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Analyzing Code Stability Using Control Theoretic Techniques

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Overview

- We Will Discuss the Application of Control Theory to Software
  - Control Theory studies the behavior of dynamical systems. For example, control theory describes the conditions under which an inverted pendulum will not fall over
  - Software describes a dynamical system – can we apply control theory?
- Controller-Oriented Programming (COP) is a New Programming Language Paradigm Developed to Enable Software that is Efficient & Adaptable
  - Adds two key language constructs: Partitions and Controllers
    - Partitions capture sets of implementation options that can be treated as equivalent
    - Controllers dynamically select among these options and manage side effects and other couplings to enable systems to act like they are decoupled
  - Separates *action flow,* which specifies the essential tasks necessary to provide the required functionality, from *controller flow*, which restores necessary pre-conditions
  - Hypothesis: partitions and controllers and the resulting separation of action and controller flow may lead to ability analyze more easily
  - SymLang is the first instance of a COP Language
- We will describe an example control theory-based analysis of SymLang code

Simple Example Problem: Data Stores Utilizes Feedback Control to Ensure Sufficient Resources

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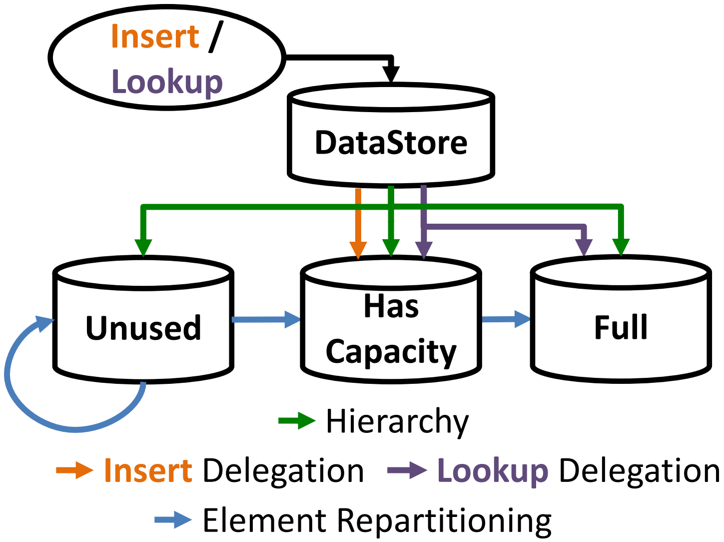
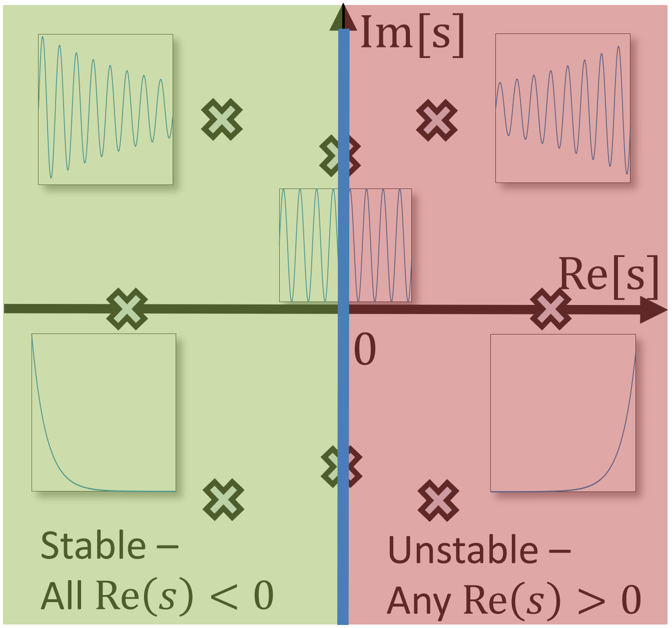
- Data Store Problem: given an unbounded stream of integer values, support lookup (true iff the value has been seen previously) in bounded time
- DataStore Implementation uses an unbounded set of atomic stores
  - Stores are organized into 3 partitions
    - Unused: new stores; uses a feedback controller to spin up additional stores as current stores are depleted
    - HasCapacity: stores with capacity; supports insert and lookup
    - Full: full stores; supports lookup
  - Insert is implemented by...
    - Inserting the value into a store in HasCapacity, which also triggers a controller to
      (a) move the store to Full, if it not longer has remaining capacity, and
      (b) take a store from Unused if HasCapacity becomes empty as a result
    - A controller also spins up new stores in Unused in anticipation of future needs



