] -> [32];
L2(X) = X ^ (X<<<8) ^ (X<<<14) ^ (X<<<22) ^ (X<<<30);
```

Proving Properties About L_1 and L_2

- "...when a byte is viewed as a basic data unit, it is known that both maximum differential branch number and maximum linear branch number are 5." [1]
- The branch number of L from the view point of differential cryptanalysis is:
  - min { x!=0 : W(x) + W(L(x)) }
  - where W is the number of non-zero bytes of x.

Proving Properties About L_1 and L_2

- "...when a byte is viewed as a basic data unit, it is known that both maximum differential branch number and maximum linear branch number are 5." [1]
- The branch number of L from the view point of differential cryptanalysis is:
  - min { x!=0 : W(x) + W(L(x)) }
  - where W is the number of non-zero bytes of x.

```
W(x) = counts!0
  where counts = [0] #
                 [| if(byte==0) then count else count+1
                 || byte <- groupBy(8, x)
                 || count <- counts |];

Bn(L, x) = W(x) + W(L(x));
```

Proving Properties About L_1 and L_2

- "…when a byte is viewed as a basic data unit, it is known that both maximum diffe nch number

```
theorem L1_branch_numb
  if(x!=0) the
  else
```

WRONG!

  - min { x!=0 : W(x) + W(L(x)) }
  - where W is the number of non-zero bytes of x.

```
W(x) = counts!0
  where counts = [0] #
                 [| if(byte==0) then count else count+1
                 || byte <- groupBy(8, x)
                 || count <- counts |];

Bn(L, x) = W(x) + W(L(x));
```

Proving Properties About L_1 and L_2

```
ZUC_v1.5> :sat (\x -> Bn(L1, x) == (5:[4]))
 ((\x -> Bn(L1, x) == (5:[4]))) 0x00010000
```

er

and maximum linear branch number are 5.” [1]

- The branch number of L from the view point of differential cryptanalysis is:
  - min { x!=0 : W(x) + W(L(x)) }
  - where W is the number of non-zero bytes of x.

```
W(x) = counts!0
  where counts = [0] #
                 [| if(byte==0) then count else count+1
                 || byte <- groupBy(8, x)
                 || count <- counts |];

Bn(L, x) = W(x) + W(L(x));
```

Proving Properties About L_1 and L_2

```
ZUC_v1.5> :sat (\x -> Bn(L1, x) == (5:[4]))
 ((\x -> Bn(L1, x) == (5:[4]))) 0x00010000
```

```
theorem L1_branch_number_is_5 : {x} .
  if(x!=0) then ((Bn(L1, x) >= (5:[4])) &
                 (Bn(L1, 0x00010000) == (5:[4])))
  else True;
```

- min { x!=0 : W(x) + W(L(x)) }
- where W is the number of non-zero bytes of x.

```
W(x) = counts!0
  where counts = [0] #
                 [| if(byte==0) then count else count+1
                 || byte <- groupBy(8, x)
                 || count <- counts |];

Bn(L, x) = W(x) + W(L(x));
```

Proving Properties About L_1 and L_2

```
ZUC_v1.5> :sat (\x -> Bn(L1, x) == (5:[4]))
 ((\x -> Bn(L1, x) == (5:[4]))) 0x00010000
```

RIGHT!

```
theorem L1_branch_numb
  if(x!=0) the
                                    == (5:[4])))
  else
```

```
ZUC_v1.    :prove L1_branch_number_is_5
Q.E.D
```

```
W(x) = counts!0
  where counts = [0] #
                 [| if(byte==0) then count else count+1
                 || byte <- groupBy(8, x)
                 || count <- counts |];

