Power Indices and Security Investment Games

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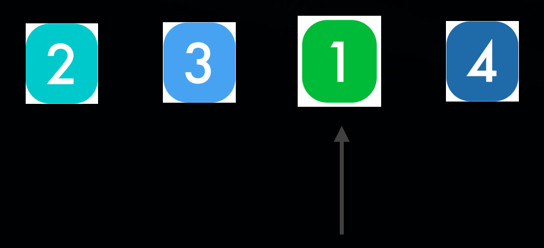
Project Objective: Run a series of experiments to better understand strategies and the requisite tradeoffs that arise when making network security investment decisions – how best to allocate finite resources to protect a portfolio of assets. In particular, we wish to assess the relative performance and economic efficiency of two allocation strategies inspired by concepts from cooperative game theory: the Shapley-Shubik Power Index and the Banzhaf Power Index.

Shapley –Shubik Power Index (SSPI)

- You have n assets; each asset has a corresponding value or weight w_n
- Construct the n! possible orderings of weight sequences.
- For each sequence, identify the pivotal weight – the one that puts the sequence over a given threshold.
- The resulting Shapley value of an asset is the number of times that asset is pivotal, divided by n!

Example: 4 assets; threshold of 6 (majority of the 10 total)

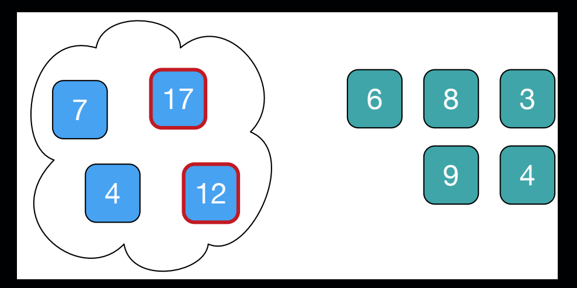


In this particular ordering, the 1 is pivotal; we do a similar calculation for all Possible orderings, and then divide by n!

Banzhaf Power Index (BPI)

- You have n assets; each asset has a corresponding value or weight w_n
- Construct all possible winning coalitions.
- For each winning coalition, identify the critical assets – the ones that, if they left the coalition, would turn it from winning to losing.
- The resulting Banzhaf value of an asset is the ratio of the number of times it is critical divided by the total number of critical assets (calculated across all winning coalitions)

Example: in the following winning coalition(40 points of 70 with winning threshold of 36, the 17 and the 12 are critical:



Calculating the Economic Efficiency of an Index

In assessing how efficient an allocation strategy is, we consider two kinds of "waste" – the total waste is the sum of the two:

- Spending more than the bare minimum needed to claim an asset (as either attacker or defender)
- Any amount spent on an asset that is awarded to an adversary.

- Example: Suppose Attacker and Defender are contending for a single asset
  - Attacker allocates .4 to the asset, Defender allocates .3 to the asset
  - This asset is awarded to Attcker (.4 > .3)
  - Attacker waste is .4 - .3 = .1
  - Defender waste is .3

Project Implementation: To assess the performance of these two power indices (SSPI, BPI) we constructed a tournament that pitted the two against each other, in addition to two baseline strategies: random allocation, and linear allocation.

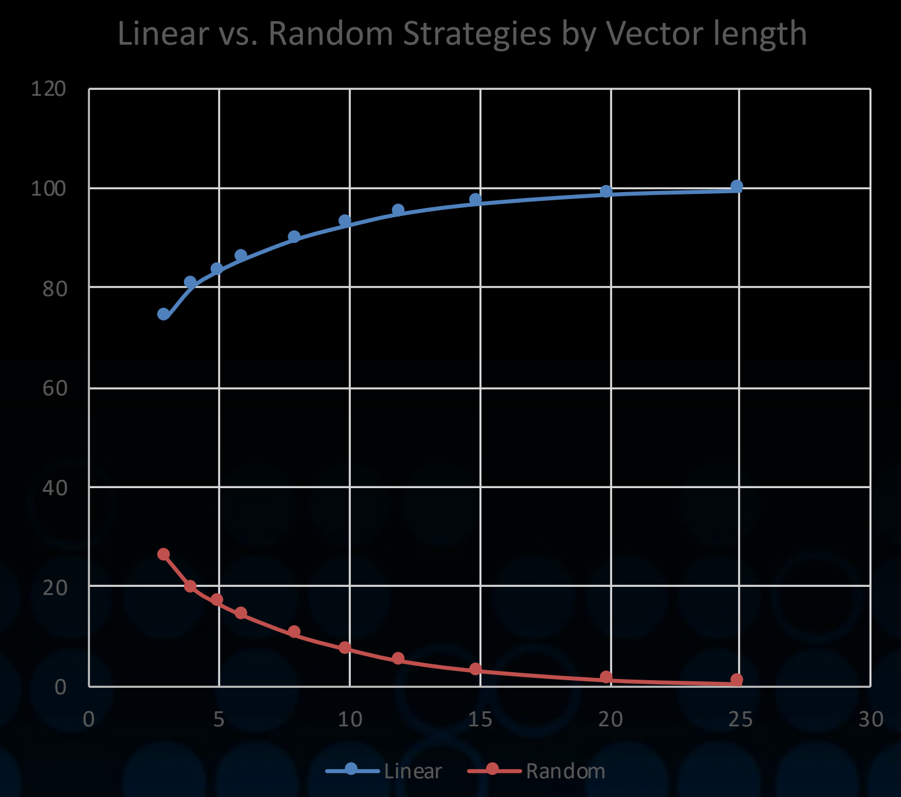
In this tournament, we varied the following key parameters:

- Number of assets under contention in a single contest
- Range of possible asset values
- The value of epsilon, which represents the amount of measurement noise in the system.