A Brief Overview of Types of Stability

- BIBO (bounded-input, bounded-output) Stability: System is bounded by a finite output for a finite input
  - Example: Ideal oscillator – when displaced, oscillates with finite amplitude around its equilibrium
  - Asymptotic stability: System returns to equilibrium when displaced
    - Example: pendulum with friction, when displaced will always trend back to its downward position
    - Condition for LTI systems: all poles have Re(s)<0
- Marginal stability: Displaced system does not explode but also does not return to equilibrium
  - Example: mass on a surface with friction – when impacted, it will travel and stop eventually but won't return to its original position
- Unstable: Displaced system explodes
  - Example: Mic and Speaker – the roar of positive feedback when a mic picks up the speaker output



Poles of the Transfer Function Indicate Stability

- Transfer functions describe Input-Output relationships of a system:

$$H(s) = \frac{output(s)}{input(s)} = \frac{N(s)}{D(s)}$$

- Poles are $s \in \mathbb{C}$ s.t. $D(s) = 0$

Control Theoretic Approach to Stability: Transfer Function Analysis

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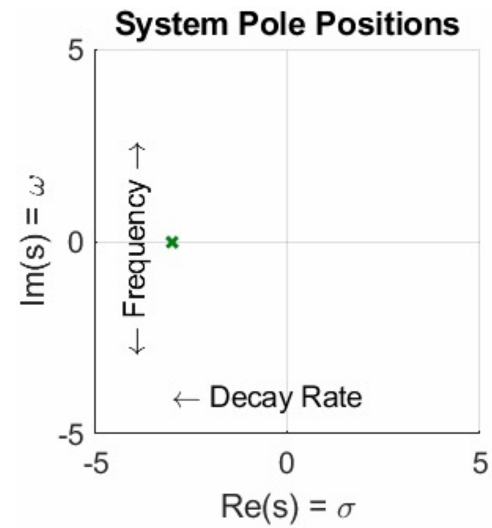
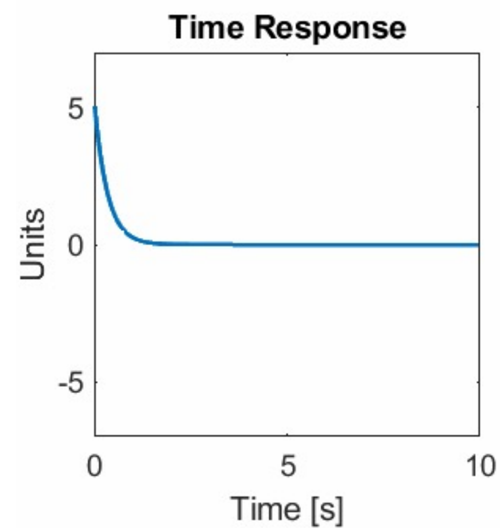
- Transfer Function Analysis Provides a Simple Way to Analyze Stability of Linear Time-Invariant Systems
- Step 1: Create a block diagram capturing system dynamics
  - Block diagrams live in the Laplace domain
  - Fourier transforms decomposes a signal into frequency components (sines and cosines): $e^{i\omega t}$
  - Laplace transforms include both real and imaginary components to capture signal growth in addition to oscillations: $e^{(\sigma + i\omega)t}$
    - $\sigma > 0$: signal blows up: $for\ \sigma > 0, t \to \infty\ e^{\sigma t} \to \infty$ (unstable)
    - $\sigma < 0$: signal decays: $for\ \sigma > 0, t \to \infty\ e^{\sigma t} \to 0$ (stable)
    - $\sigma = 0$: signal oscillates forever (neither stable nor unstable)
- Step 2: Solve for the Transfer Function, $H(s) = \frac{output(s)}{input(s)}$
  - Relates the output signal to the input signal
  - Derived by reducing block diagram (well-understood in Control Theory)
- Step 3: Analyze the poles of the transfer function
  - A system is stable if and only if all poles have negative real part
  - Otherwise, the system does not converge (conceptually, the output blows up for a non-decaying input)





Control Theory is Designed to Analyze Stability… Why Not Apply to Code?

