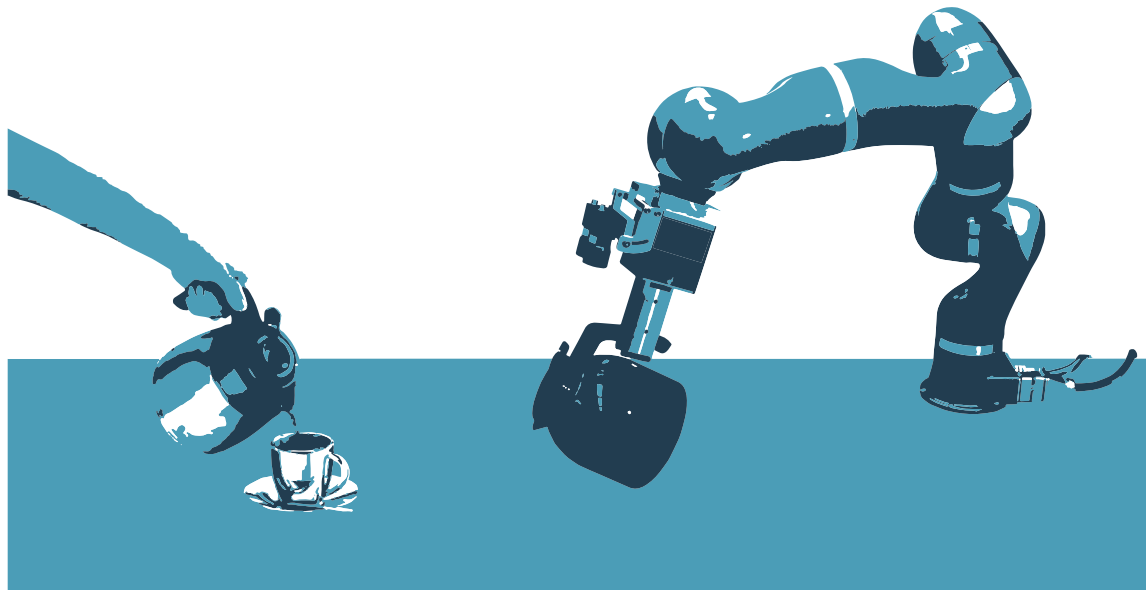


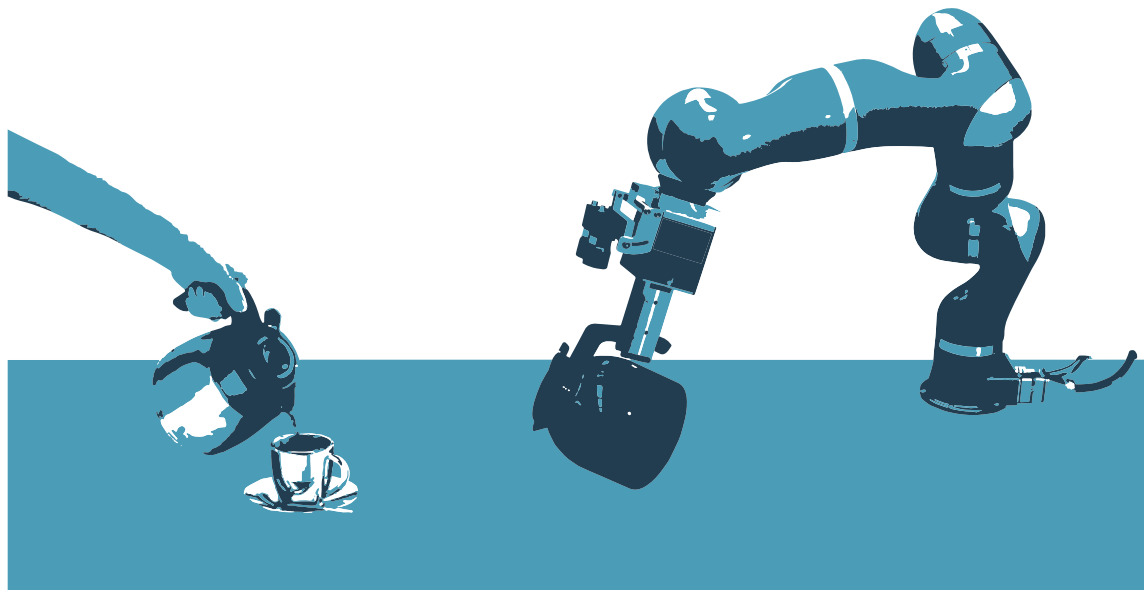
# Safety-constrained Reinforcement Learning for MDPs



**HCSS 2016**

**Nils Jansen**

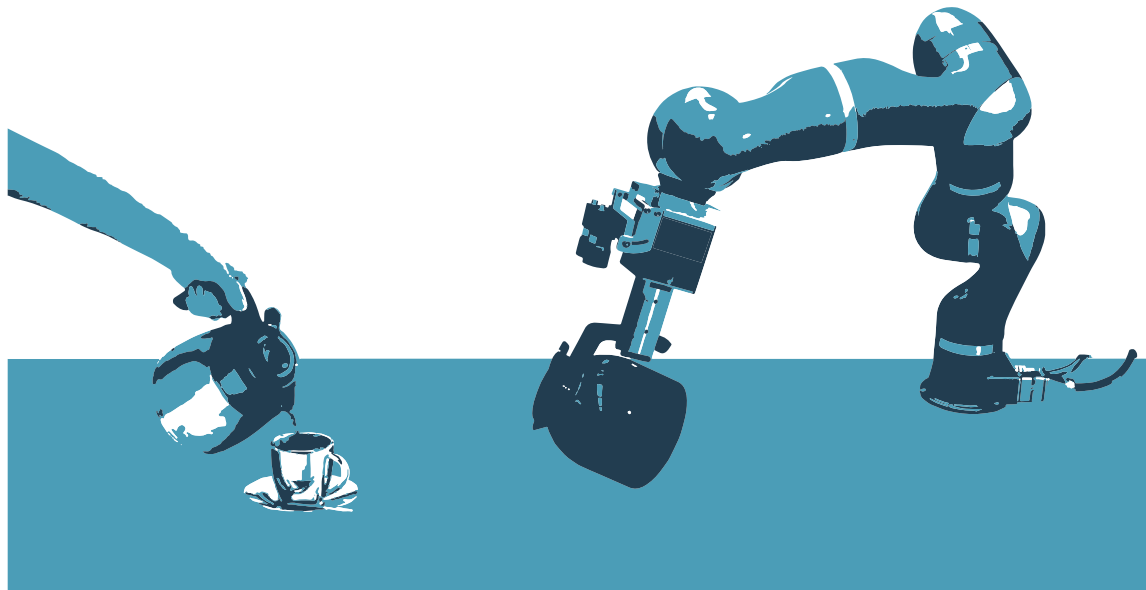
# Some results on Controller Synthesis for Probabilistic Systems



**HCSS 2016**

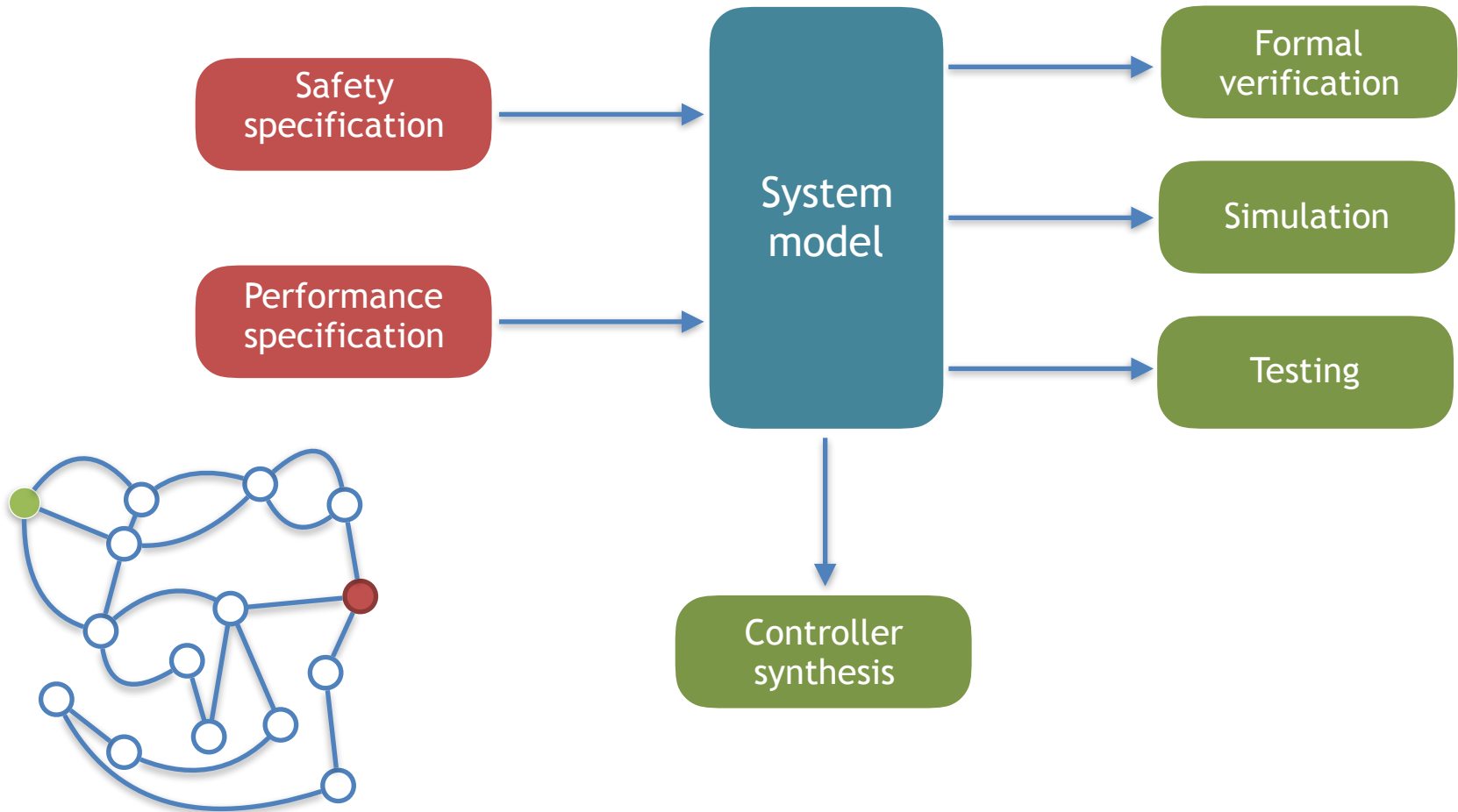
**Nils Jansen**

# Safety-constrained Reinforcement Learning for MDPs

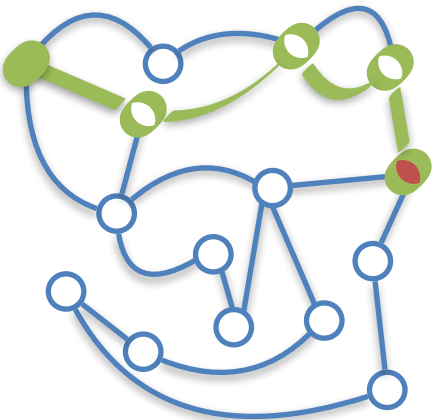
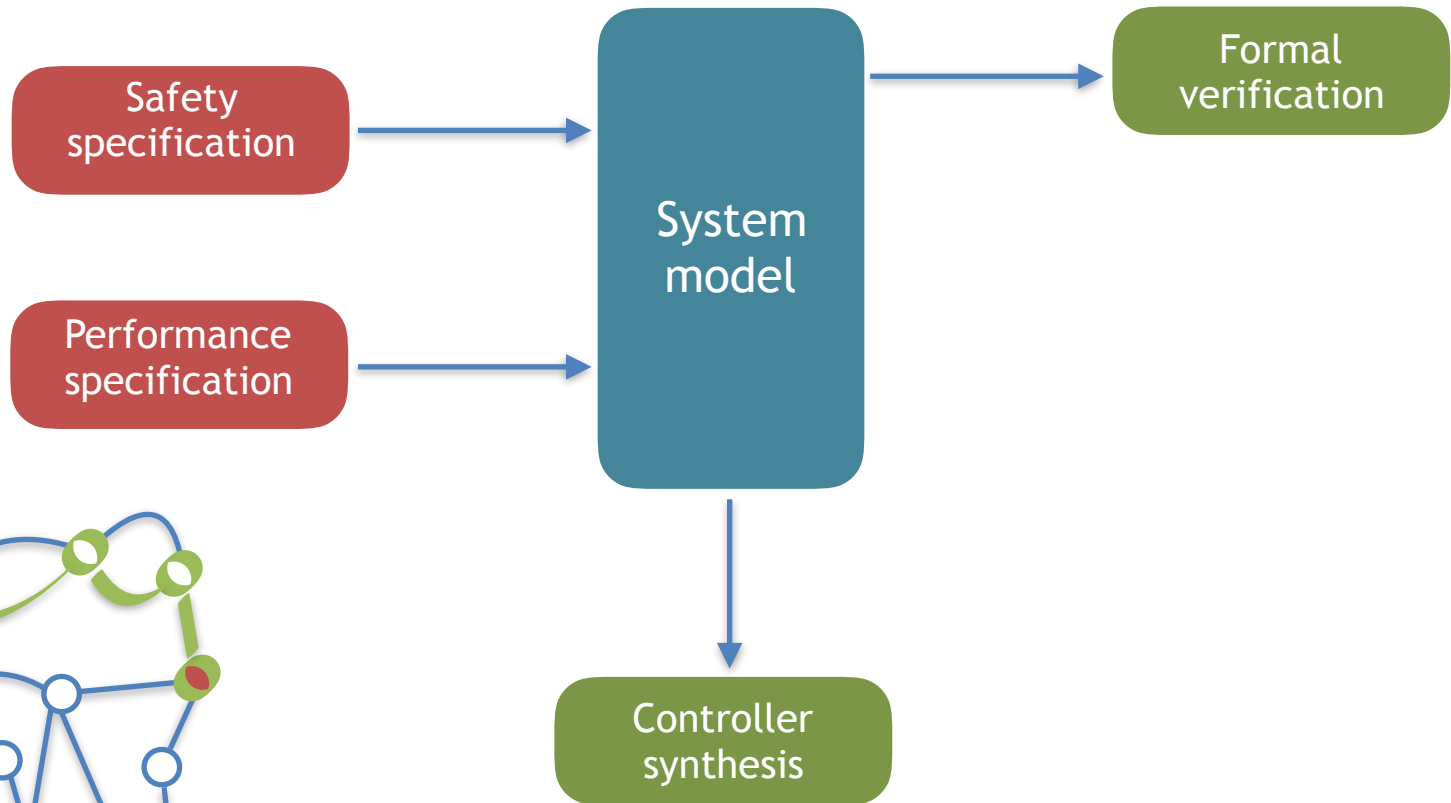


