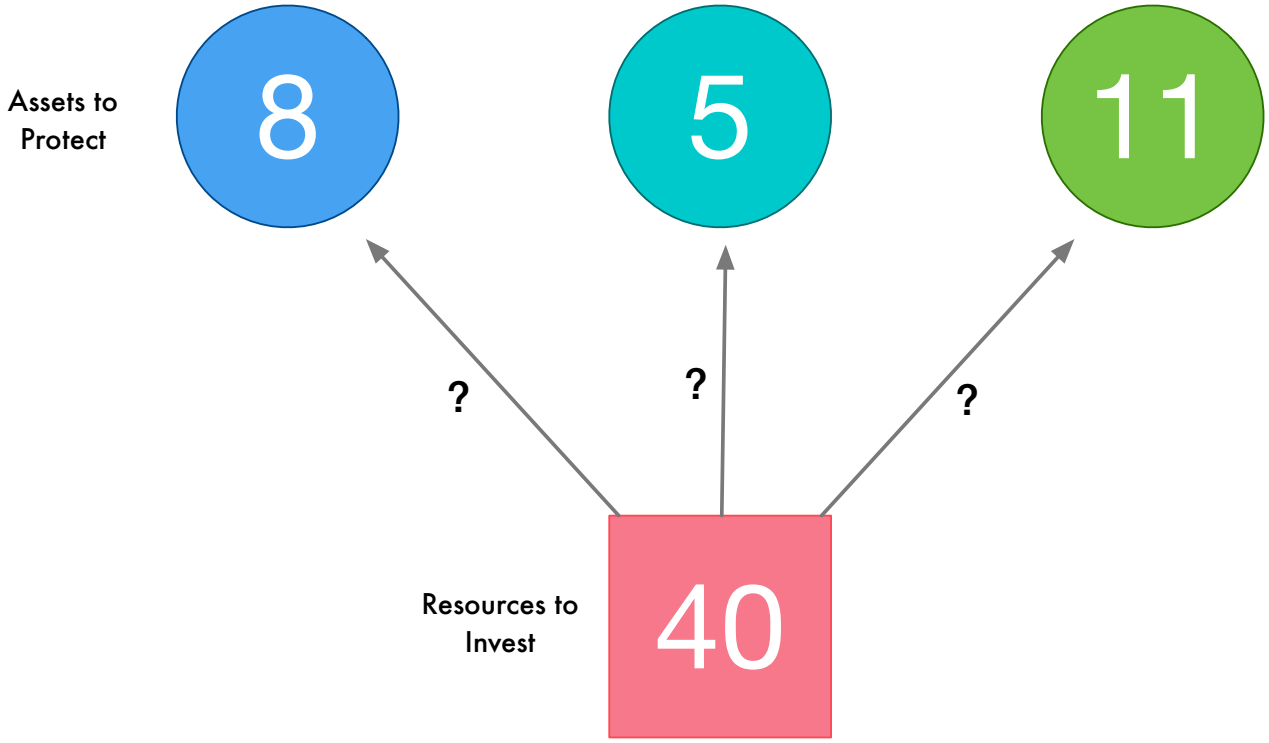
Power Indices and Security Investment Games