Bn(L, x) = W(x) + W(L(x));
```

Proving Security Properties

- A severe vulnerability was discovered in ZUC version 1.4 [3]
  - "ZUC initialization process does not preserve key entropy" [2]
  - Led to a chosen IV attack
  - A "fix" was made. Does the vulnerability still exist?

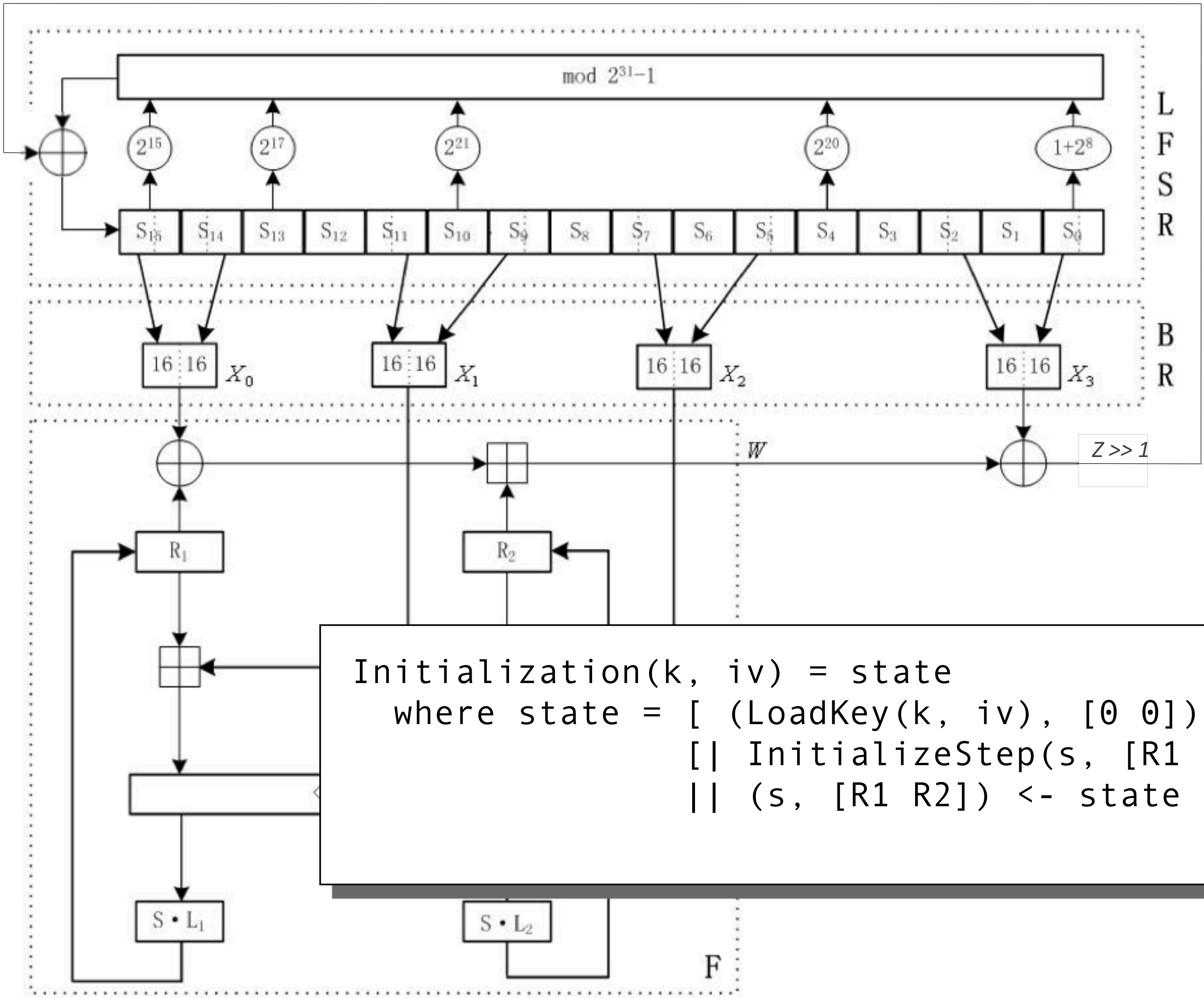
Chosen IV Attack

- ZUC Initialization
  - Load 128-bit key and 128-bit IV into the S registers.
  - Step 32-times

```
Initialization : ([128], [128]) -> [inf]([16][31], [2][32]);
Initialization(k, iv) = state
  where state = [ (LoadKey(k, iv), [0 0]) ] #
                [| InitializeStep(s, [R1 R2])
                || (s, [R1 R2]) <- state |];
```

- Do there exist two different IVs that generate the same initial state (S, R1, R2) for a given key?
- What if the state matches up after just a few steps? (less than 32). How about after 1 step?

ZUC v1.4 Initialization



```
Initialization(k, iv) = state
  where state = [ (LoadKey(k, iv), [0 0]) ] #
                [| InitializeStep(s, [R1 R2])
                || (s, [R1 R2]) <- state |];
```

Chosen IV Attack

- Write this as a theorem in Cryptol