**Sebastian Junges, Nils Jansen, Christian Dehnert,  
Ufuk Topcu, and Joost-Pieter Katoen**

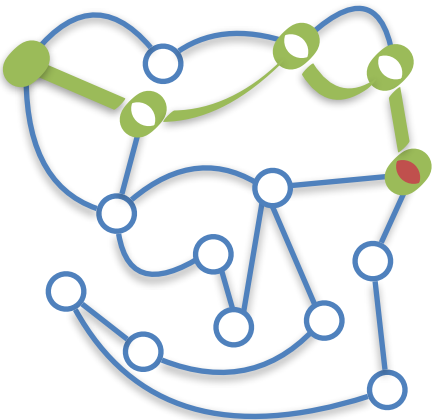
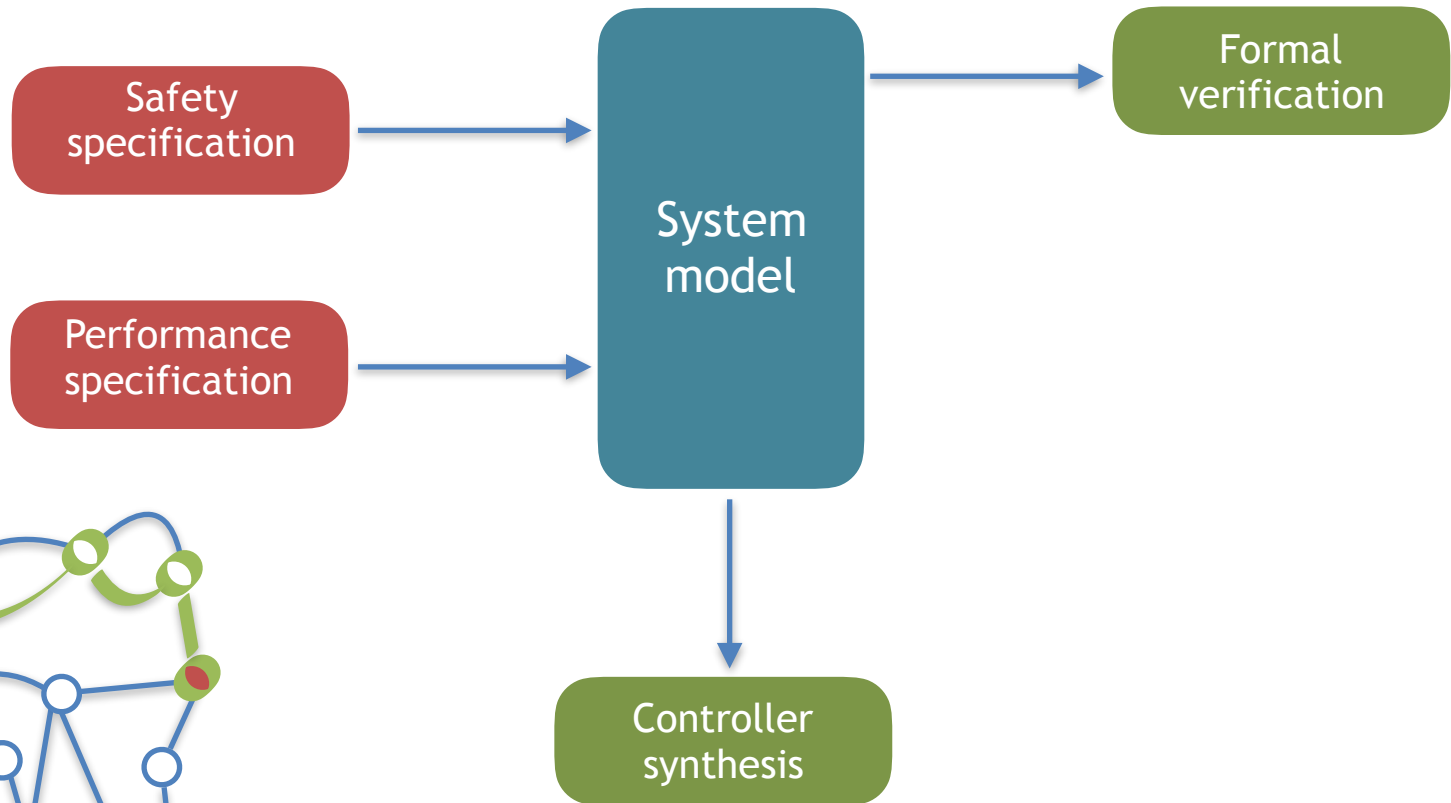
# Motivation - Safety-critical Systems



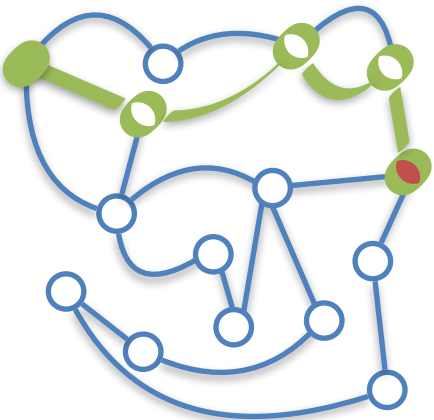
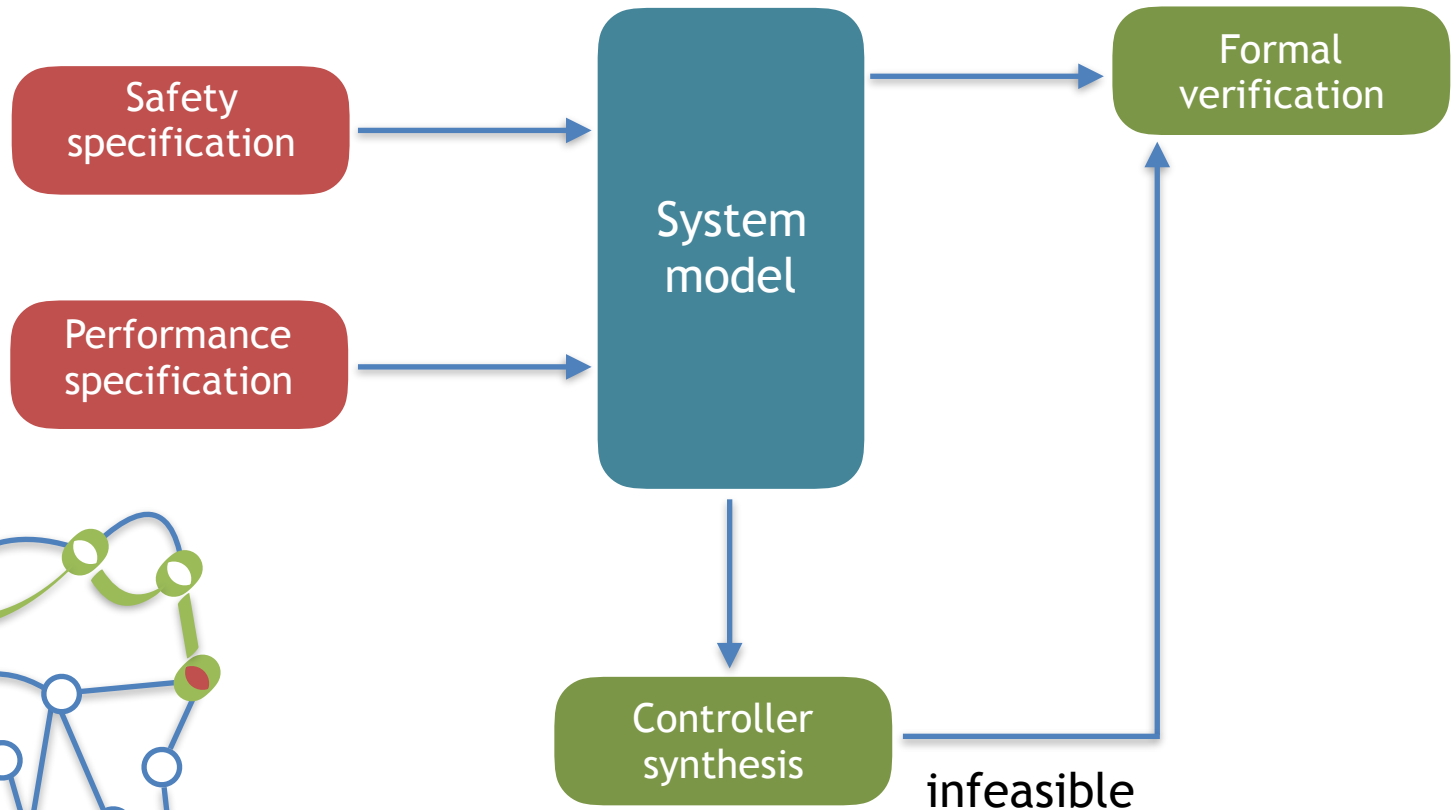
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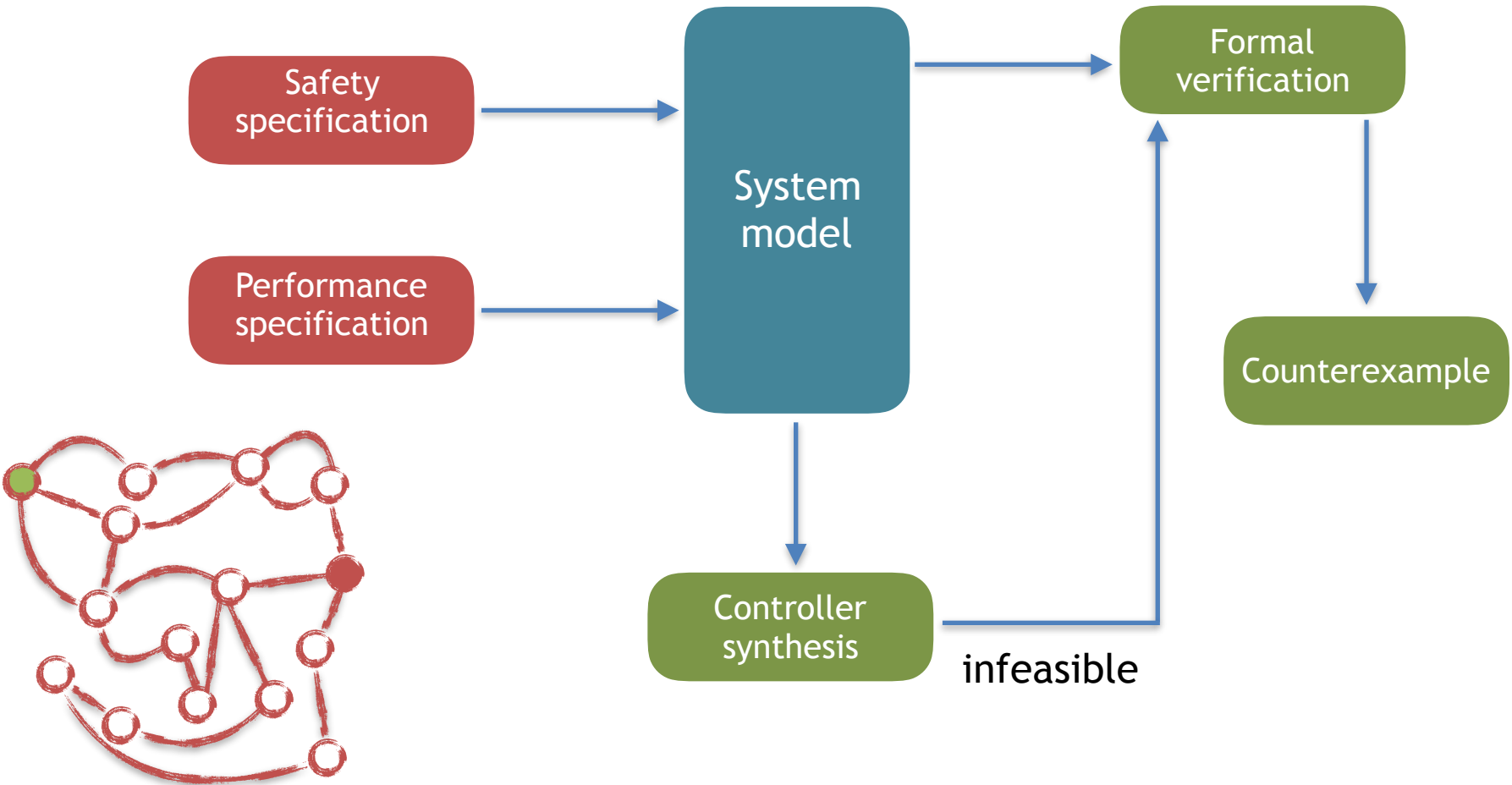
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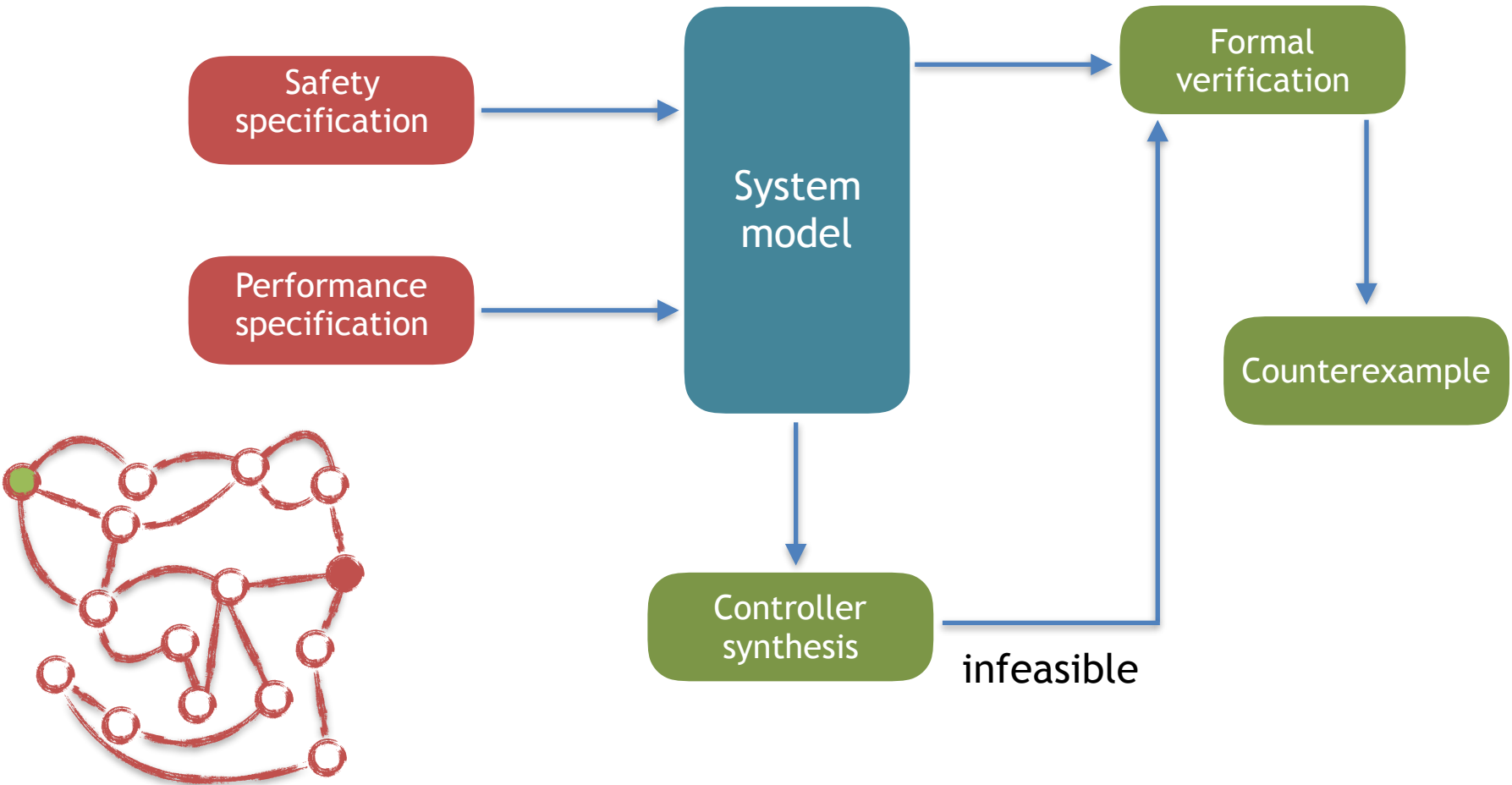