David Burke

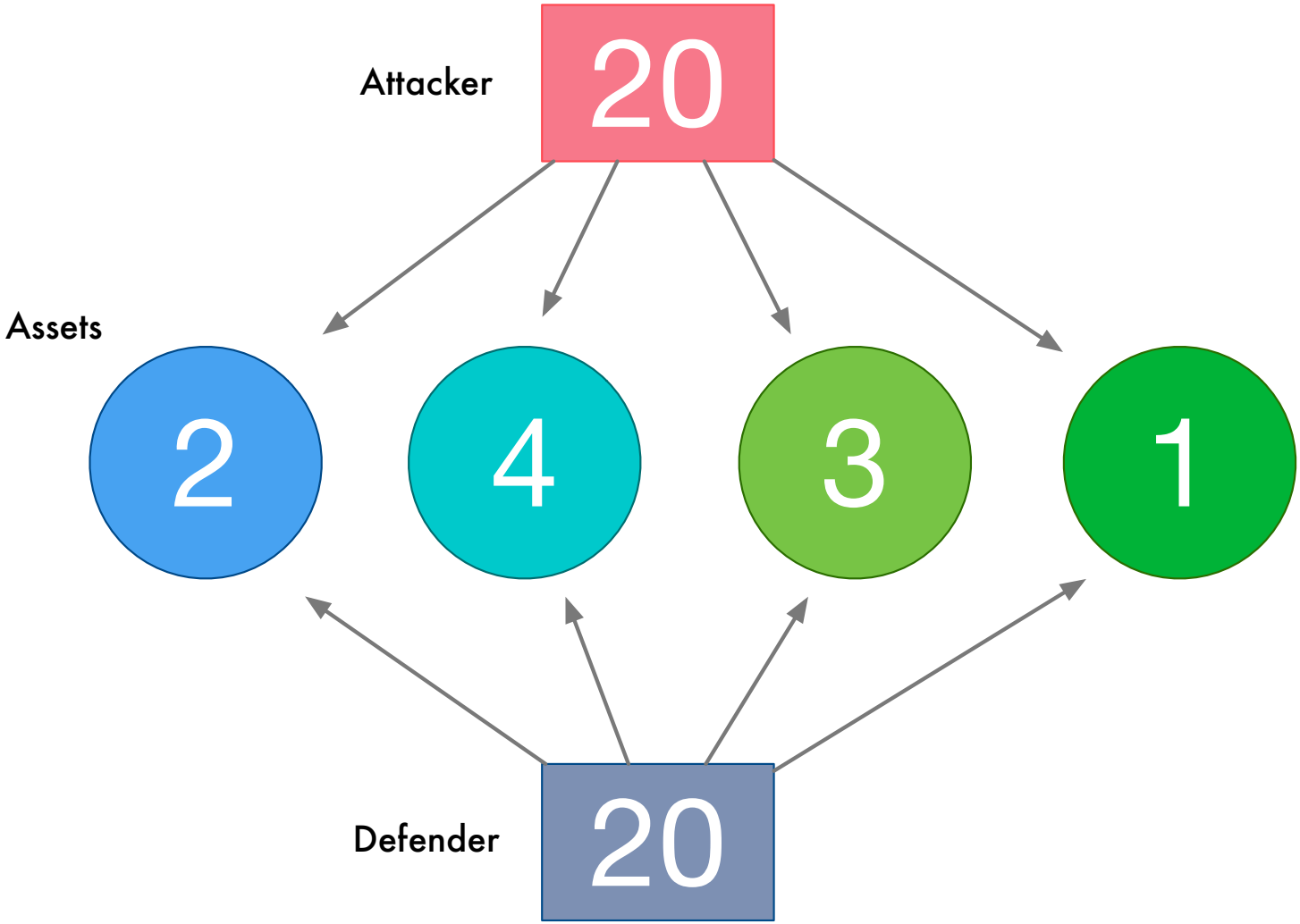
C3E 2021

Security Investment Game

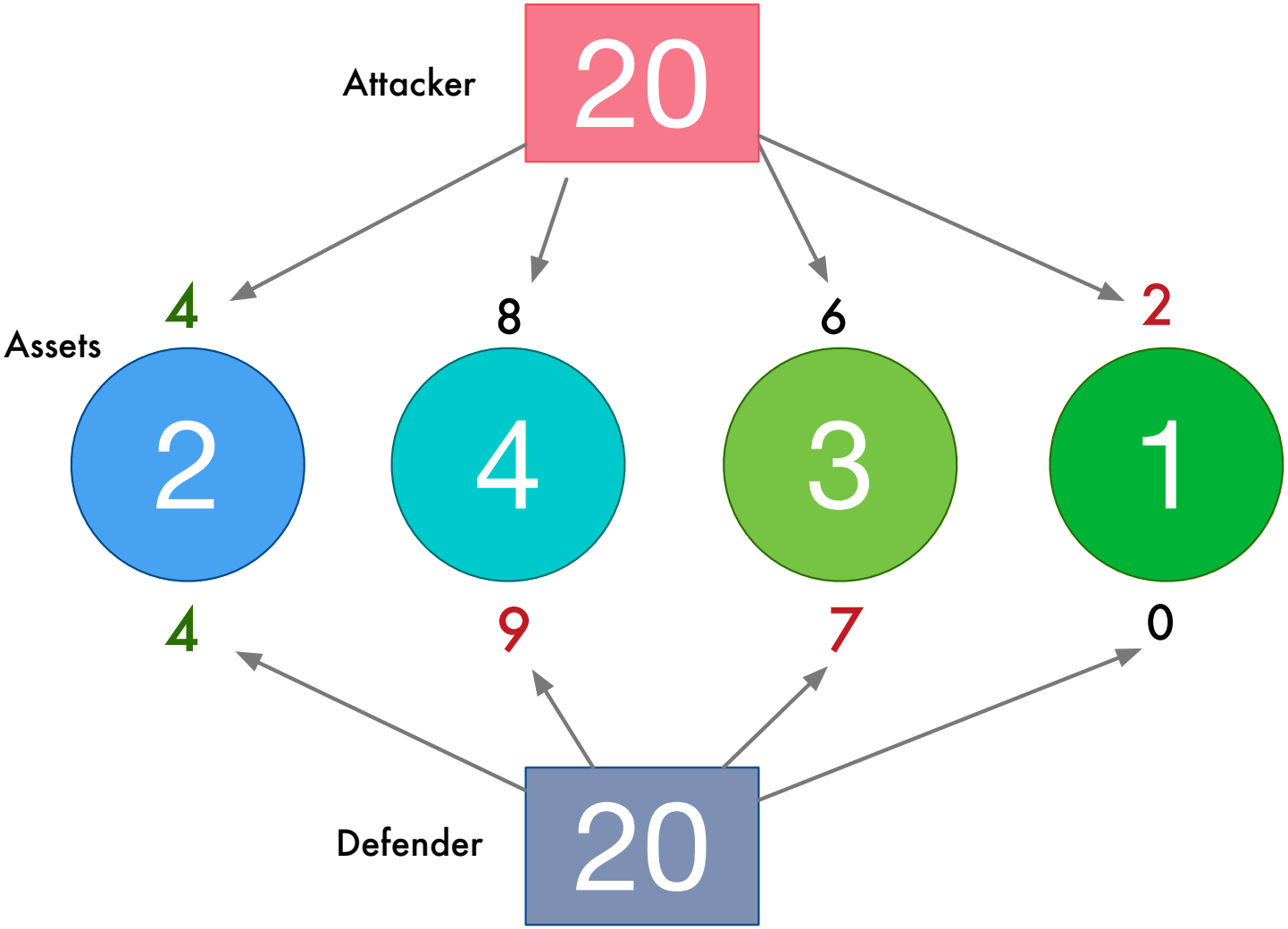


- The defender has a set of assets to protect. These assets typically have different values to the defender.
- The defender also has finite amount of resources to invest to protect those assets.

"Colonel Blotto" Games



"Colonel Blotto" Games



Allocation Strategies

4 assets, each labeled with its value:

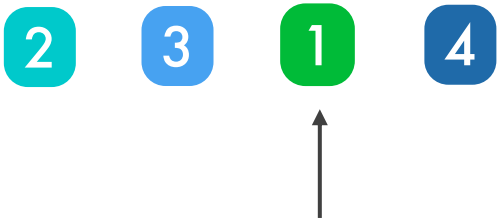
Object of the game: win the majority of assets (i.e., 6 or more out of 10)

	Asset values: 4	3	2	1
Random	.24	.03	.12	.61
Linear	.4	.3	.2	.1
Shapley-Shubik	.417	.25	.25	.083