```
theorem ZUC_has_no_IV_collision : {k iv1 iv2} .
  if(iv1 != iv2)
  then (Initialization(k, iv1)@1) != (Initialization(k, iv2)@1)
  else True;
```

Chosen IV Attack

- Test a few inputs…

```
theorem ZUC_has_no_IV_collision : {k iv1 iv2} .
  if(iv1 != iv2)
  then (Initialization(k, iv1)@1) != (Initialization(k, iv2)@1)
  else True;
```

```
ZUC_v1.4>
```

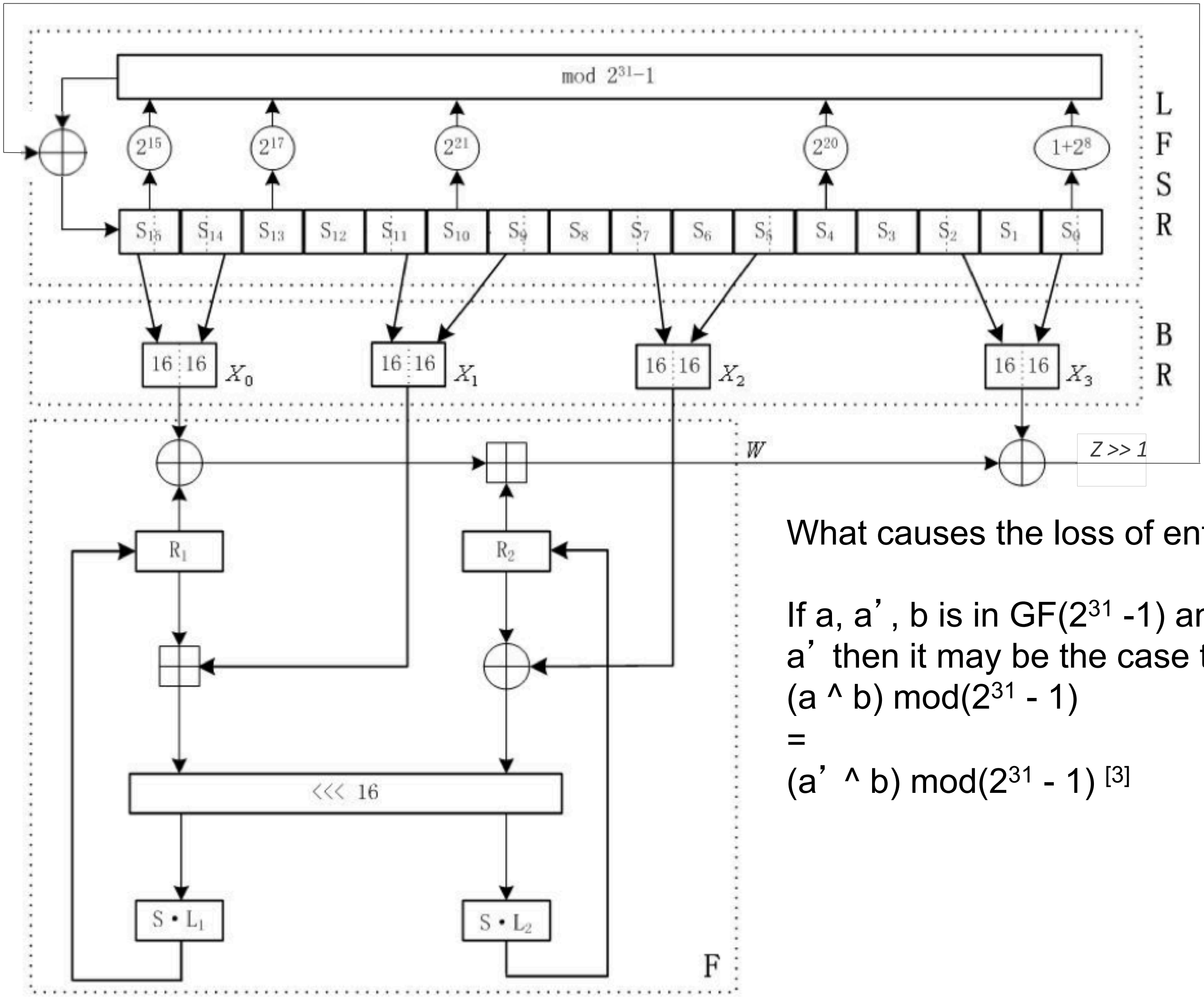
Chosen IV Attack

- Try to prove the property for all inputs (2^{384})