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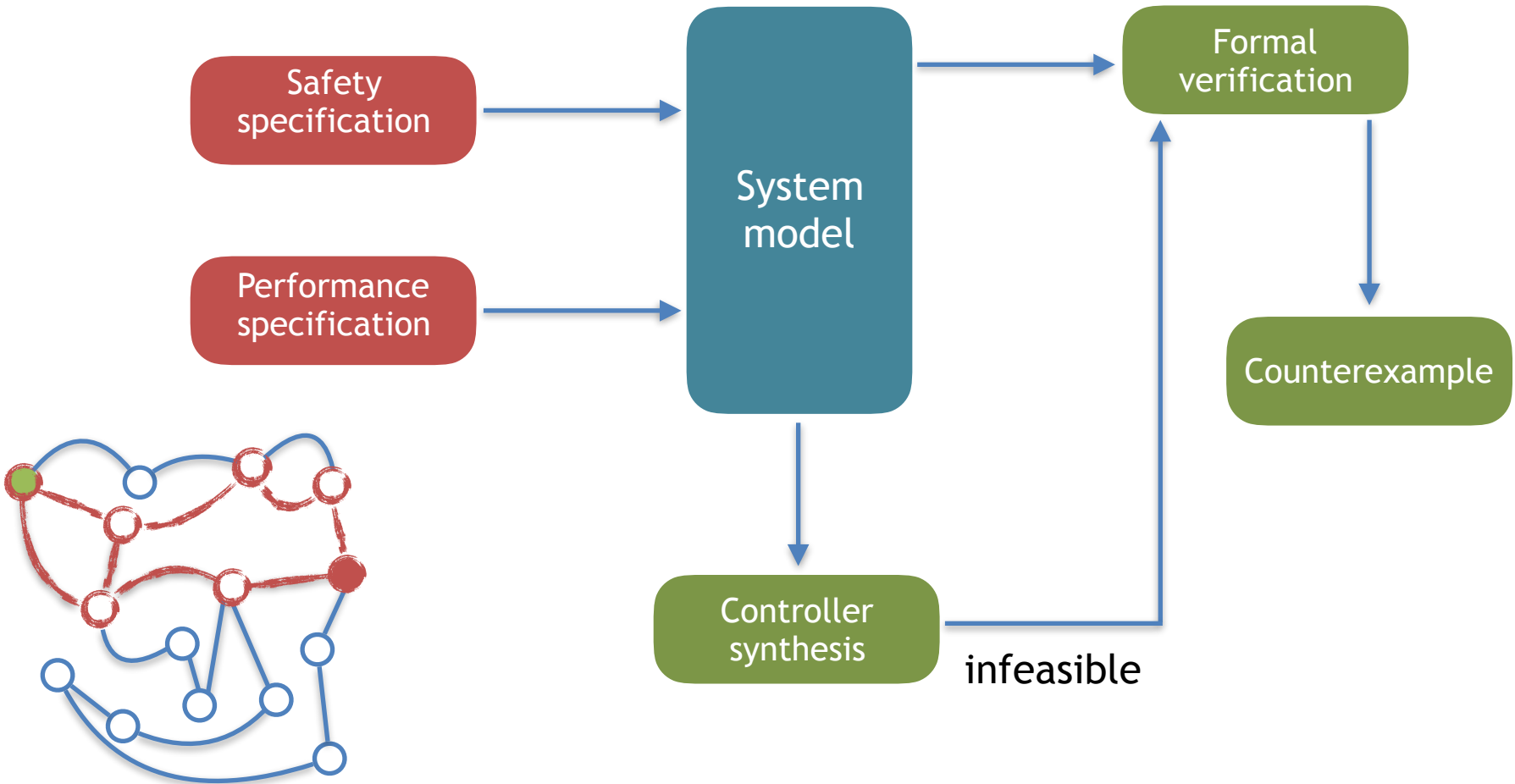




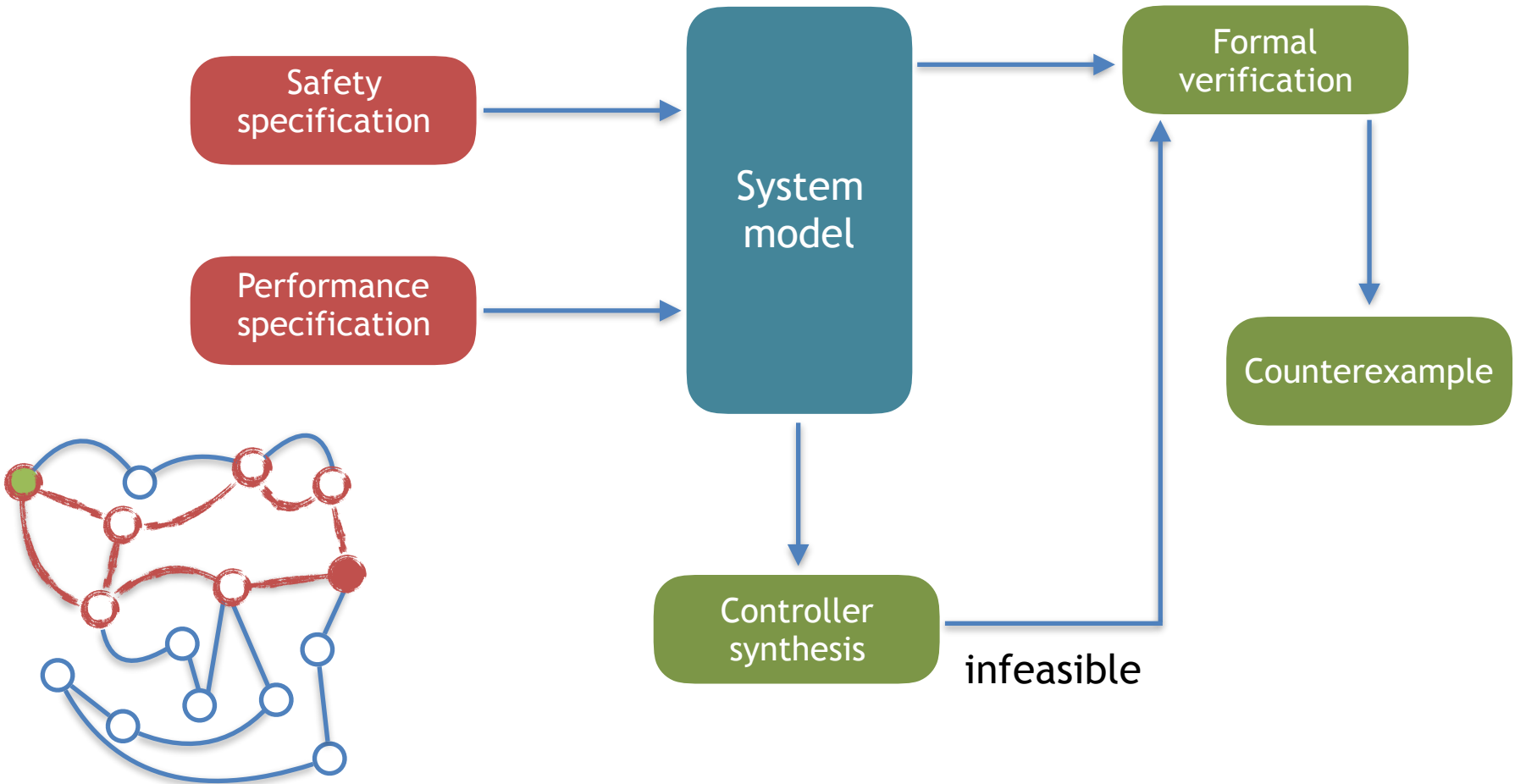
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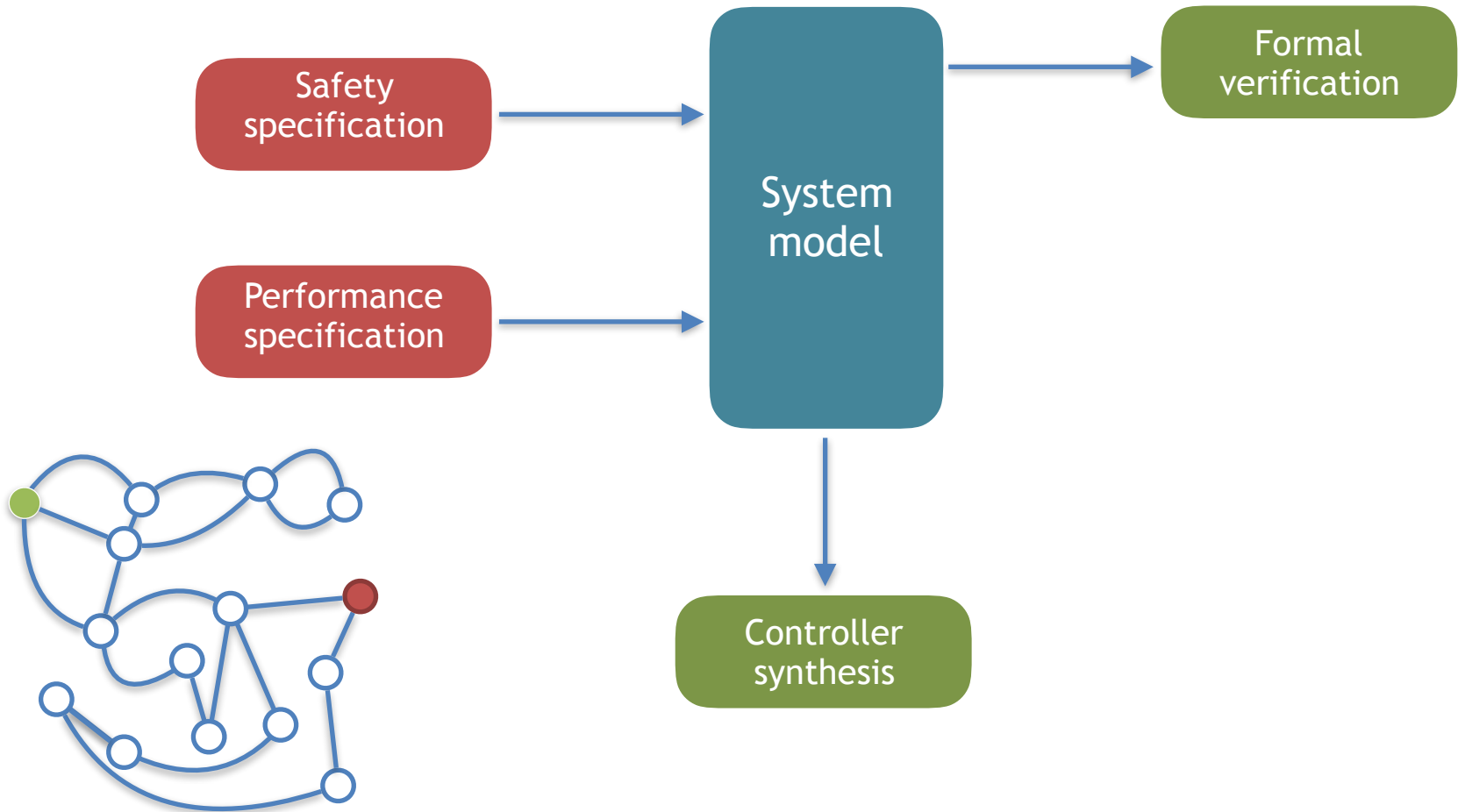
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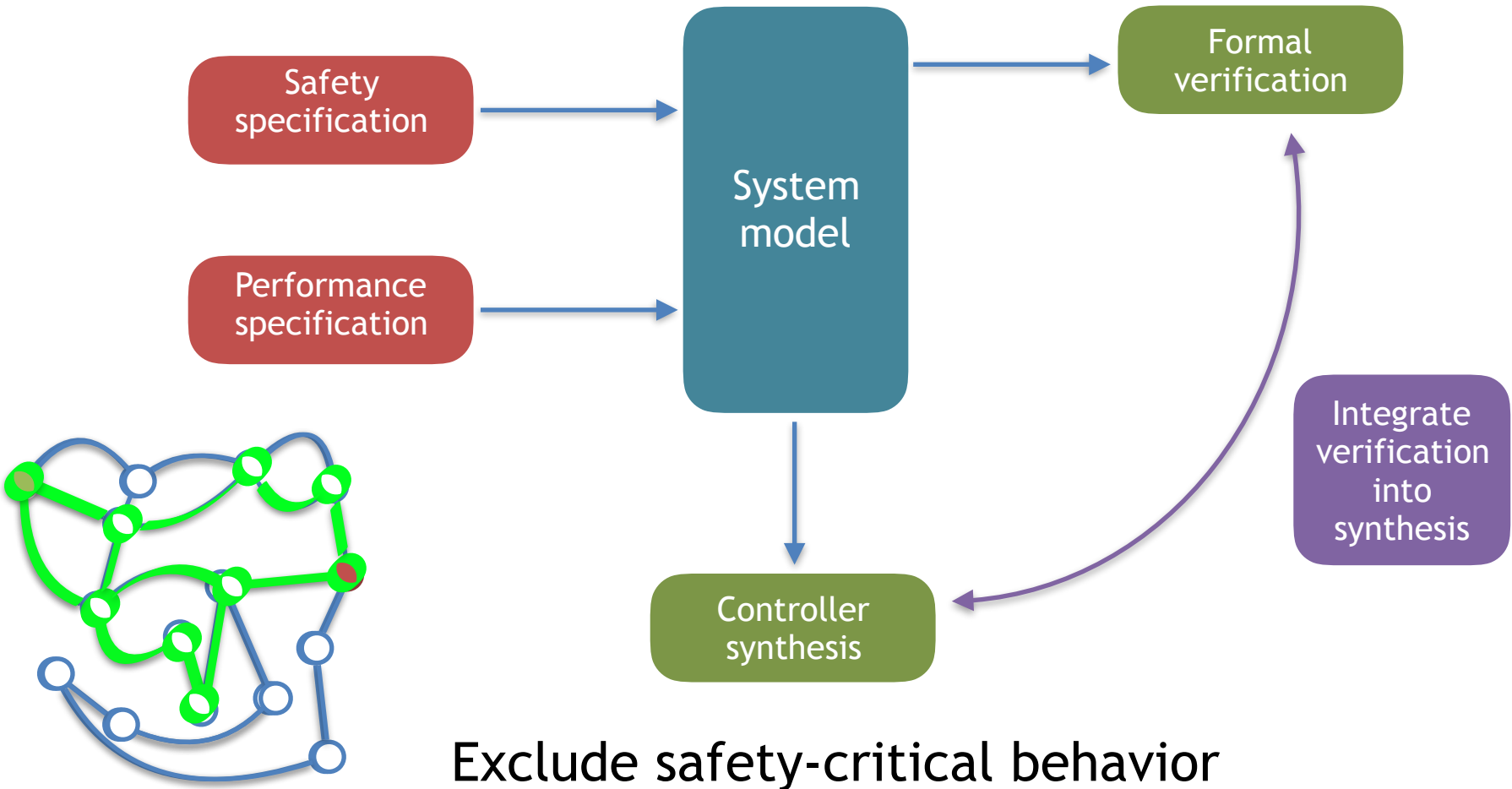
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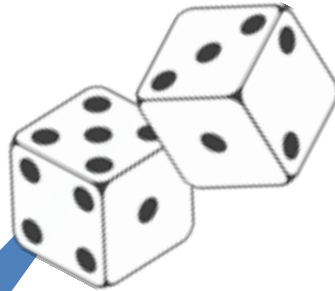
# Motivation - Safety-critical Systems