Shapley-Shubik Power Index

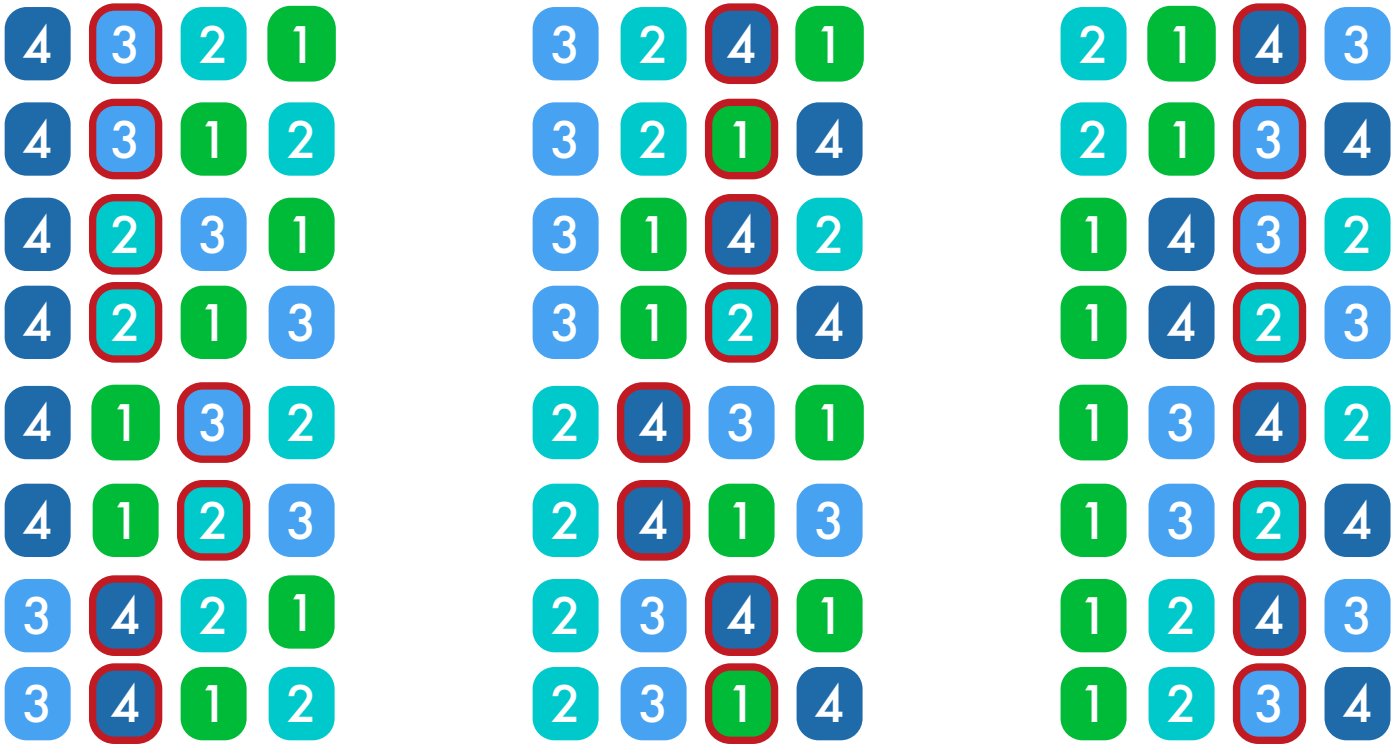
- You have n assets; each asset has a corresponding value or weight w_n
- Construct the n! possible orderings of weight sequences.
- For each sequence, identify the pivotal weight – the one that puts the sequence over a given threshold.
- The resulting Shapley value of an asset is the number of times that asset is pivotal, divided by n!