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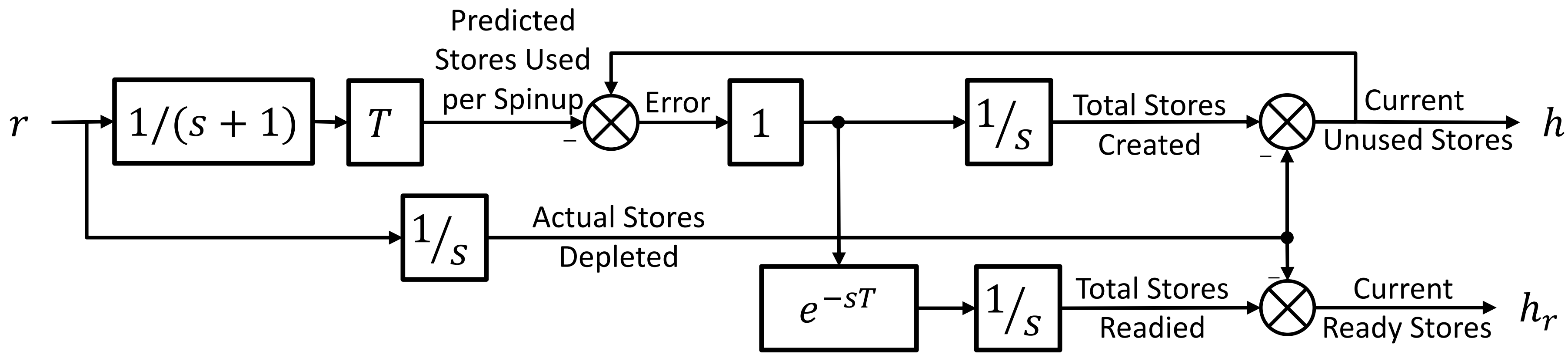
- Software Is a Dynamical System
  - Inputs are transformed into outputs
  - Software defines these transformations in code
- Failures at Cloud-Scale Often Look like Stability Issues
  - Amazon – 2021, large-scale AWS outage due to an internal migration that caused a temporary spike in network activity, which became self-perpetuating due to retry (e.g., on timeout) policies
  - Microsoft Azure – 2018, Overloaded Redis cache increased lookup latency, leading to application-level timeouts, which caused cascading failures, leading to a 17-hour downtime for multi-factor user-authentication
- Control Theory Answers Questions That Seem Relevant for Software
  - Stability: does the system have a bounded output for all sequences of bounded input?
  - Margin: does the system have sufficient resources such that future stability is guaranteed? (Is it possible for the system to run out of resources (in the future)?)
  - Note: This analysis describes the conditions under which we can guarantee that stability holds
  - Challenge is bridging the gap between control theory tools and code implementations

Controllers in (SymLang!) Code

- SymLang Code Incorporates Controllers As First-Class Language Elements, Defines a Dynamical System With Separation of Concerns
  - *Action flow* specifies the essential tasks necessary to provide the required functionality, while *Controller flow* which restores necessary pre-conditions
  - For example, in the Data Store Insert Implementation,
    Action flow specifies that the value is inserted into a store to support lookup
    Controller flow ensures that HasCapacity has a store and can support insert
- For Data Store Implementation, Want to Analyze the Stability of the Number of Unused Stores
  - Want to show that there does not exist a condition under which the number of Unused stores could become unbounded
  - Bounded input bounded output (BIBO) stability would guarantee that, for any bounded input rate, the number of unused stores is always bounded
- Question: Can We Apply Control Theoretic Techniques to the SymLang Code to Show BIBO Stability?