Establishing Relative Baseline Strategy Strength:

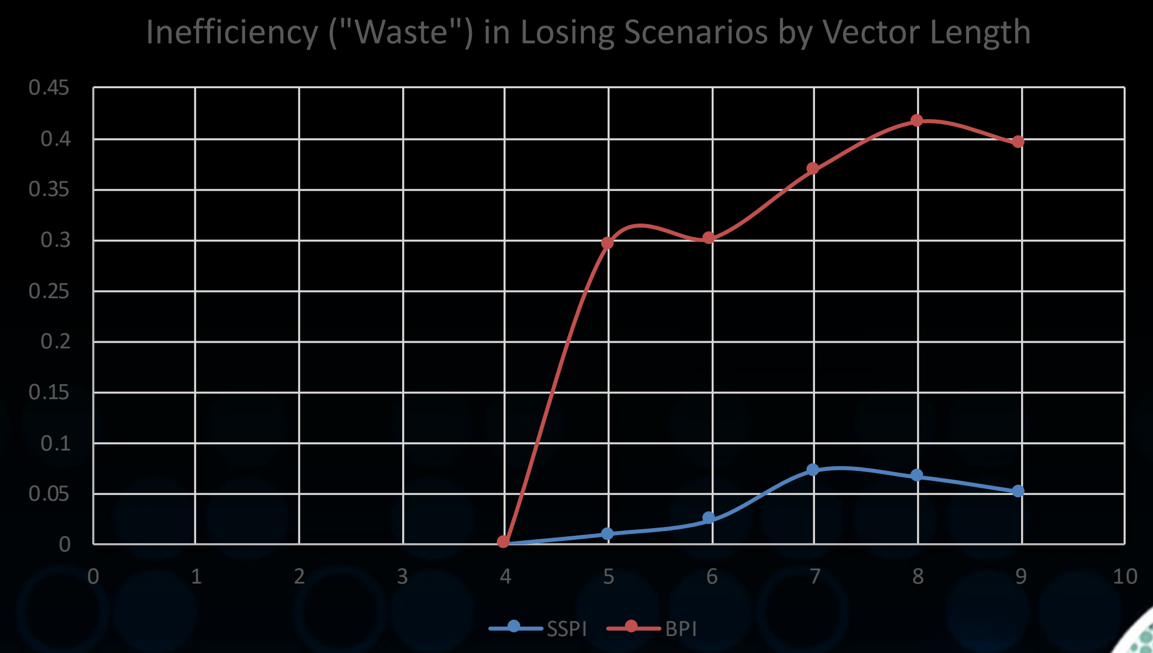
As vector length increases,
Linear strategy beats Random strategy.

However, the Random strategy is
Surprisingly successful when the
number of assets under contention
Is small!



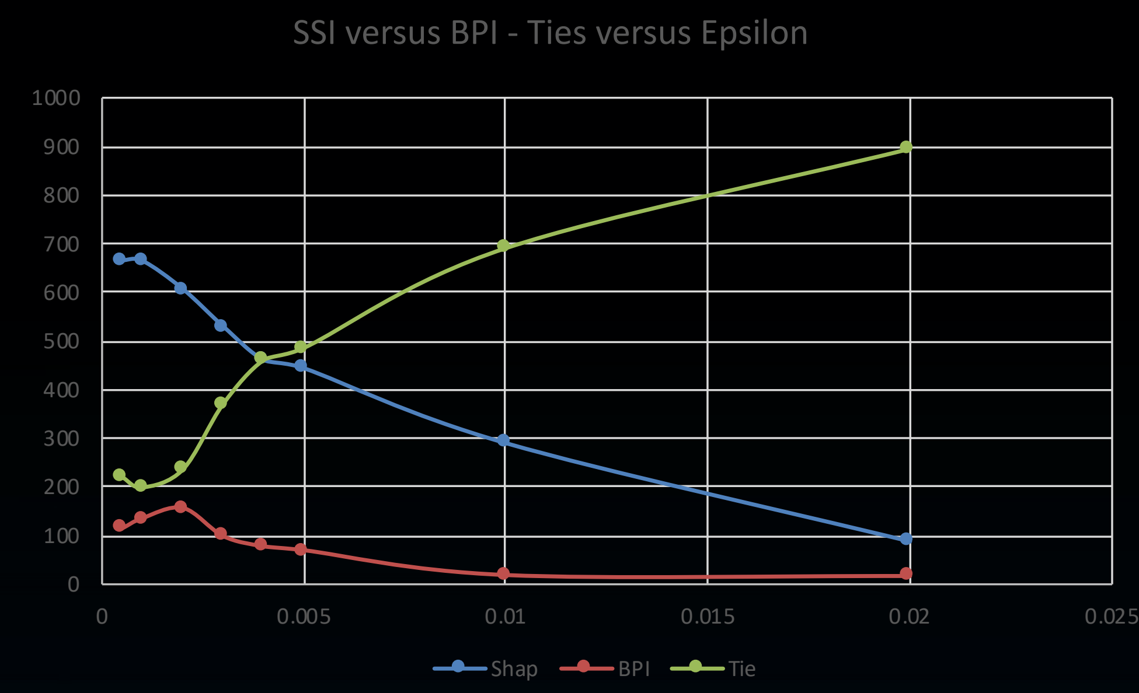
SSPI is a more efficient allocator than BPI over virtually all scenarios:

In losing scenarios, BPI average allocation waste is higher than SSPI.



Performance and Measurement Noise:

As noise increases, SSPI and BPI contests turn to ties.



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