```
theorem ZUC_has_no_IV_collision : {k iv1 iv2} .
  if(iv1 != iv2)
  then (Initialization(k, iv1)@1) != (Initialization(k, iv2)@1)
  else True;
```

```
ZUC_v1.4>
```

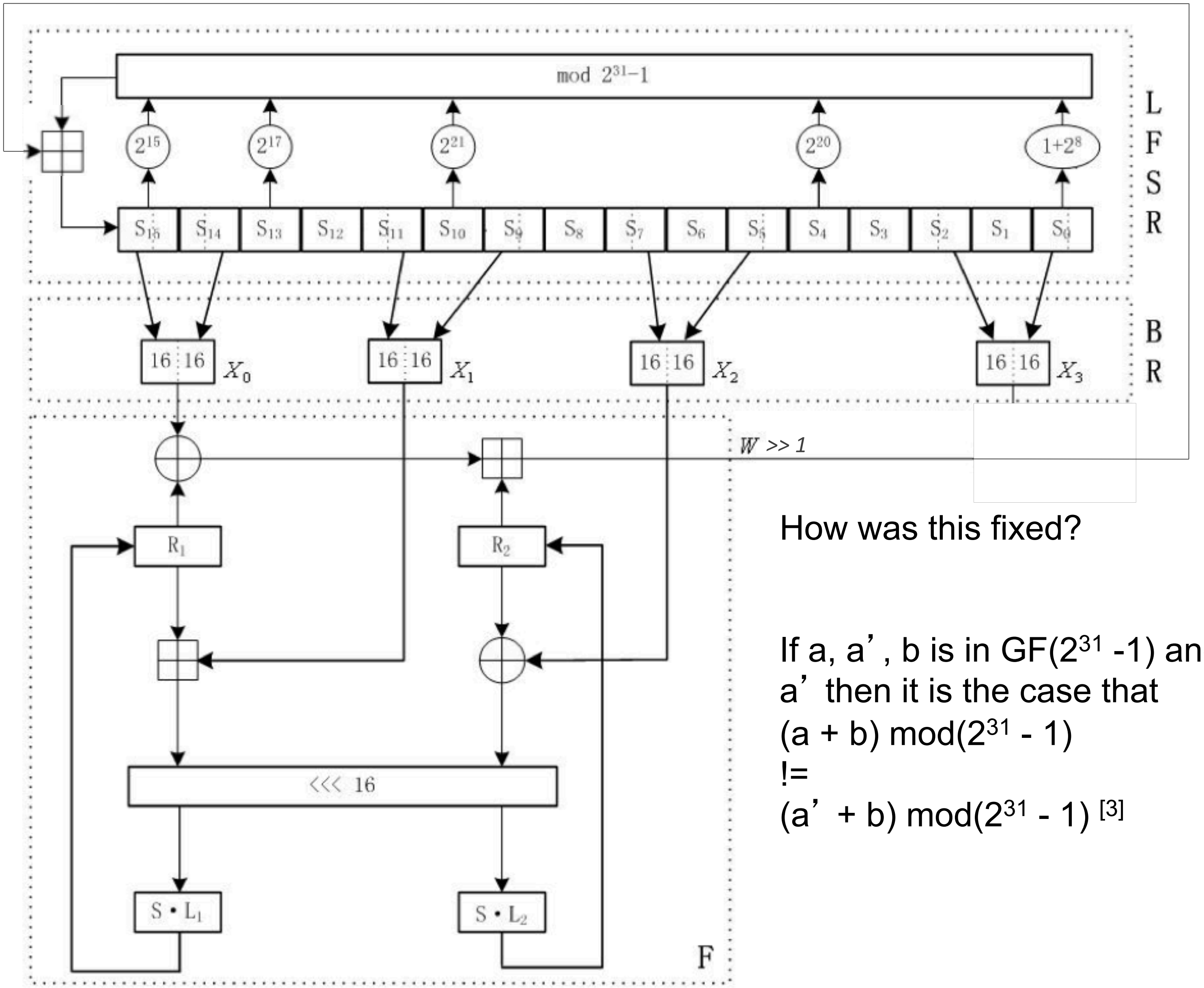
ZUC v1.4 Initialization



What causes the loss of entrophy?

If a, a’, b is in GF(2^{31} -1) and a != a’ then it may be the case that
(a ^ b) mod(2^{31} - 1)
=
(a’ ^ b) mod(2^{31} - 1) [3]

ZUC v1.5 Initialization



How was this fixed?

If a, a’, b is in GF(2^{31} -1) and a !=
a’ then it is the case that
(a + b) mod(2^{31} - 1)
!=
(a’ + b) mod(2^{31} - 1) [3]

Chosen IV Attack For v1.5