# Randomization



# Randomization



## Algorithms speed-up

- Probabilistic quicksort
- Rabin-Miller primality test
- Verification of matrix multiplication

# Randomization



## Deterministic techniques fail - symmetry breaking

- Dining philosopher problem (Lehmann & Rabin '81)
- Leader election (Angluin '80)
- Ethernet's randomized exponential backoff (IEEE 802.3)

## Algorithms speed-up

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# Randomization

Probabilistic programs are simple and intuitive to understand

```
repeat  
  c := coin_flip(0.5)  
until (c=heads)
```



Deterministic techniques fail - symmetry breaking

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Algorithms speed-up

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cryptography

biology

communication

Applications in several domains

computer vision

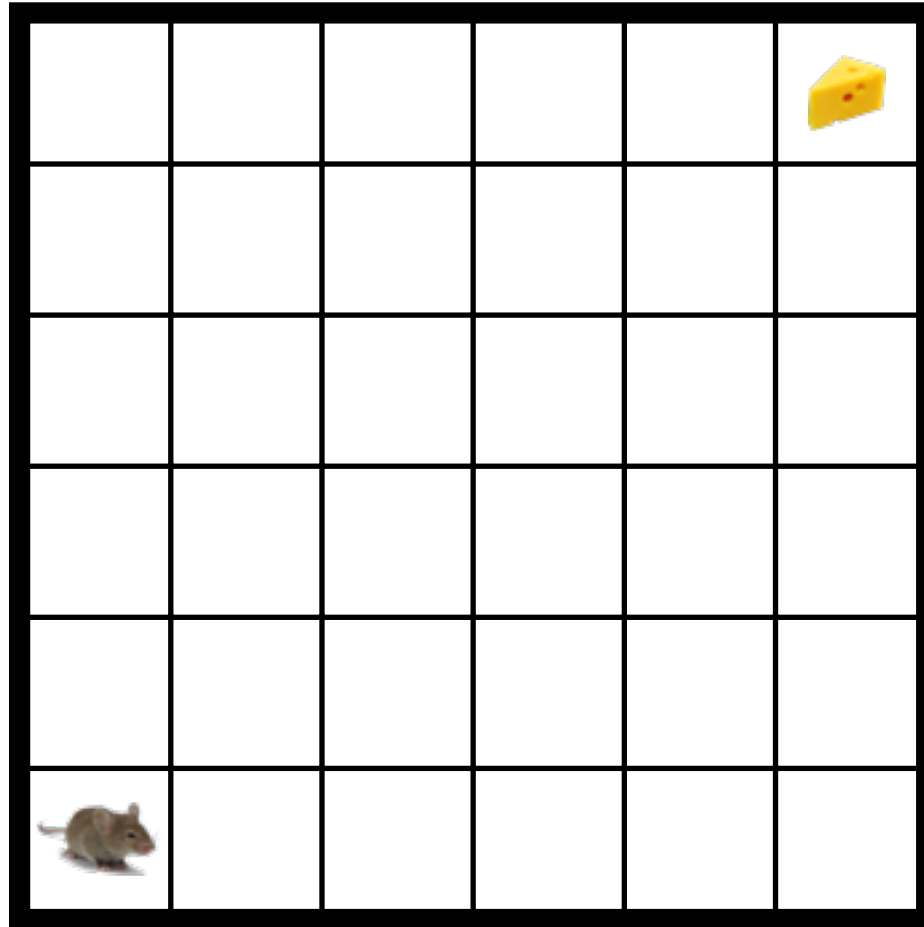
robotics

data management

optimization

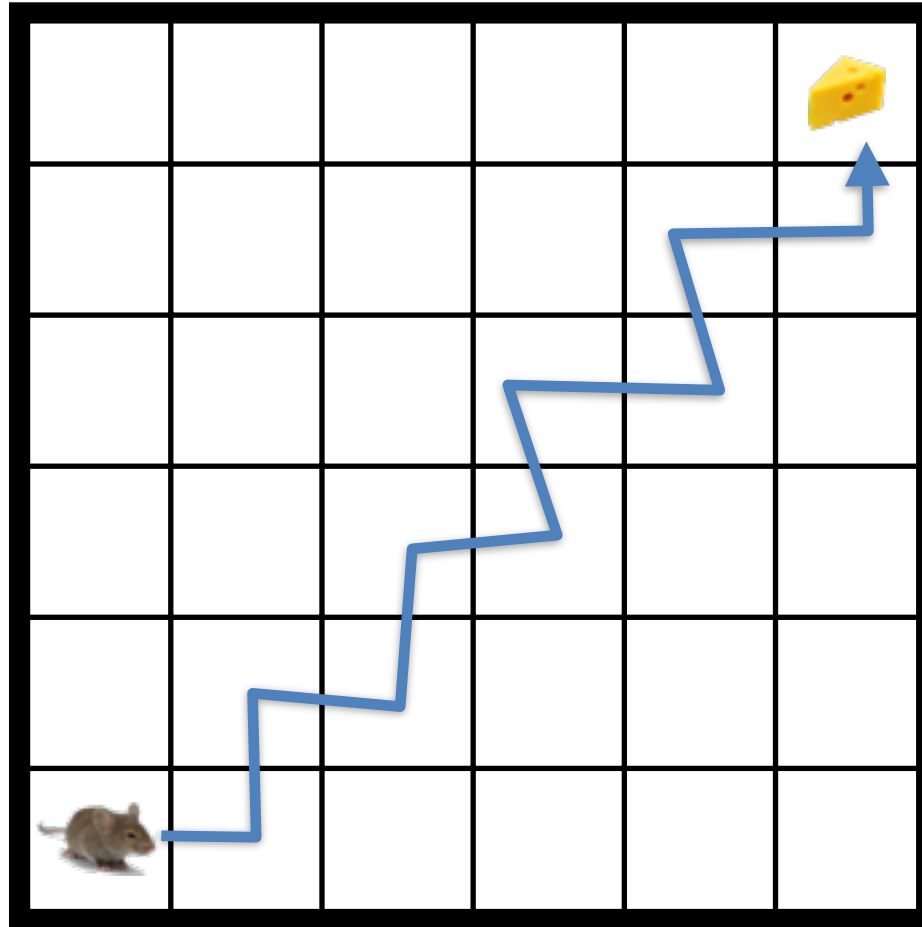
# Help the Mouse

Find the best way to  
the cheese



# Help the Mouse

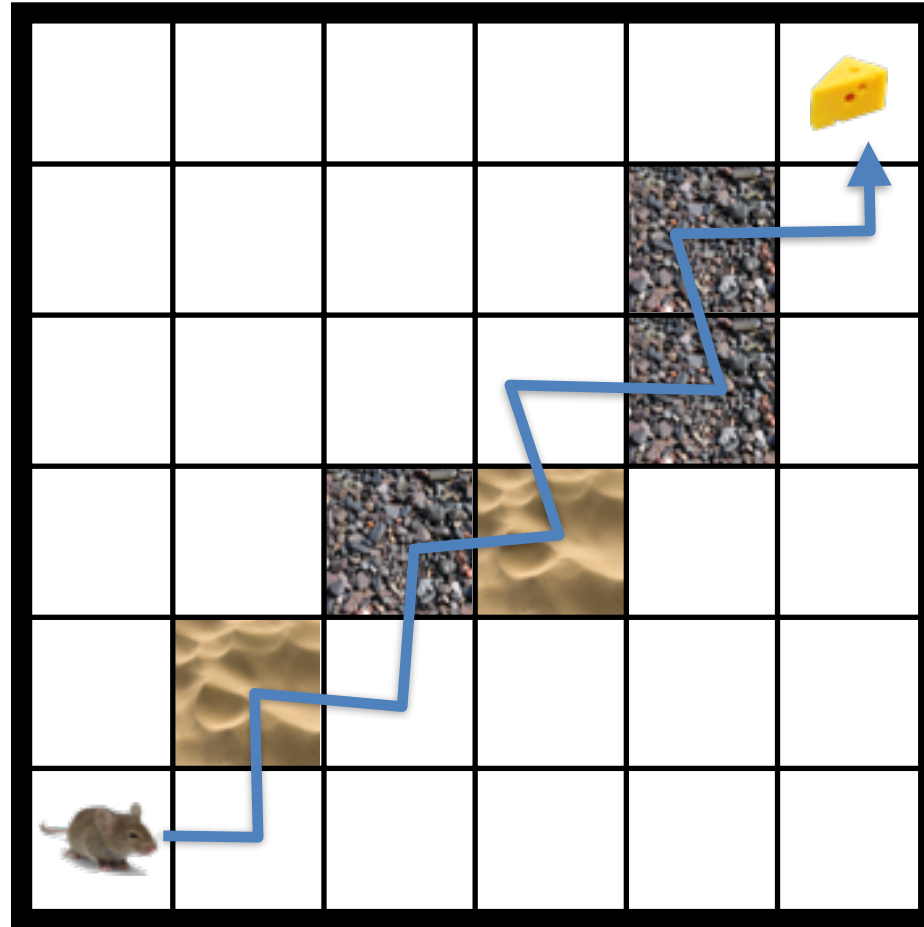
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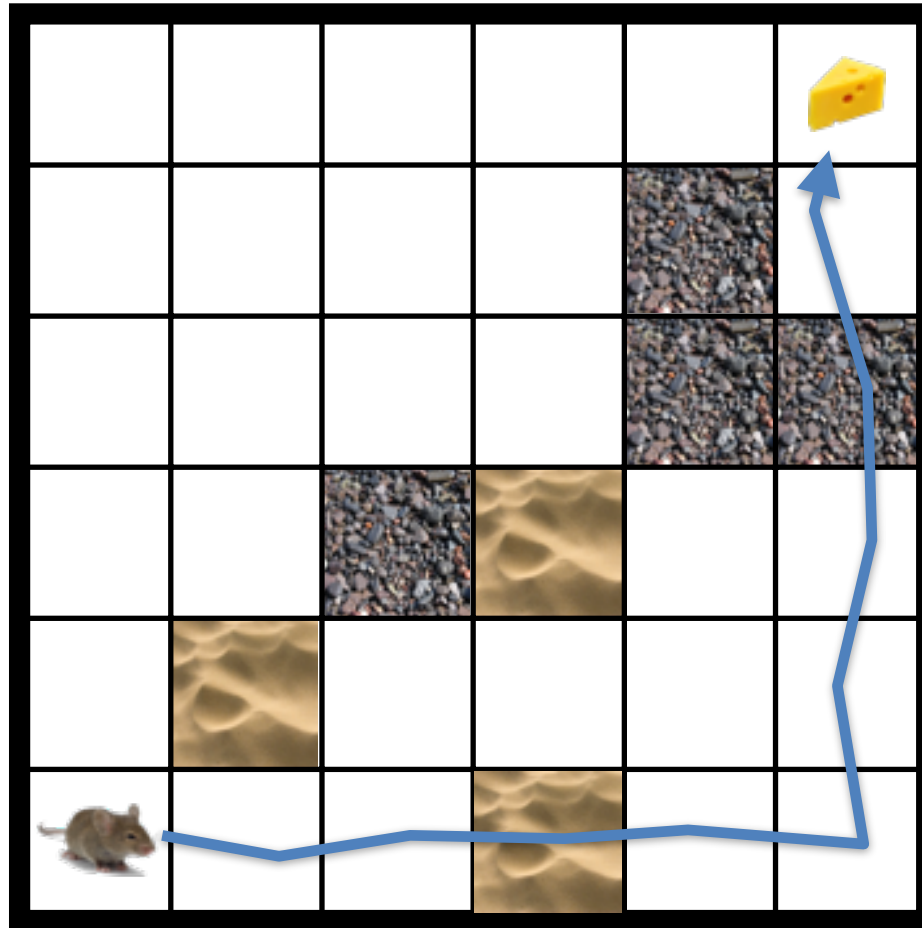
While moving mouse  
discovers exhausting  
surfaces



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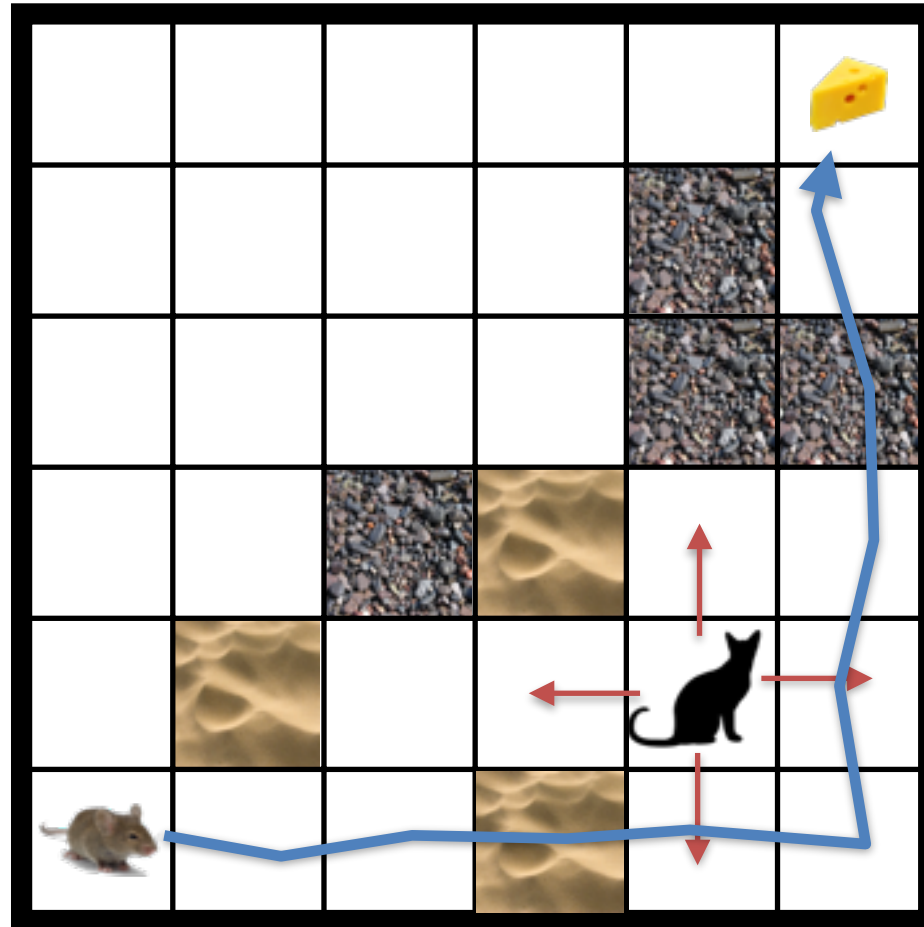