Derived Block Diagram From Code, Transfer Function for Stability Analysis

Step 1, Manually derived a Block Diagram for the DataStore Implementation



Step 2, derived a Transfer function describing the number of unused stores as a function of insert rate:

$$H(s) = \frac{output(s)}{input(s)} = \frac{h}{r} = \frac{T - (1 + s)}{(s + 1)^2}$$

Step 3, Evaluated the poles by solving $D(s) = (s + 1)^2 = 0$

$$s = -1$$

Because s has negative real part, this implies BIBO stability

$$Re(s) = -1$$

∴ for any bounded insert rate, the number of unused stores will remain bounded

Analysis Correctly Identifies Unstable Implementations: Positive Feedback Bug

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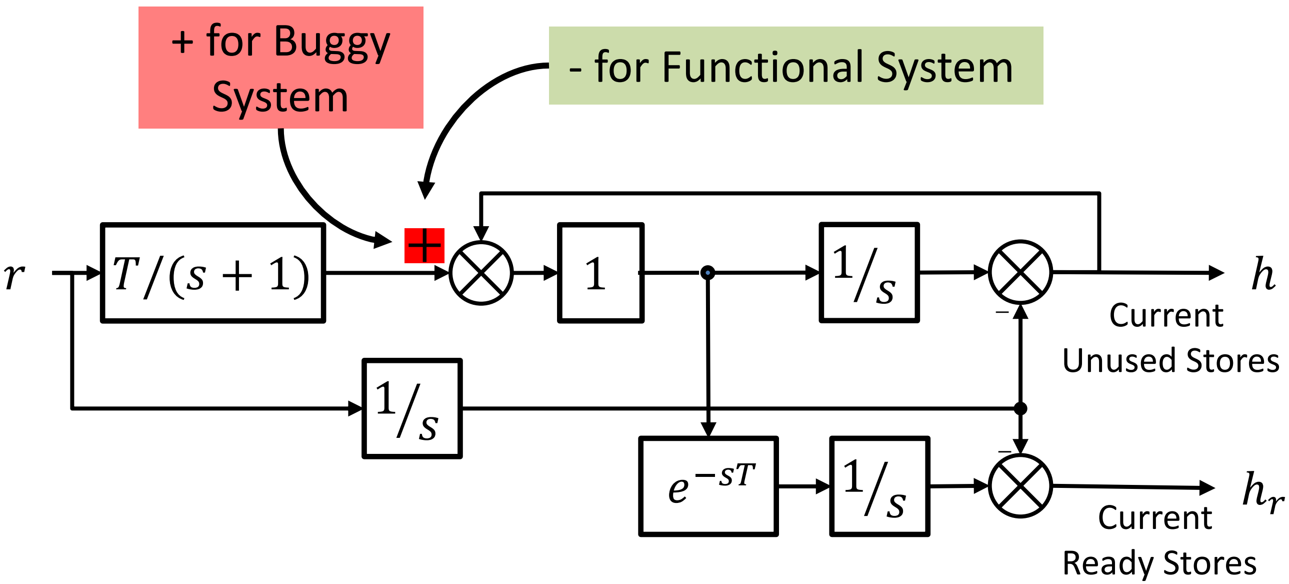
- Analyzed Two Real Bugs Introduced by Junior Devs

```
Bug #1
// creates positive feedback
val error = (setpoint + size())
```

```
Functional Implementation
internal void runPID() = Do {
  action() = {
    val error = (setpoint - size())
    var desiredUpdate = PID.execute(error);
    addOrRemoveStores(desiredUpdate);
}}
```