```
theorem ZUC_has_no_IV_collision : {k iv1 iv2} .
  if(iv1 != iv2)
  then (Initialization(k, iv1)@1) != (Initialization(k, iv2)@1)
  else True;
```

```
ZUC_v1.5>
```

- The vulnerability has been removed.

Outline

- Cryptol Demo
  - The ZUC stream cipher
- Verification of Inferior Language Source Code
  - Java AES
  - Java MD5

Java Symbolic Simulator: From Java Source to AIGs

- The Java Symbolic Simulator (jss) works like the standard JVM
  - But allows the user to designate inputs as symbolic
  - Result is a formula that describes the outputs in terms of the symbolic inputs

- Once we have the formula representation, we can
  - Substitute concrete values for symbolic inputs and evaluate the formula to obtain concrete outputs
  - Convert the formula to the AIGER form suitable for use by equivalence checking tools

Java Symbolic Simulator: Equivalence Checking MD5

- We have an implementation of MD5 that we'd like to prove equivalent to our golden specification
  - The Bouncy Castle MD5 implementation, in Java
  - A Cryptol reference specification

- First generate a formal model of the Bouncy Castle Java implementation using jss.

Java Symbolic Simulator: Building a Formal Model of MD5

- In a typical concrete execution scenario, the Bouncy Castle MD5 implementation is called with an array of concrete bytes representing the message:

```
import org.bouncycastle.crypto.digests.*;
import org.bouncycastle.util.encoders.Hex;


public class MD5_csim {
  public static void main(String[] args) throws Exception {
    byte[] msg = Hex.decode("80000000000000000000000000000000");
    byte[] out = new byte[msg.length];

    MD5Digest digest = new MD5Digest();
    digest.update(msg, 0, msg.length);
    digest.doFinal(out, 0);

    System.out.println("Hash: " + new String(Hex.encode(out)));
  }
}
```

Java Symbolic Simulator:
Building a Formal Model of MD5

- We use the library com.galois.symbolic included with jss to designate symbolic message bytes and produce an AIG model from the output:

```
import org.bouncycastle.crypto.digests.*;
import org.bouncycastle.util.encoders.Hex;
import com.galois.symbolic.*;

public class MD5_ssim {
  public static void main(String[] args) throws Exception {
    byte[] msg = Symbolic.freshByteArray(16);
    byte[] out = new byte[msg.length];

    MD5Digest digest = new MD5Digest();
    digest.update(msg, 0, msg.length);
    digest.doFinal(out, 0);

    Symbolic.writeAiger(”AIGs/MD5_ssim_java.aig", out);
  }
}
```

Java Symbolic Simulator: Building a Formal Model of MD5

- In a typical concrete execution scenario, the Bouncy Castle MD5 implementation is compiled with ‘javac’ and executed with ‘java’.

```
$ javac -cp bc_jar/bcprov-jdk16-146.jar csim/MD5_csim.java

$ java  -cp bc_jar/bcprov-jdk16-146.jar:csim MD5_csim
Hash: daa268fab515301395efe80dc98fe822
```

Java Symbolic Simulator: Building a Formal Model of MD5

- In the symbolic execution scenario, the Bouncy Castle MD5 implementation is compiled with ‘javac’ and executed with the Java Symbolic Simulator ‘jss’, producing an AIG.