# Help the Mouse

Find the best way to  
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While moving mouse  
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Avoid randomly  
moving cat



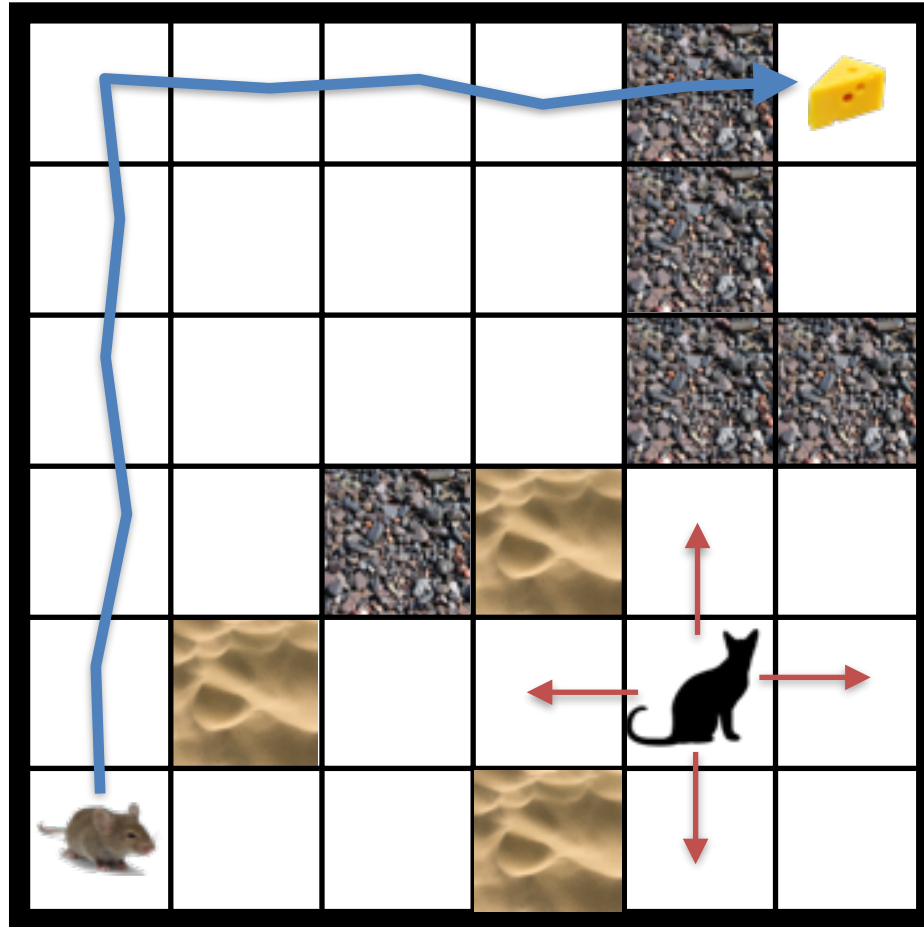
Find safe and  
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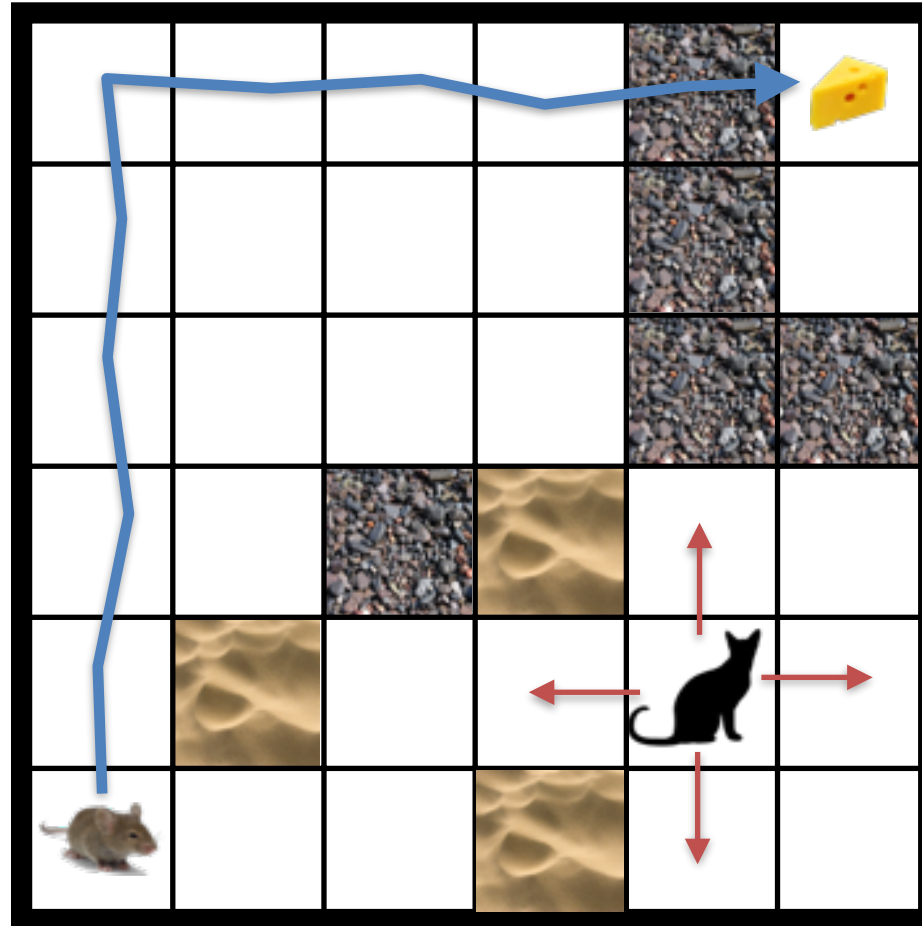


# Help the Mouse

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While moving mouse discovers exhausting surfaces

Avoid randomly moving cat



Find safe and cost-optimal strategy to get to the cheese

Cost is not known prior to exploring the grid

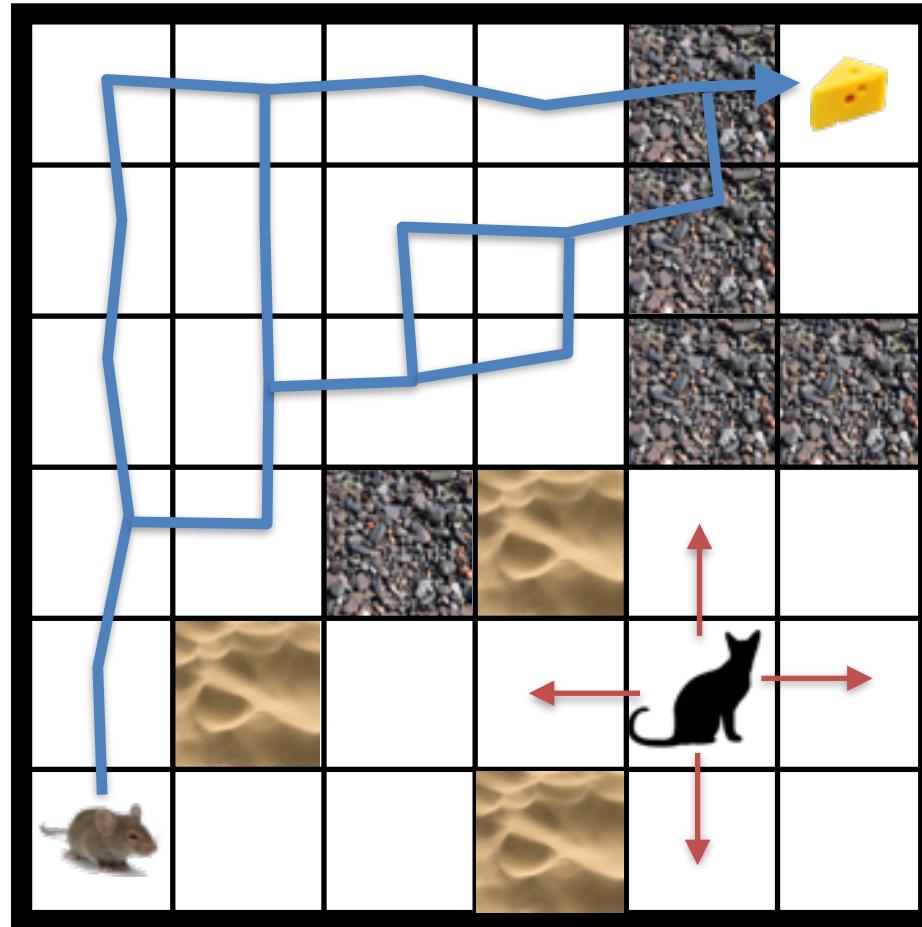
# Help the Mouse

Find the best way to the cheese

While moving mouse discovers exhausting surfaces

Avoid randomly moving cat

Try all safe ways to the cheese (for future mice)

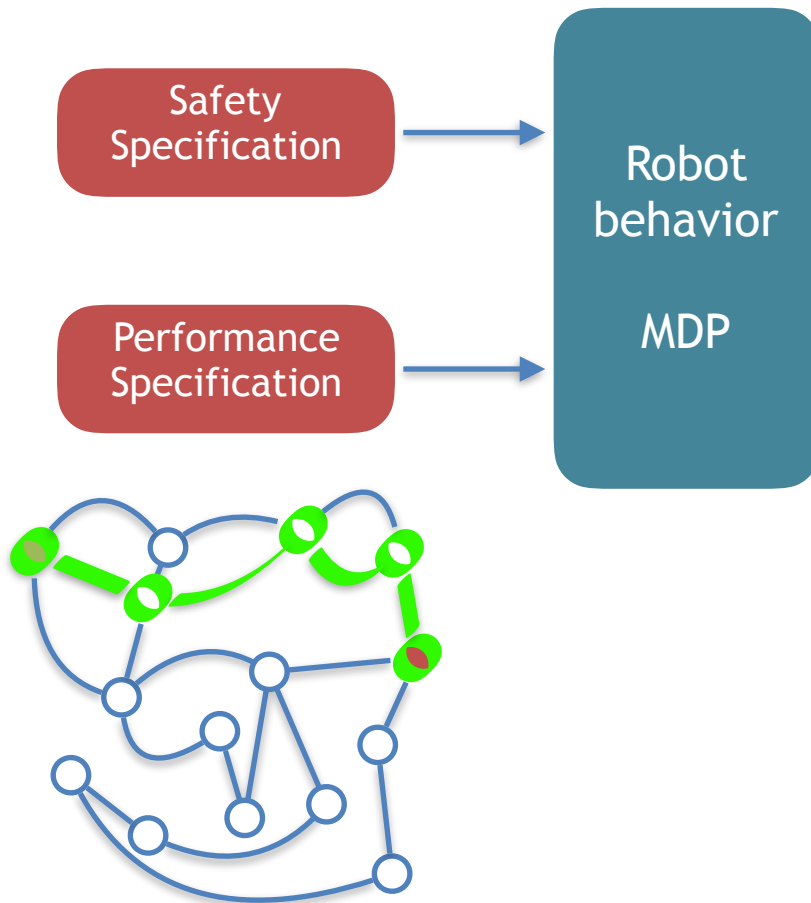


Find safe and cost-optimal strategy to get to the cheese

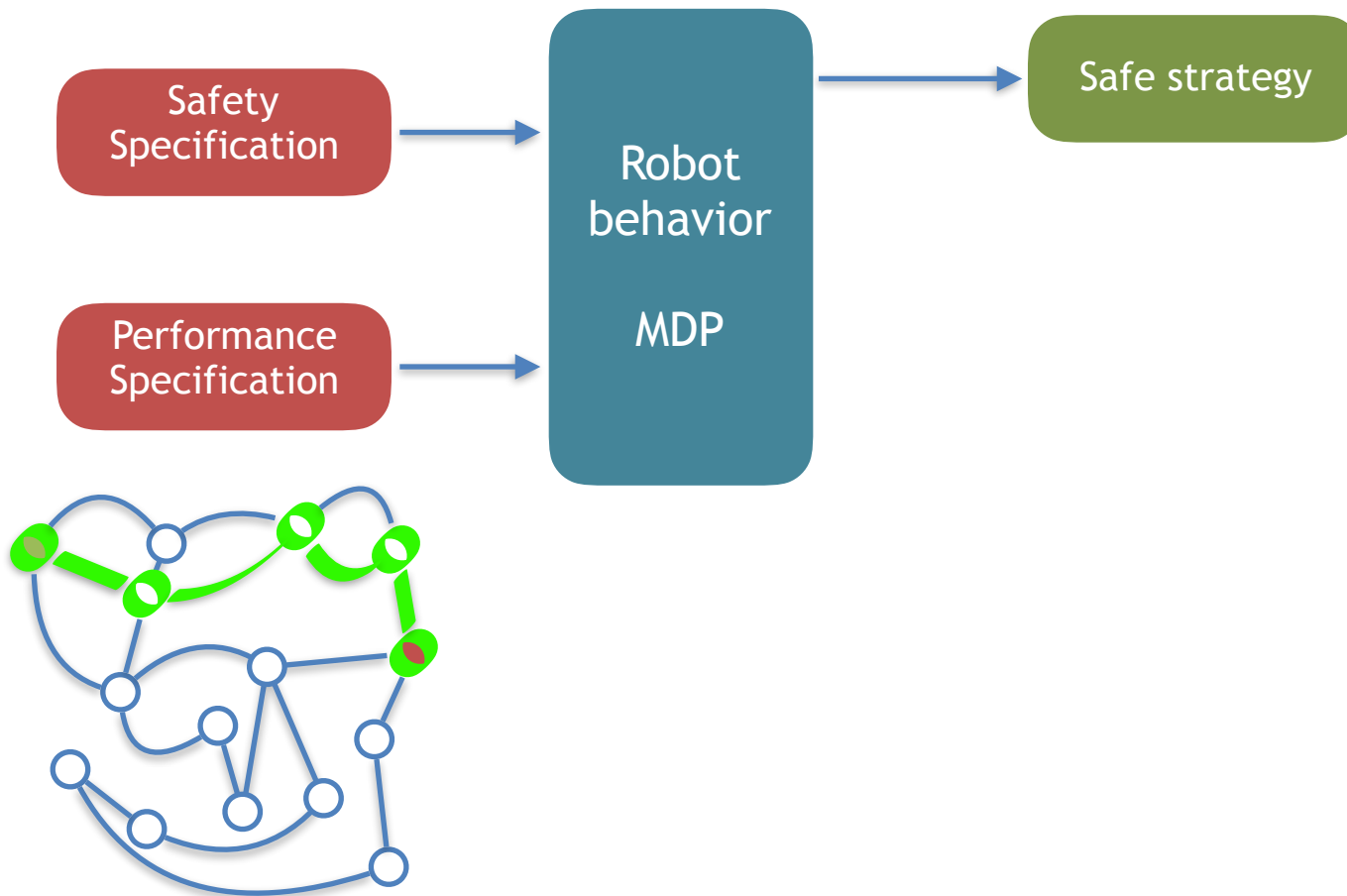
Cost is not known prior to exploring the grid