Transfer Function

$$\frac{h}{r} = \frac{T-(1+s)}{(s+1)(s-1)}$$

Pole at $s = +1$, so System is UNSTABLE

Transfer function analysis shows positive feedback creates instability

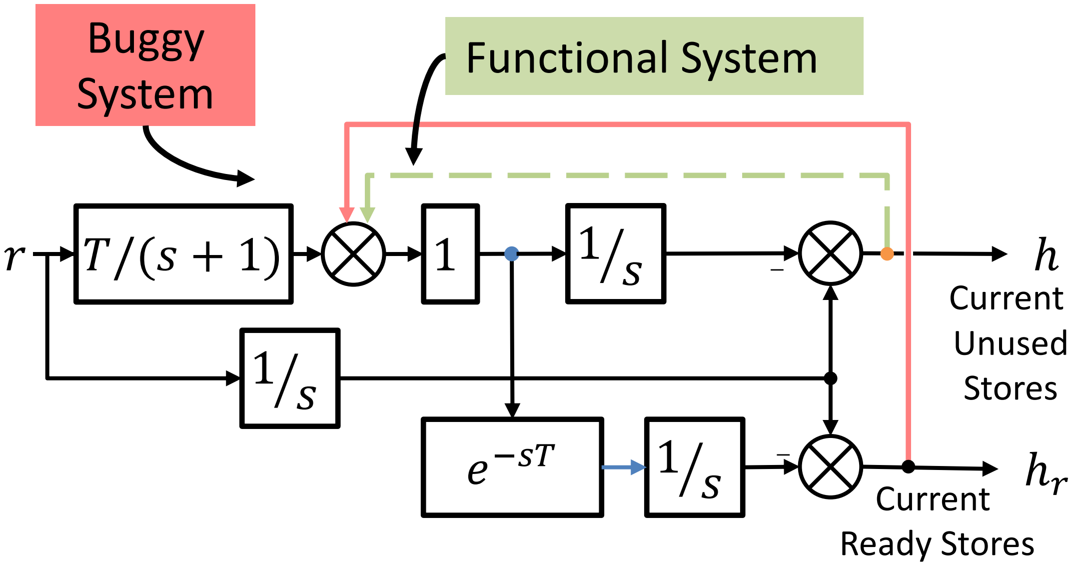
Analysis Correctly Identifies Unstable Implementations: Latency Bug

Bug #2

```
// introduces latency by only
// counting Ready stores
val error = (setpoint – Ready.size());
```

Functional Implementation

```
internal void runPID() = Do {
  action() = {
    val error = (setpoint - size())
    var desiredUpdate = PID.execute(error);
    addOrRemoveStores(desiredUpdate);
}}
```