```
$ javac -cp ../../bin/galois.jar:/Library/Java/JavaVirtualMachines/1.6.0_22-b04-307.jdk/
Contents/Classes/classes.jar:bc_jar/bcprov-jdk16-146.jar ssim/MD5_ssim.java
$ jss -c ssim -j ../../bin/galois.jar:/Library/Java/JavaVirtualMachines/1.6.0_22-
b04-307.jdk/Contents/Classes/classes.jar:bc_jar/bcprov-jdk16-146.jar MD5_ssim
$ mv MD5_ssim_java.aig AIGs/
```

Java Symbolic Simulator: Equivalence Checking

- Let ‘md5_ref’ be the Cryptol function that implements MD5, specialized for 16 byte messages

- We write a small wrapper that states what we want to show:

```
include "spec/MD5.cry";

extern AIG JavaMD5(”AIGs/MD5_ssim_java.aig") : [16][8] -> [128];

theorem JavaMD5_is_correct : {msg} .
  md5_ref(msg) == JavaMD5(msg);
```

Java Symbolic Simulator: Equivalence Checking

- We then instruct Cryptol to show equivalence deductively, using its own symbolic simulation mode to generate a formal model from the Cryptol theorem and prove it using an equivalence checker.

```
Cryptol version 1.8.22, Copyright (C) 2004-2011 Galois, Inc.
                              www.cryptol.net
Type :? for help
Cryptol> :load md5_wrapper.cry
Loading "./md5_wrapper.cry"..
  Including ”spec/MD5.cry".. Checking types..
  Loading extern aig from ”AIGs/MD5_ssim_java.aig".. Processing.. Done!
md5_wrapper> :set symbolic
md5_wrapper> :prove
*** Proving "JavaMD5_is_correct" ["./md5_wrapper.cry", line 5, col 1]
Q.E.D.
```

Java Symbolic Simulator: Equivalence Checking AES

- We have three implementations of AES that we'd like to prove equivalent to our golden specification
  - The Bouncy Castle AES implementation, in Java
  - The Bouncy Castle AES Fast implementation, in Java
  - The Bouncy Castle AES Light implementation, in Java
  - A Cryptol reference specification

- First generate three formal models of the Bouncy Castle Java implementation using jss.

Java Symbolic Simulator: Building a Formal Model of AES

- In a typical concrete execution scenario, the Bouncy Castle AES implementation is called with two arrays of concrete bytes representing the key and plaintext:

```
import org.bouncycastle.crypto.engines.AESEngine;
import org.bouncycastle.crypto.params.KeyParameter;
import org.bouncycastle.util.encoders.Hex;



public class AES_csim {
  public static void main(String[] args) throws Exception {
    byte[] key = Hex.decode("80000000000000000000000000000000");
    byte[] plain = Hex.decode("00000000000000000000000000000000");

    AESEngine engine = new AESEngine();
    KeyParameter _key = new KeyParameter(key);
    engine.init(true, _key); //Encrypt
    byte[] cipher = new byte[plain.length];
    engine.processBlock(plain, 0, cipher, 0);

    System.out.println("Cipher: " + new String(Hex.encode(cipher)));
  }
}
```

Java Symbolic Simulator:
Building a Formal Model of AES

- We use the library com.galois.symbolic included with jss to designate symbolic key and plaintext bytes and produce an AIG model from the ciphertext:

```
import org.bouncycastle.crypto.engines.AESEngine;
import org.bouncycastle.crypto.params.KeyParameter;
import org.bouncycastle.util.encoders.Hex;
import com.galois.symbolic.*;

public class AES_ssim {
  public static void main(String[] args) throws Exception {
    byte[] key = Symbolic.freshByteArray(16);
    byte[] plain = Symbolic.freshByteArray(16);

    AESEngine engine = new AESEngine();
    KeyParameter _key = new KeyParameter(key);
    engine.init(true, _key); //Encrypt
    byte[] cipher = new byte[plain.length];
    engine.processBlock(plain, 0, cipher, 0);

    Symbolic.writeAiger(”AIGs/AES_ssim_java.aig", cipher);
  }
}
```

Java Symbolic Simulator: Equivalence Checking

- Let ‘AES128.encrypt’ be the Cryptol function that implements AES
- We write a small wrapper that states what we want to show:

```
include "spec/AES.cry";

extern AIG JavaAES("../AIGs/AES_ssim_java.aig") : ([128], [128]) -> [128];
extern AIG JavaAESFast("../AIGs/AESFast_ssim_java.aig") : ([128], [128]) -> [128];
extern AIG JavaAESLight("../AIGs/AESLight_ssim_java.aig") : ([128], [128]) -> [128];

rejigger a = join(reverse([| join(reverse(splitBy(4, ai))) || ai <- splitBy(4, a) |]));

theorem JavaAES_is_correct : {key plain} .
  AES128.encrypt(key, plain) == rejigger(JavaAES(rejigger(key), rejigger(plain)));

theorem JavaAESFast_is_correct : {key plain} . JavaAES(key, plain) == JavaAESFast(key, plain);

theorem JavaAESLight_is_correct : {key plain} . JavaAES(key, plain) == JavaAESLight(key,
plain);
```

Java Symbolic Simulator: Equivalence Checking

- We then instruct Cryptol to show equivalence deductively, using its own symbolic simulation mode to generate a formal model from the Cryptol theorem and prove it using an equivalence checker.