Deploy multiple strategies for safe exploration (permissive strategy)

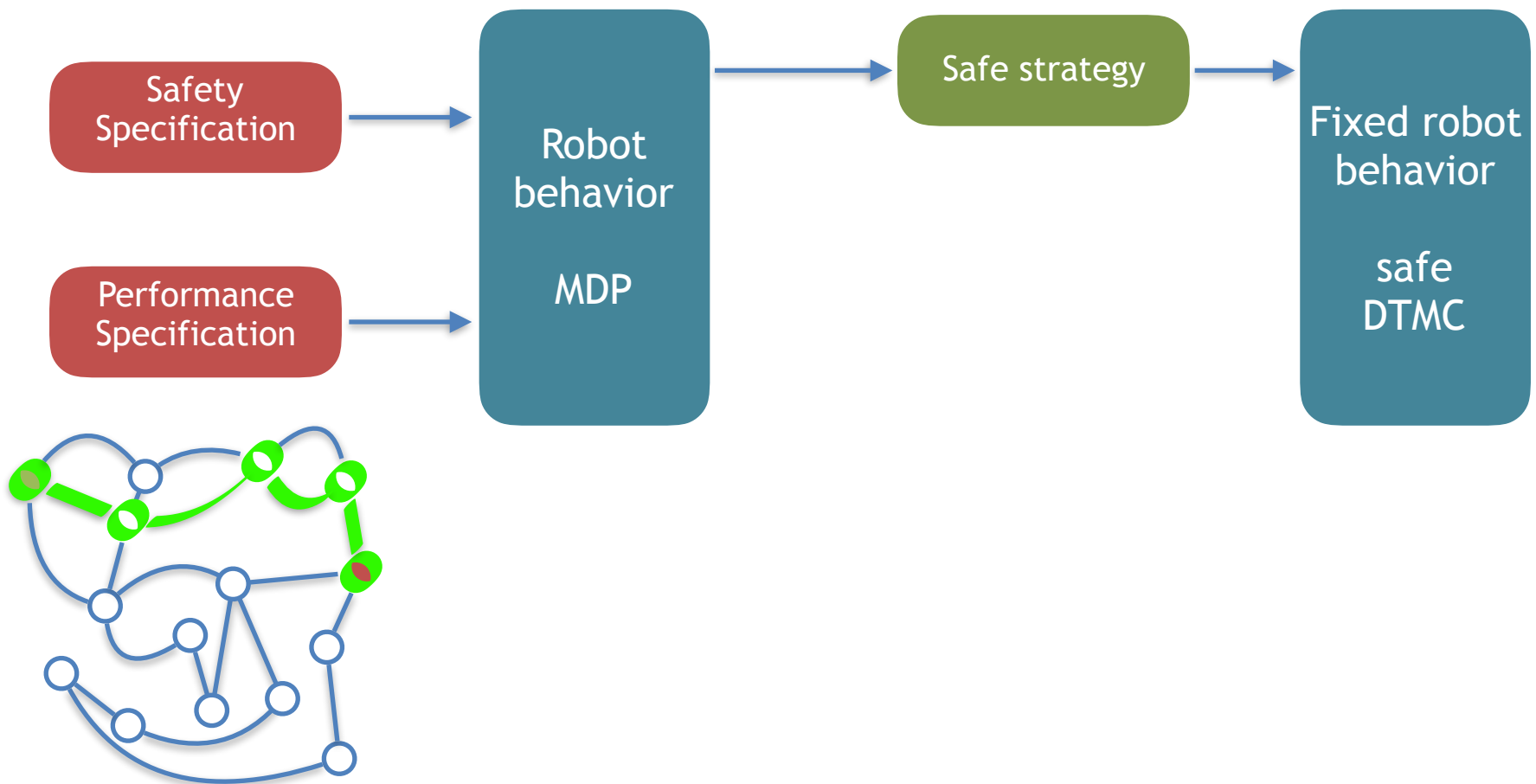
# Overview - Naive Approach



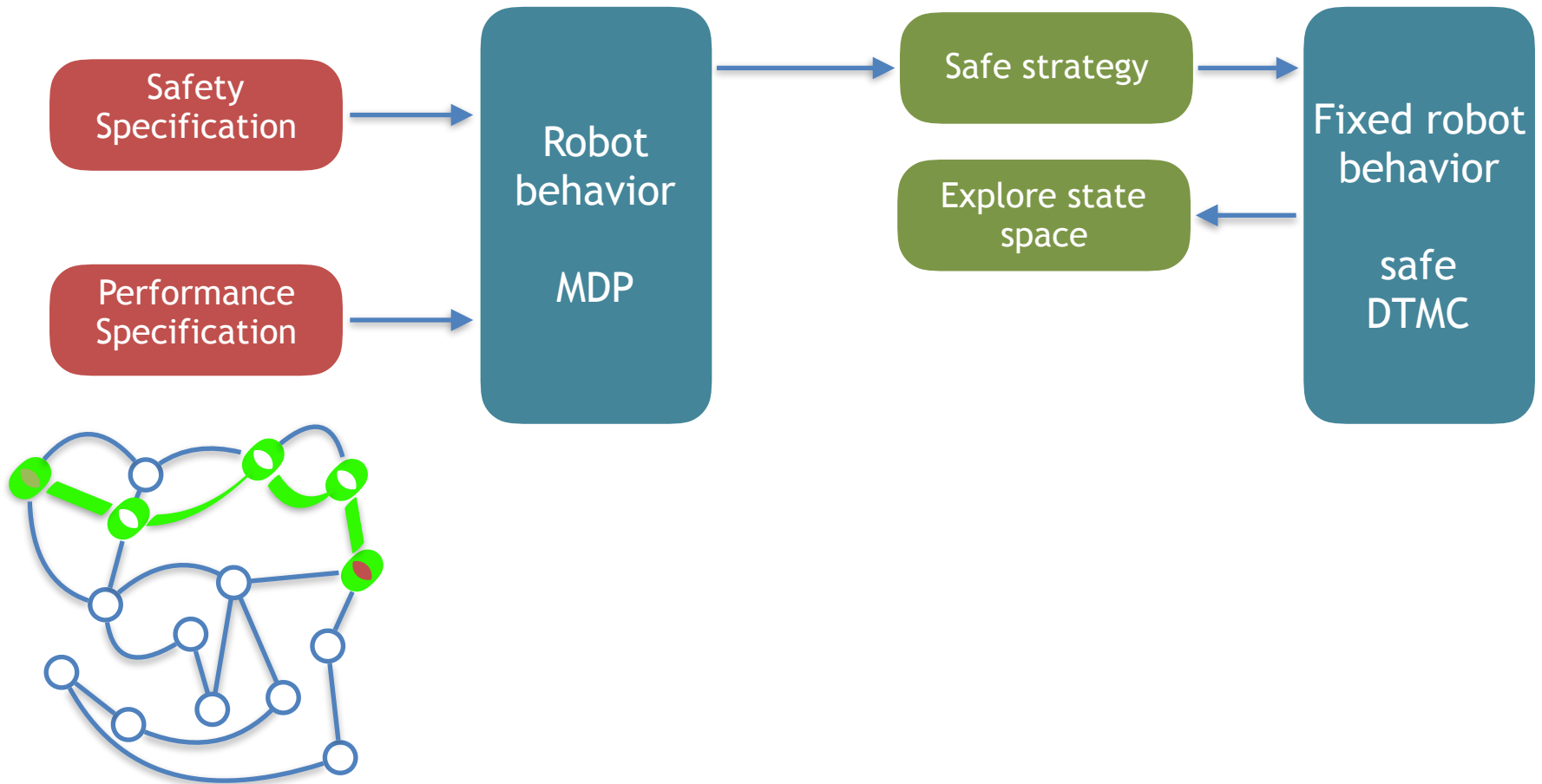
# Overview - Naive Approach



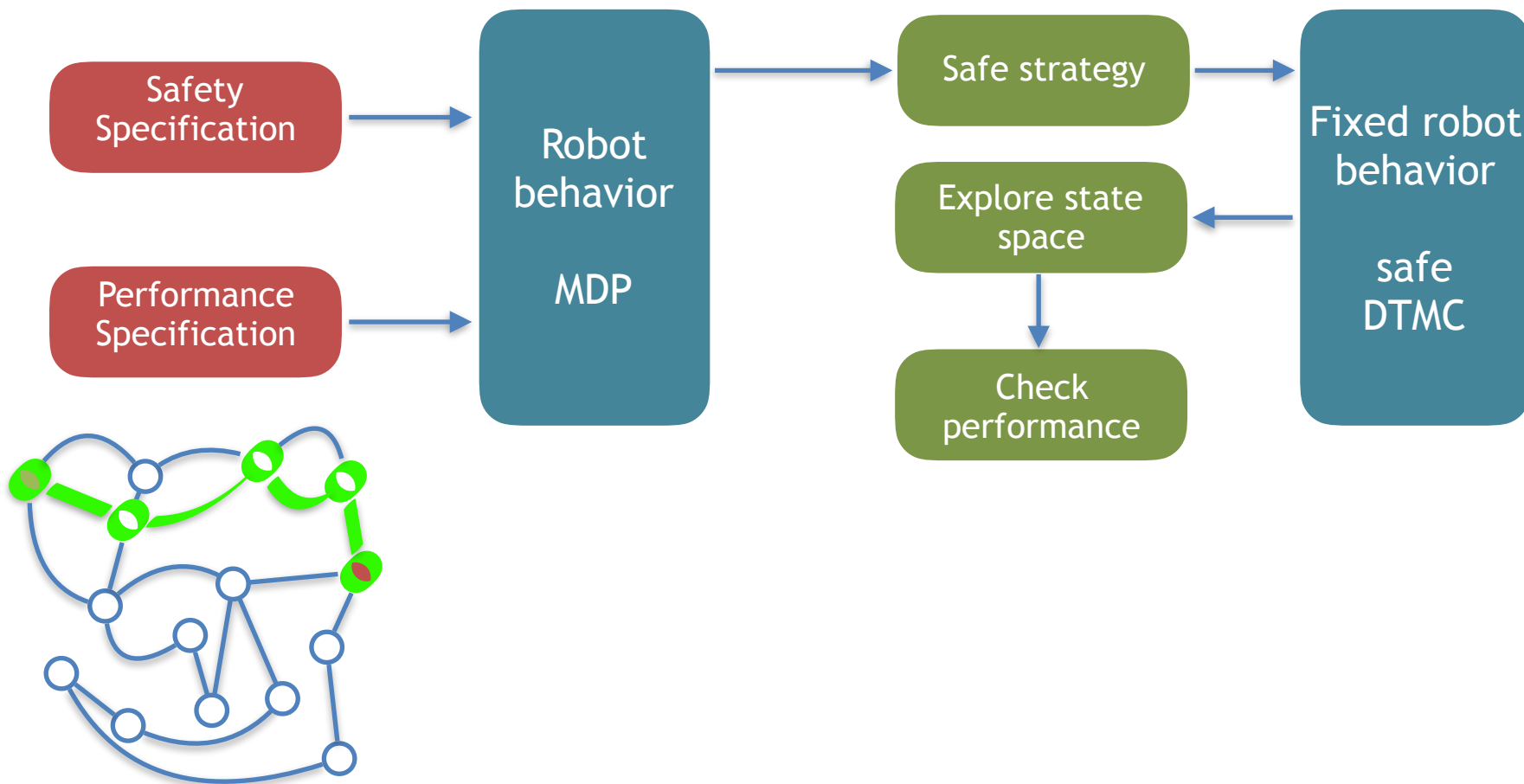
# Overview - Naive Approach



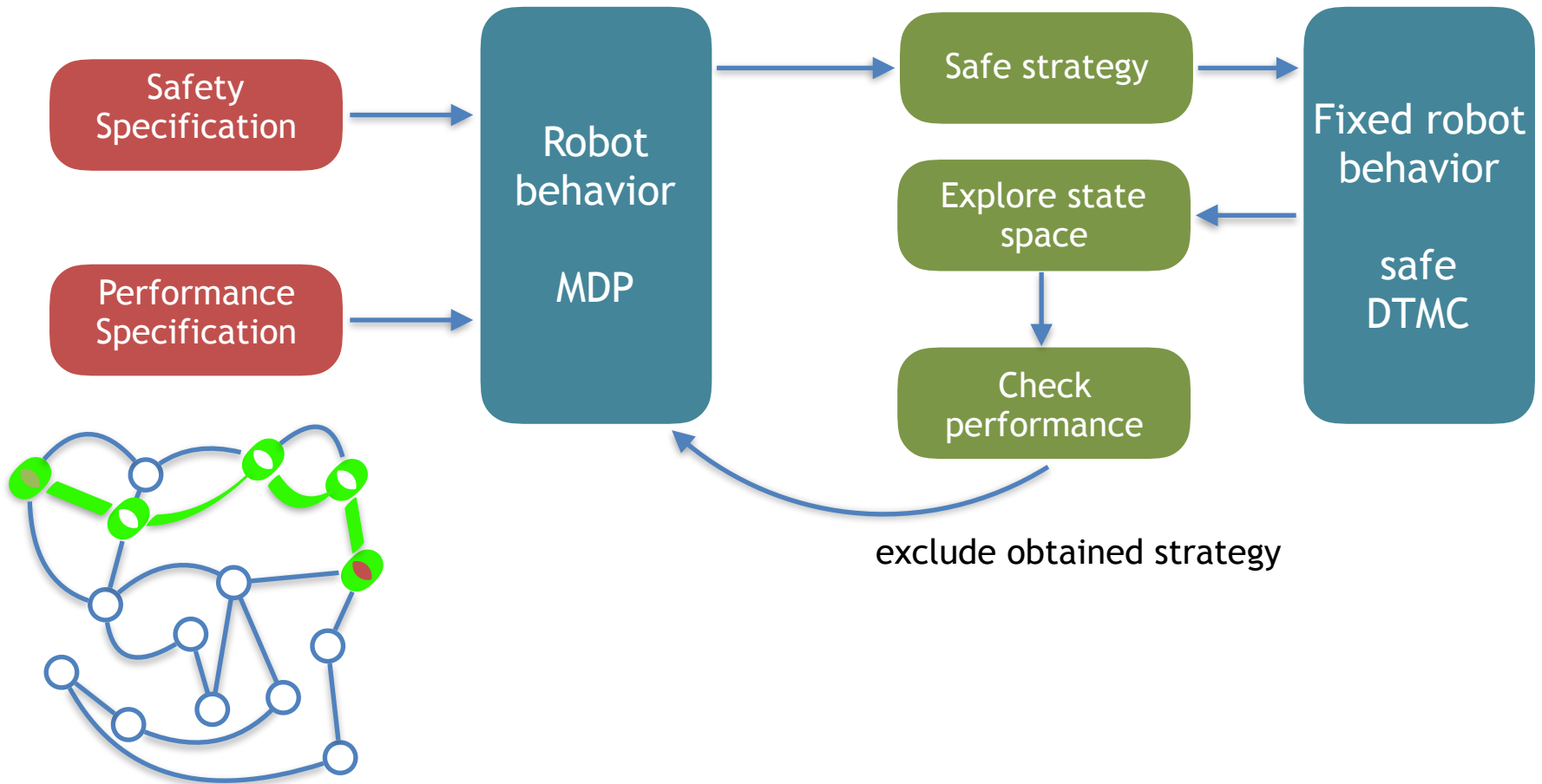
# Overview - Naive Approach



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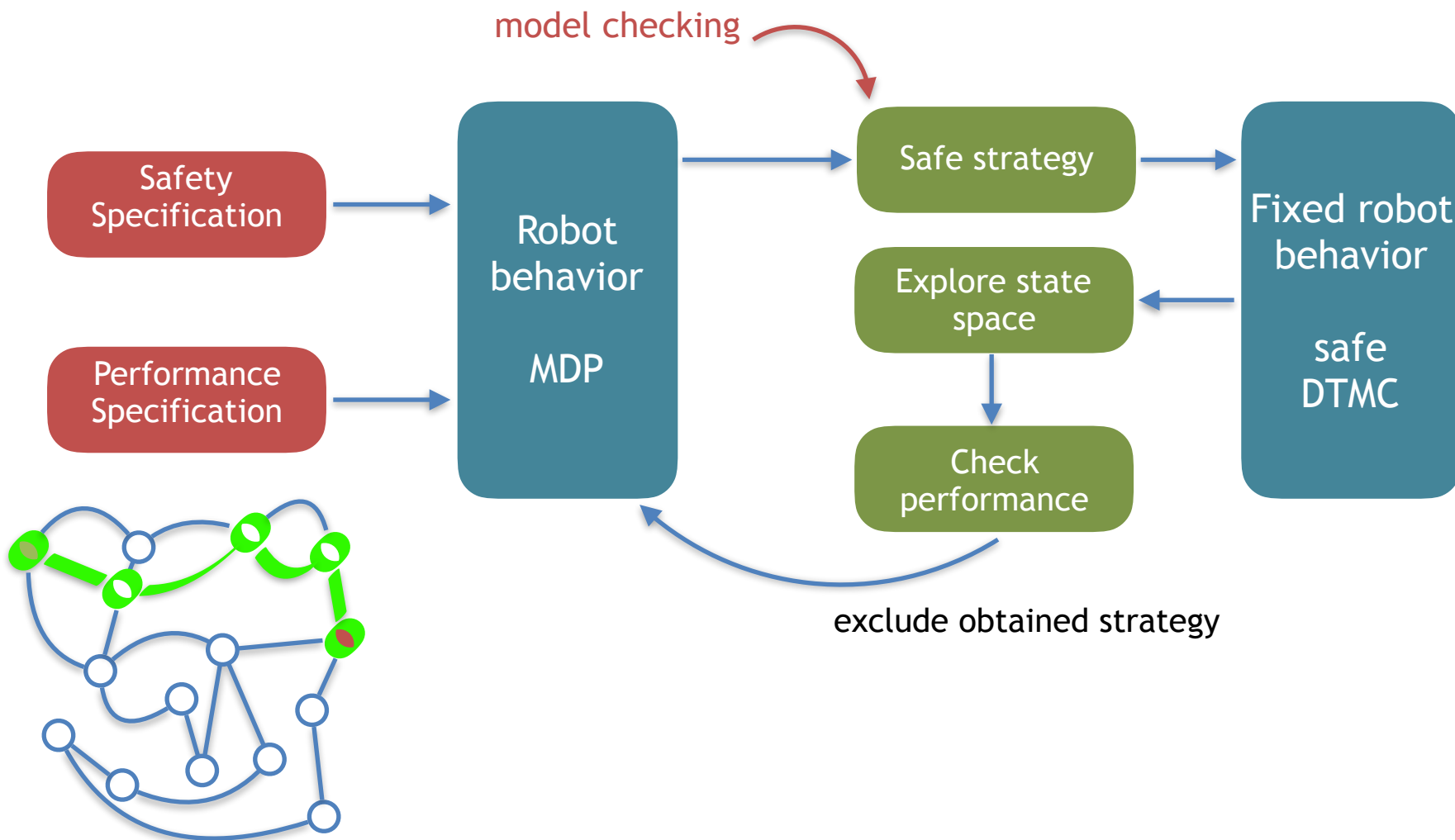