Transfer Function

$$\frac{h}{r} = \frac{sT + (s+1)(1 - s - e^{-sT})}{s(s + e^{-sT})(s+1)}$$

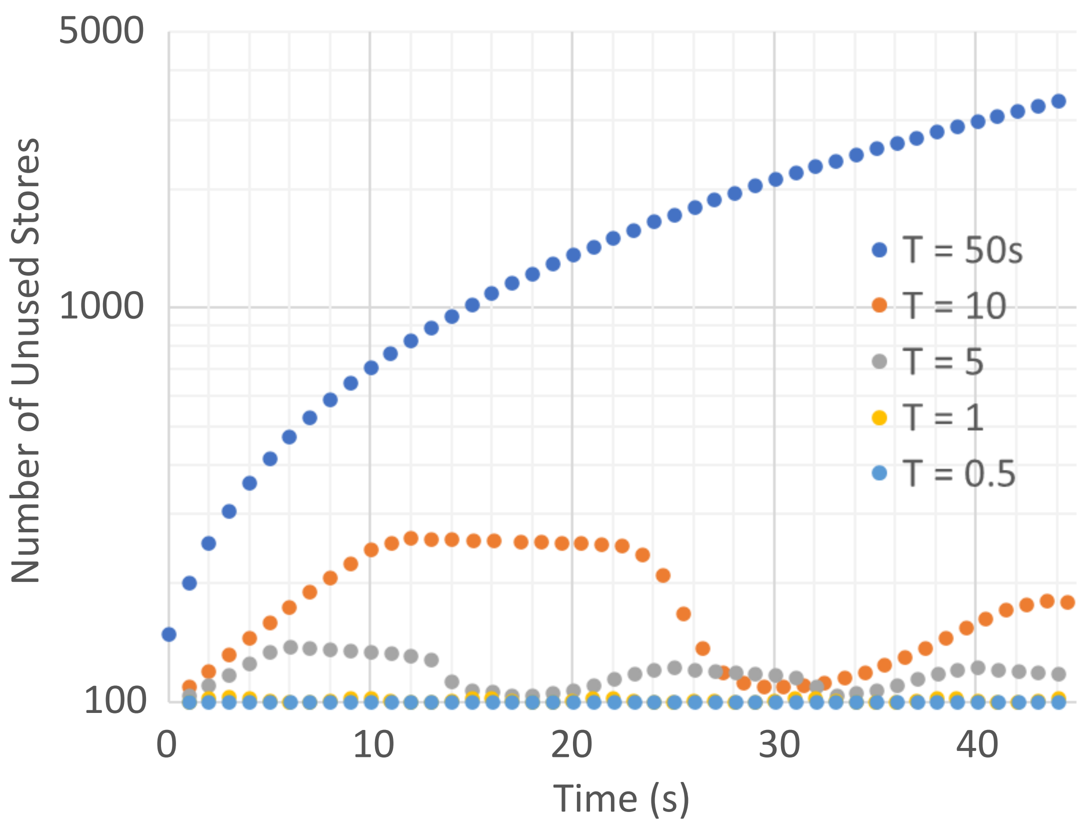
Poles at $s = 0, -1$ and s s.t. $s + e^{-sT} = 0$

For $T \geq \frac{\pi}{2}$, $s + e^{-sT} = 0$ has roots with positive real part, so System is UNSTABLE

Transfer function analysis shows introduced latency creates instability

Control Theory Identifies Regions of Stability and Instability

- With Bug #2, DataStore has regions of stability, instability
  - When T (spin-up time) is small, DataStore appears stable
  - For large T, number of stores grows without bound
  - T is the time to spin up a new store – an external parameter. If cloud outage causes an increase in latency, do not want the system diverge unrecoverably!



- Control Theory Reasons over the Range of Possible Spin up Delays
  - Static analysis that identifies potential instabilities due to non-syntactic errors
  - Provides stronger confidence than running a small sample of points in the config space

Moving Towards Automated Analysis

- Automated Stability Analysis Combines Techniques from Code Analysis and Control Theory
  - Tool uses data flow, control flow, and Laplace transforms of controller functionalities to derive the block diagram
  - Analysis of stability from a block diagram is well-understood in control theory
- Support Controller Analysis for a Limited Subset of Language
  - Automated controller analysis not possible in general, e.g., code must be analyzable
  - Defined a restricted set of primitive operations such that anything written in this subset of the language can be analyzed. Next step: formalize as a restricted DSL
  - SymLang also provides Control Theory Libraries for standard functionality, e.g., PID controllers, that include Laplace Transforms to enable analysis
- Implemented Working Prototype of Automated Stability Analysis
  - Analyzes example DataStore implementation, and we believe the prototype will extend to other relevant cases
  - Happy to provide both the SymLang and analysis code to those interested (conditioned on government approval for release)

Conclusion

- Demonstrated Analysis of Stability of SymLang Code
  - Derived transfer function from SymLang code
  - Pole analysis correctly identified stability and instability of implementations
  - Automated analysis of SymLang implementation for example Data Store problem
- Practically, Stability Bugs are Not Easy to Catch with Current Tools
  - Bugs are semantic, not syntactic – code will compile because syntax validation, type-checking, and other common code analysis techniques do not reason about stability
  - Require reasoning over a very large (possibly infinite) state space
- Early Work – Lots More to Do!
  - Extend and improve automated analysis tool
  - Margin analysis: in addition to stability, want to know if sufficient stores available
  - Approaches for scalability: can we use compositional approaches to achieve scalability, e.g., by characterizing the gain and phase lag of each module

Questions?