Example: 4 assets; threshold of 6 (majority of the 10 total)



In this particular ordering, the 1 is pivotal

Full Shapley-Shubik Example



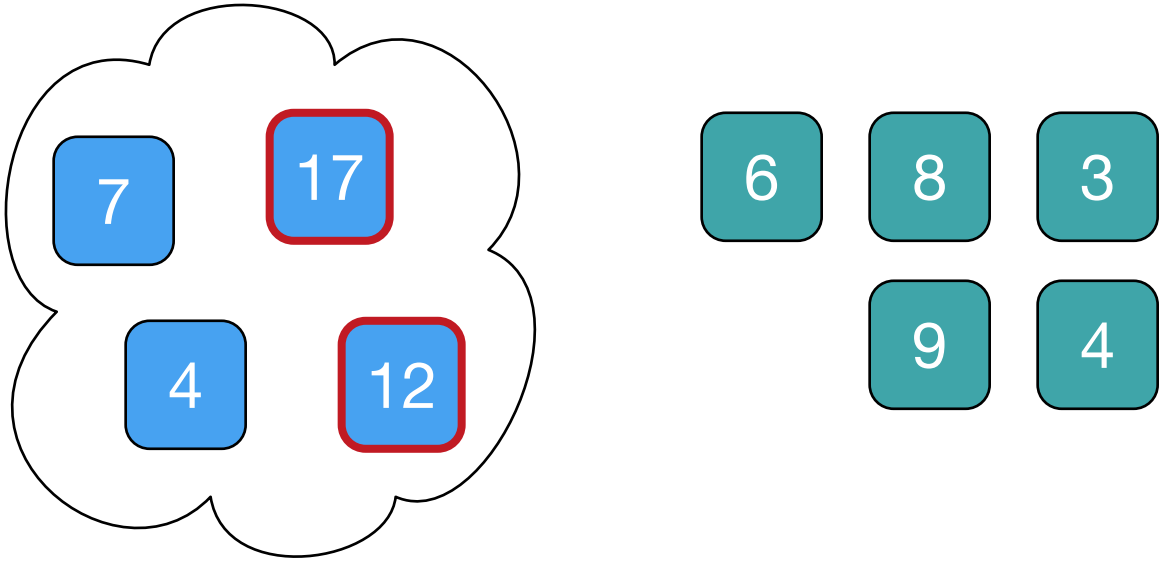
4 = 10 / 24 3 = 6 / 24 2 = 6 / 24 1 = 2 / 24

Banzhaf Power Index (BPI)

Total = 70
Threshold = 36

Construct all winning coalitions. For each winning coalition, identify the critical assets – the ones that, if they were to defect from the coalition, turn it from winning to losing.

An example:



Coalition total = 40, so 17 and 12 are critical – if either defect, we're under the threshold of 36.

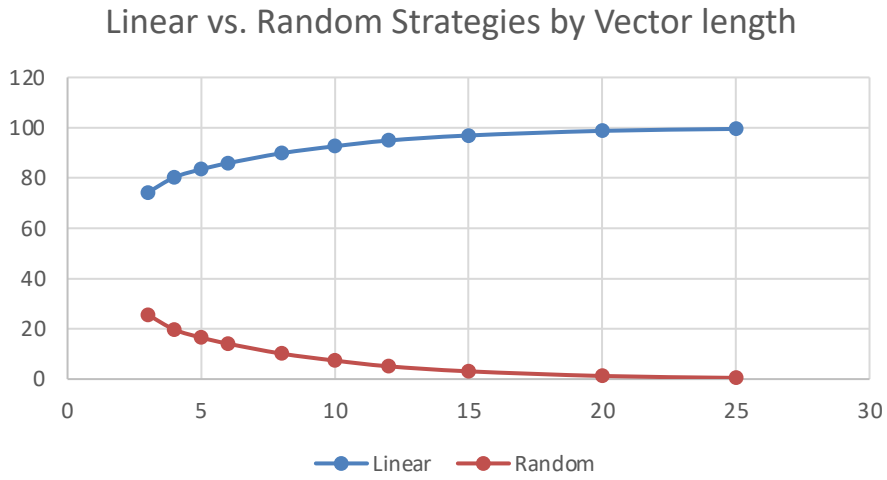
An asset's BPI is the percentage of cases it is critical after considering all winning coalitions.