# Overview - Naive Approach

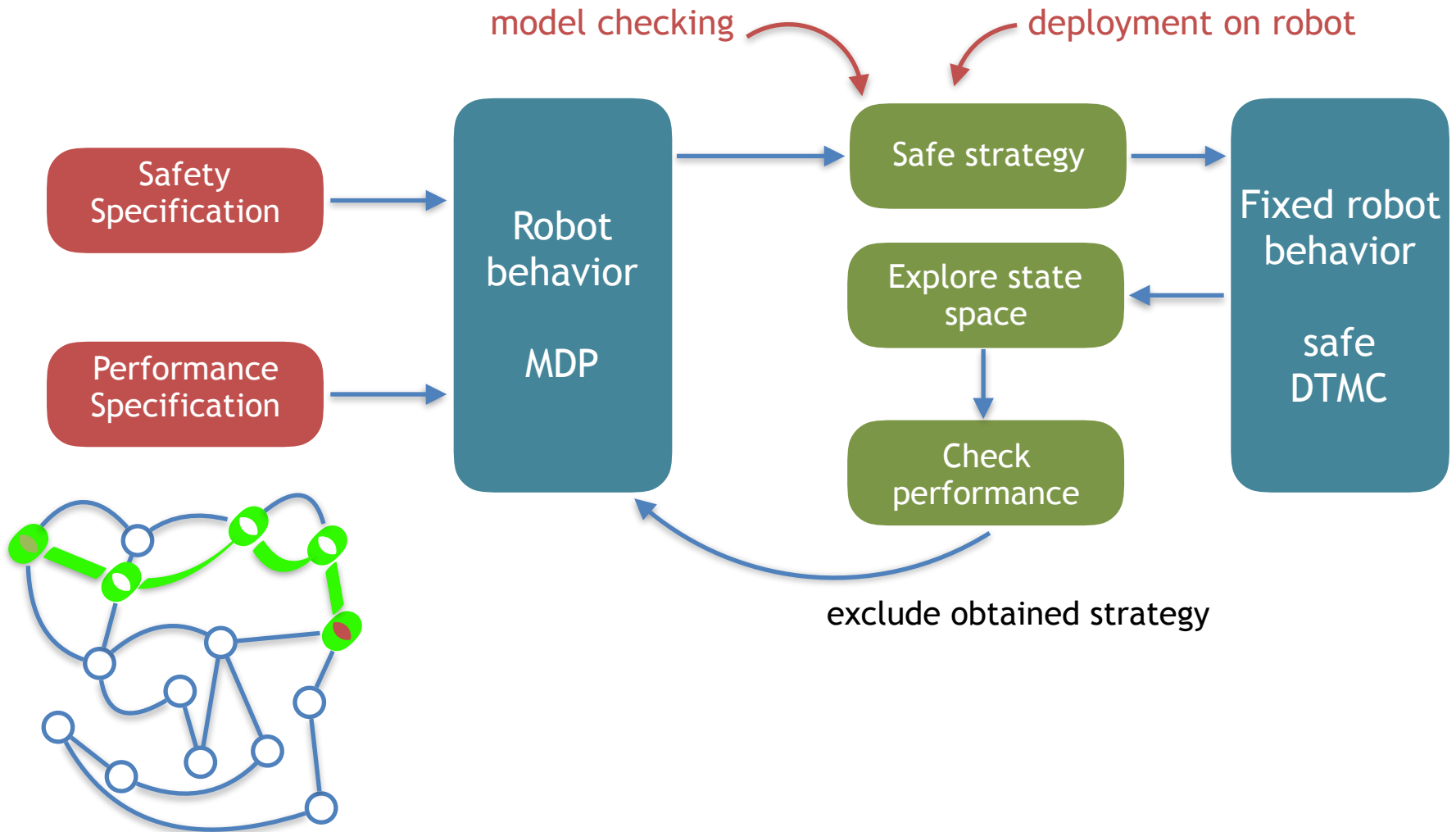




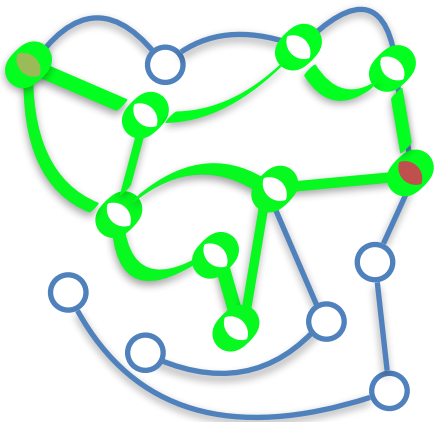
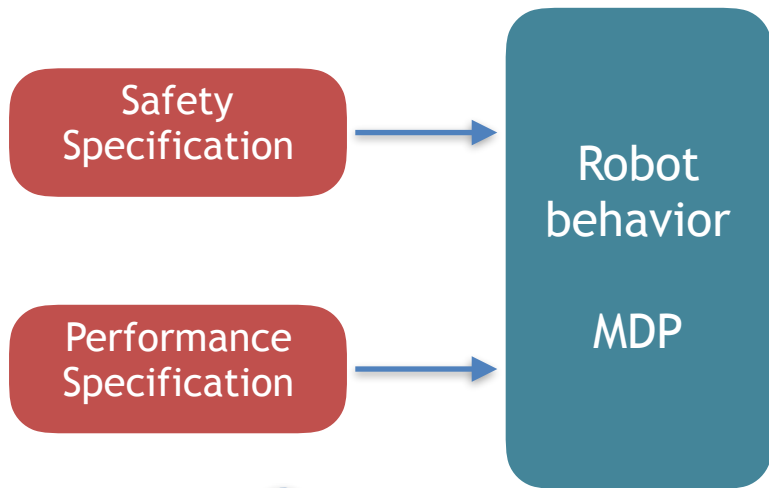
# Overview - Naive Approach



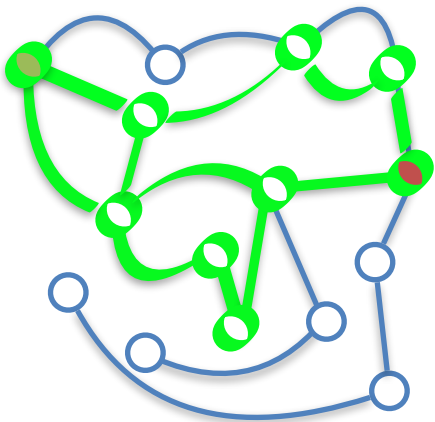
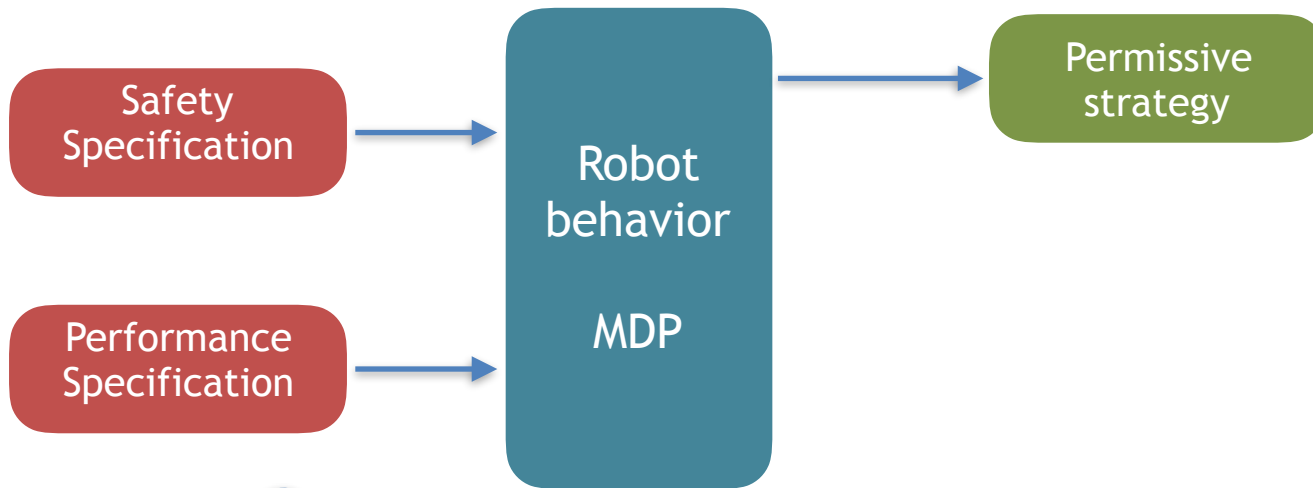
# Overview - Naive Approach



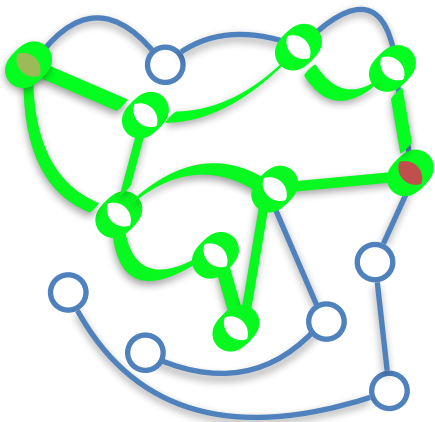
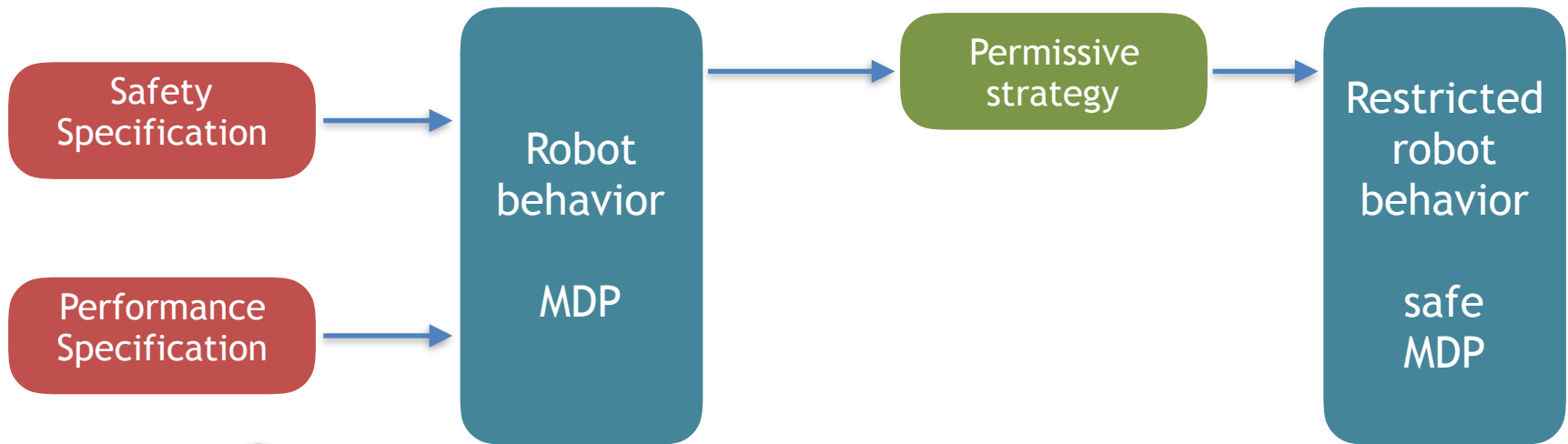
# Overview - Permissive Approach