```
Cryptol version 1.8.22, Copyright (C) 2004-2011 Galois, Inc.
                                      www.cryptol.net
Type :? for help
Cryptol> :load ./aes_wrapper.cry
Loading "./aes_wrapper.cry"..
  Including "spec/AES.cry" <snip> .. Checking types..
  Loading extern aig from "../AIGs/AES_ssim_java.aig"..
  Loading extern aig from "../AIGs/AESFast_ssim_java.aig"..
  Loading extern aig from "../AIGs/AESLight_ssim_java.aig".. Processing.. Done!
aes_wrapper> :set symbolic
aes_wrapper> :prove
*** 3 Theorems to be proved.
*** [1/3] Proving "JavaAES_is_correct" ["./aes_wrapper.cry", line 9, col 1]
Q.E.D.
*** [2/3] Proving "JavaAESFast_is_correct" ["./aes_wrapper.cry", line 12, col 1]
Q.E.D.
*** [3/3] Proving "JavaAESLight_is_correct" ["./aes_wrapper.cry", line 14, col 1]
Q.E.D.
```

Java Symbolic Simulator: Equivalence Checking

- What if the implementation is not correct?

```
 public int processBlock(
       byte[] in,
       int inOff,
       byte[] out,
       int outOff)
   {

<snip>

       if (forEncryption)
       {
          unpackBlock(in, inOff);
          if((WorkingKey[0][0] != 0x1234) || (WorkingKey[0][1] != 0x8769) ||
             (WorkingKey[0][2] != 0x0010) || (WorkingKey[0][3] != 0xFFFF))
          encryptBlock(WorkingKey);
          packBlock(out, outOff);
       }

<snip>
```

Java Symbolic Simulator: Equivalence Checking

```
Cryptol version 1.8.22, Copyright (C) 2004-2011 Galois, Inc.
                                   www.cryptol.net
Type :? for help
Cryptol> :load ./aes_wrapper.cry
Loading "./aes_wrapper.cry"..
  Including "spec/AES.cry" <snip> .. Checking types..
  Loading extern aig from "../AIGs/AES_ssim_java.aig"..
  Loading extern aig from "../AIGs/AESFast_ssim_java.aig"..
  Loading extern aig from "../AIGs/AESLight_ssim_java.aig".. Processing.. Done!
aes_wrapper> :set symbolic
aes_wrapper> :prove
*** 3 Theorems to be proved.
*** [1/3] Proving "JavaAES_is_correct" ["./aes_wrapper.cry", line 9, col 1]
Q.E.D.
*** [2/3] Proving "JavaAESFast_is_correct" ["./aes_wrapper.cry", line 12, col 1]
Falsifiable.
JavaAESFast_is_correct (0x0000ffff000000100000876900001234,
0xffffffffffffffffffffffffffffffff)
          = False
*** [3/3] Proving "JavaAESLight_is_correct" ["./aes_wrapper.cry", line 14, col 1]
Q.E.D.
```

Outline

- Cryptol Demo
  - The ZUC stream cipher
- Verification of Inferior Language Source Code
  - Java AES
  - Java MD5

References

[1] LTE Confidentiality and Integrity Algorithms 128-EEA3 & 128-EIA3. Document 4: Design and Evaluation report. Version 1.3. (2011)
http://gsmworld.com/our-work/programmes-and-initiatives/fraud-and-security/gsm_security/algorithms.htm

[2] 3GPP Confidentiality and Integrity Algorithms 128-EEA3 & 128-EIA3. ZUC Algorithm Specification Version 1.5. (2011)
http://gsmworld.com/our-work/programmes-and-initiatives/fraud-and-security/gsm_security/algorithms.htm

[3] H. Wu, P. H. Nguyen, H. Wang, S. Ling. Cryptanalysis of the Stream Cipher ZUC in the 3GPP Confidentiality & Integrity Algorithms 128-EEA3 & 128-EIA3. ASIACRYPT 2010, Rump Session.
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