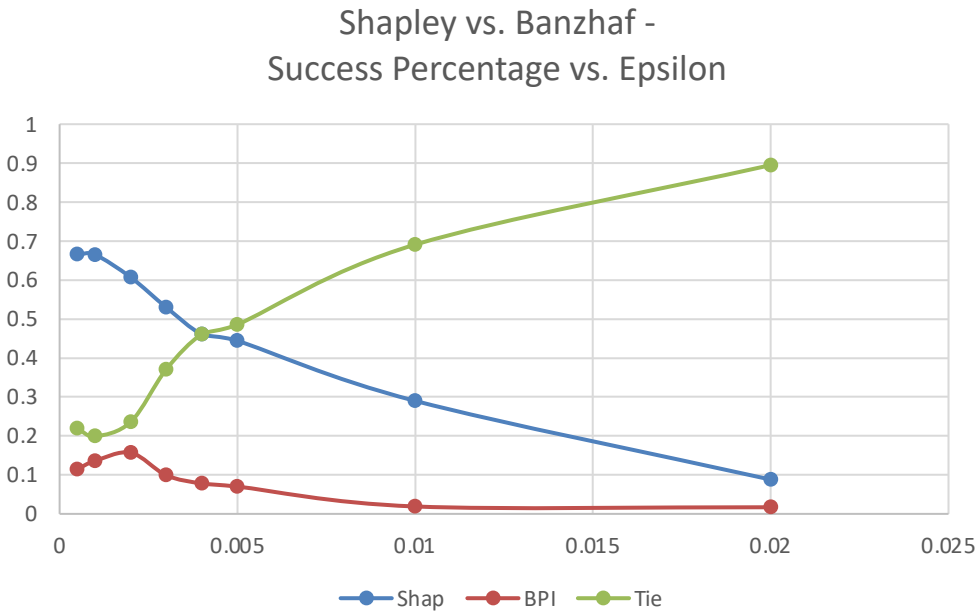
Run a Tournament!

- Start with four strategies:
  - Random
  - Linear
  - Shapley-Shubik
  - Banzhaf
- Vary key parameters:
  - Number of assets under contention
  - Range of possible asset values
  - ”Epsilon” value to capture the concept of noise in the system
- Key challenges:
  - Shapley-Shubik and Banzhaf are exponential algorithms – what if asset values are inexact?
  - Tension between performance and efficiency (are some strategies more likely to waste resources, whether they win or lose?)

Example Results

No surprises: As the vector length (number of assets) increases, the linear strategy is a much better than the random one.





Shapley generally beats Banzhaf.

As epsilon (a measure of system noise) increases, Shapley vs. Banzhaf tends to produce ties.

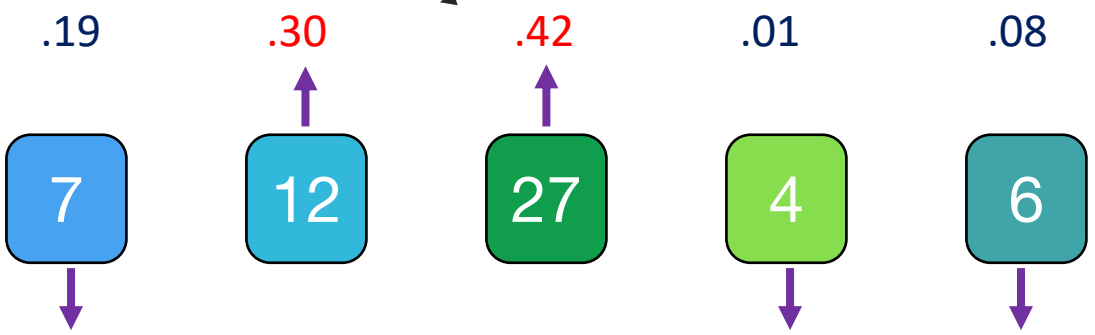
Shapley is superior in its usage of resources – much less waste.

Questions? Comments?

davidb@galois.com

Efficiency Example

Player 1 wins with 39 out of 56



						Totals
Player 1 Allocation	.19	.30	.42	.01	.08	
Player 2 Allocation	.21	.25	.24	.14	.16	
Player 1 Waste	.19	.05	.18	.01	.08	.51
Player 2 Waste	.02	.25	.24	.13	.08	.72