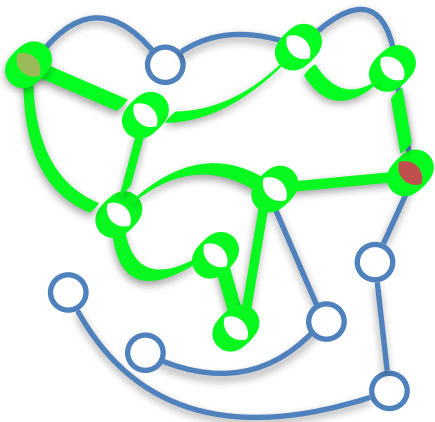
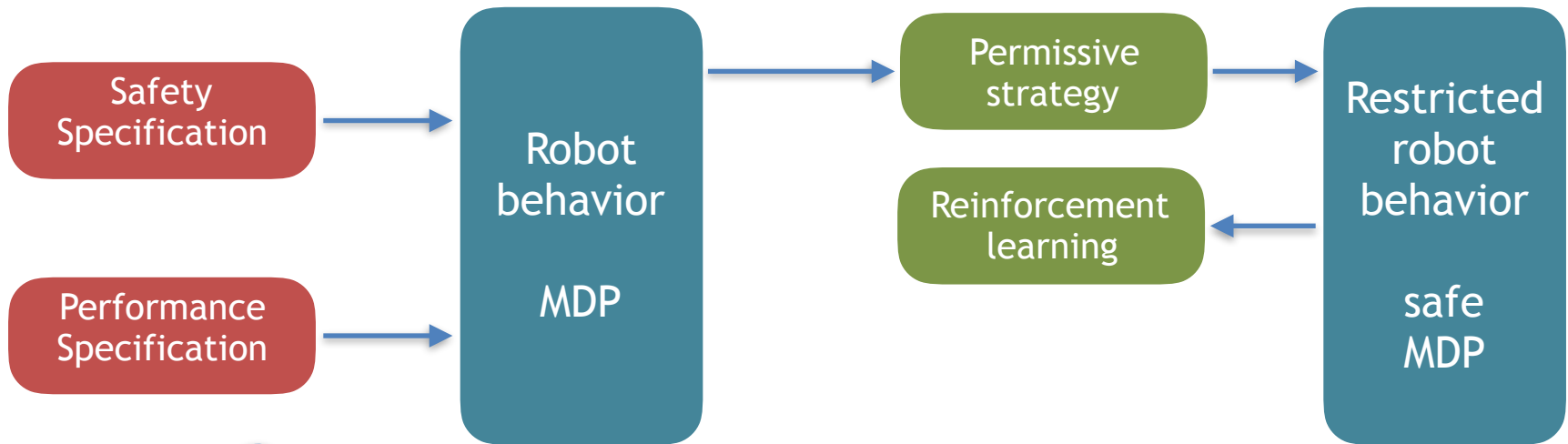
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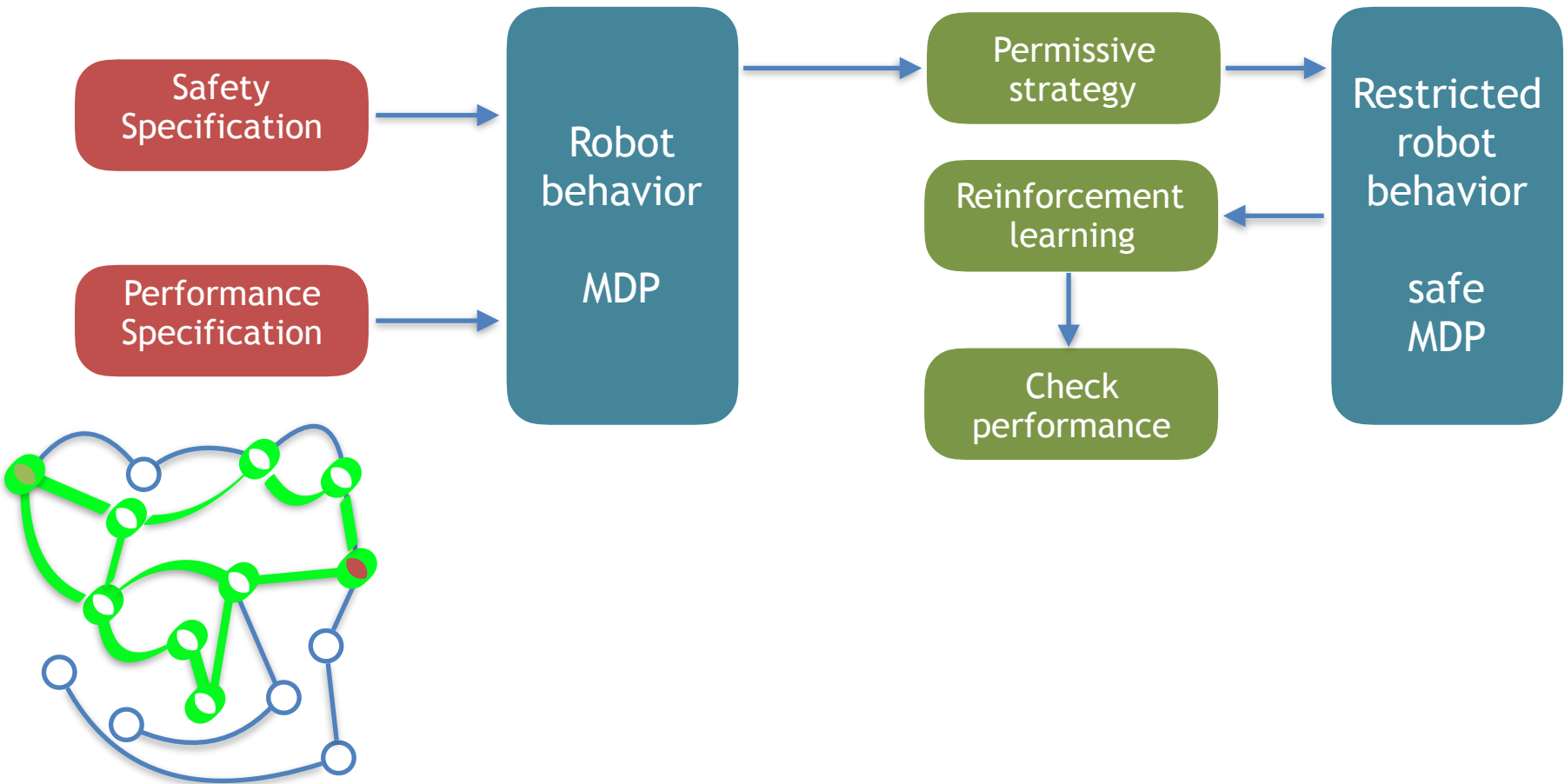
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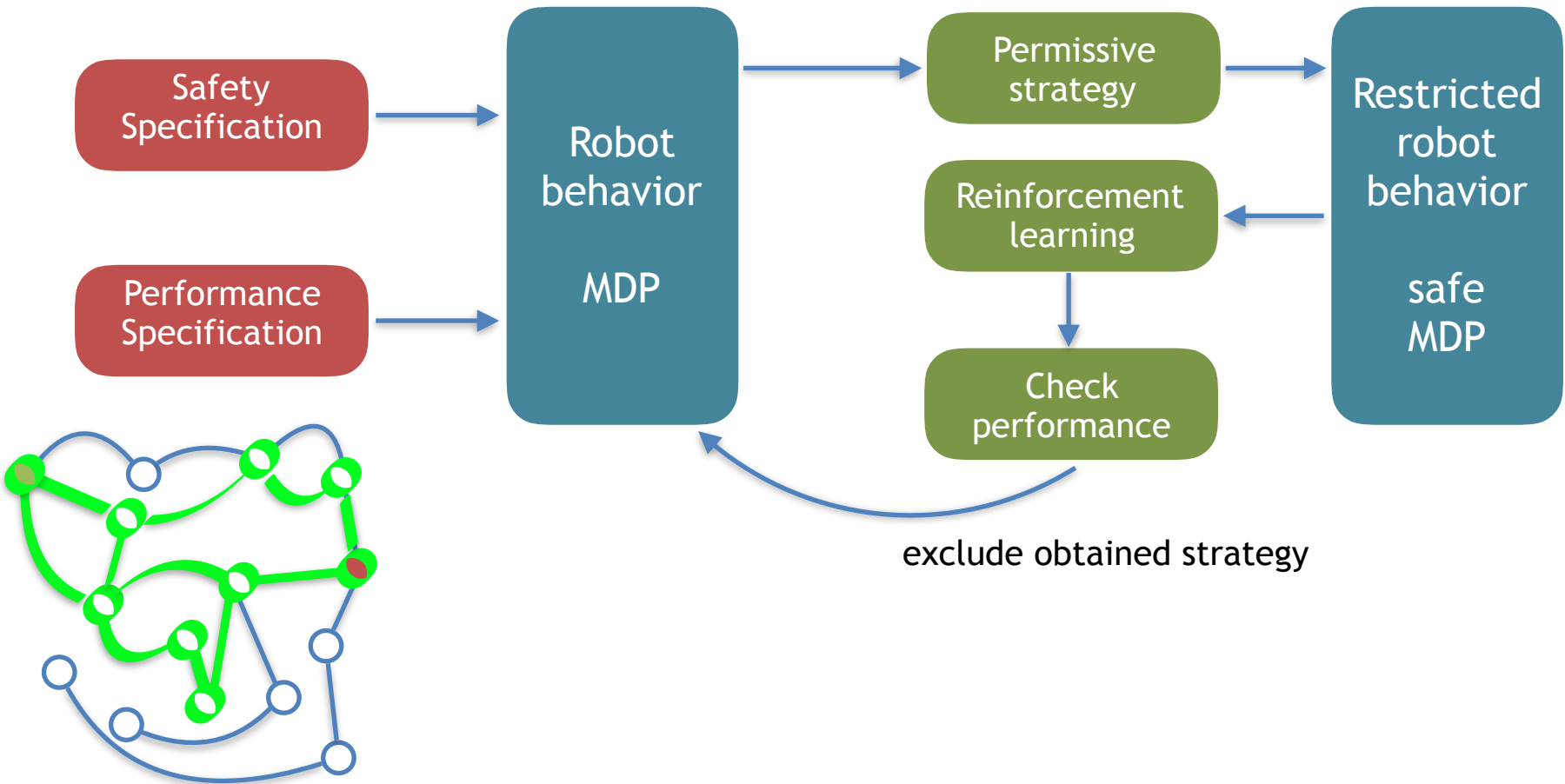
# Overview - Permissive Approach



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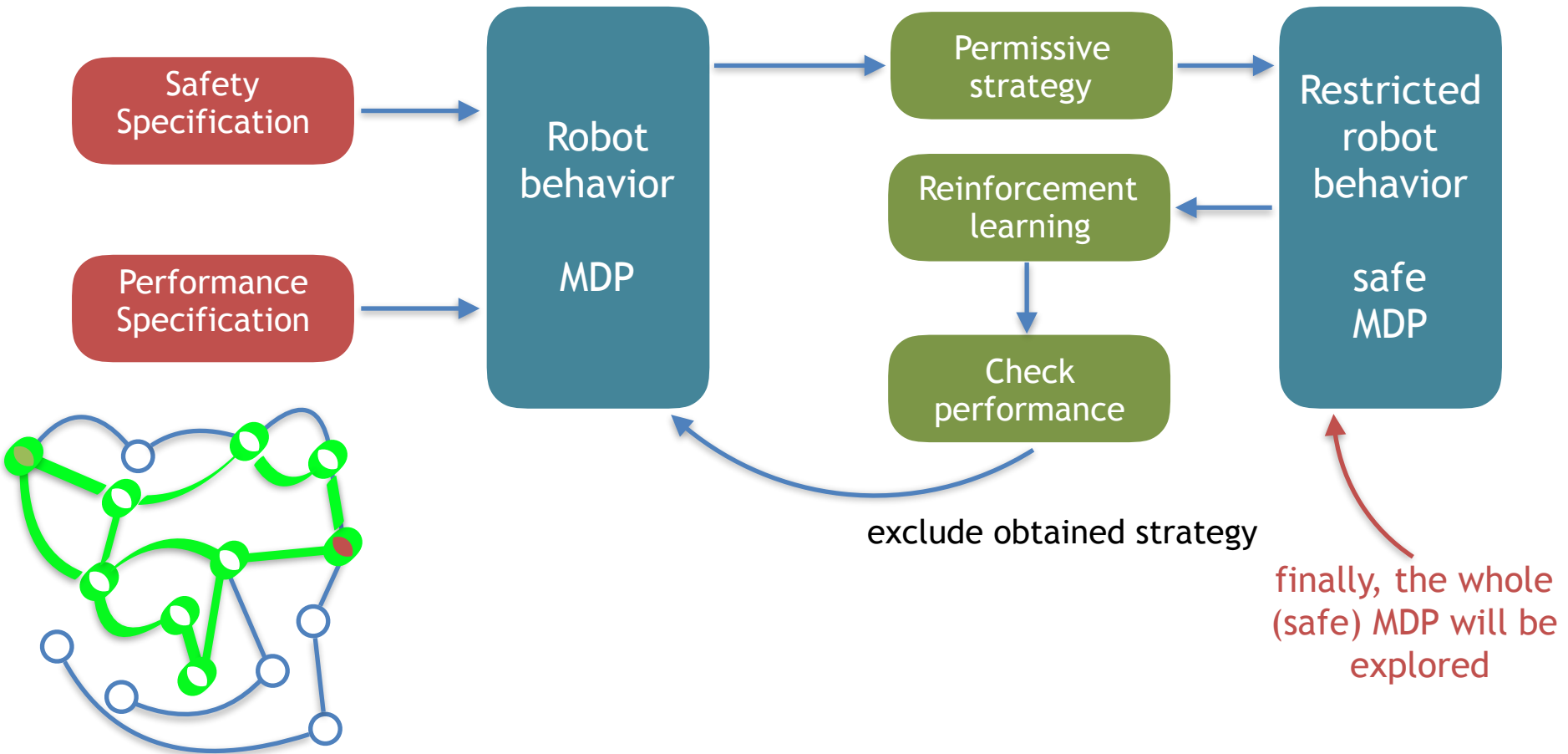


# Overview - Permissive Approach





# Overview - Permissive Approach



# Related Work

Klaus Dräger, Vojtech Forejt, Marta Z. Kwiatkowska, David Parker, Mateusz Ujma:  
**Permissive Controller Synthesis for Probabilistic Systems.** LMCS 2015

Klaus Dräger, Vojtech Forejt, Marta Z. Kwiatkowska, David Parker, Mateusz Ujma:  
**Permissive Controller Synthesis for Probabilistic Systems.** TACAS 2014

# Related Work

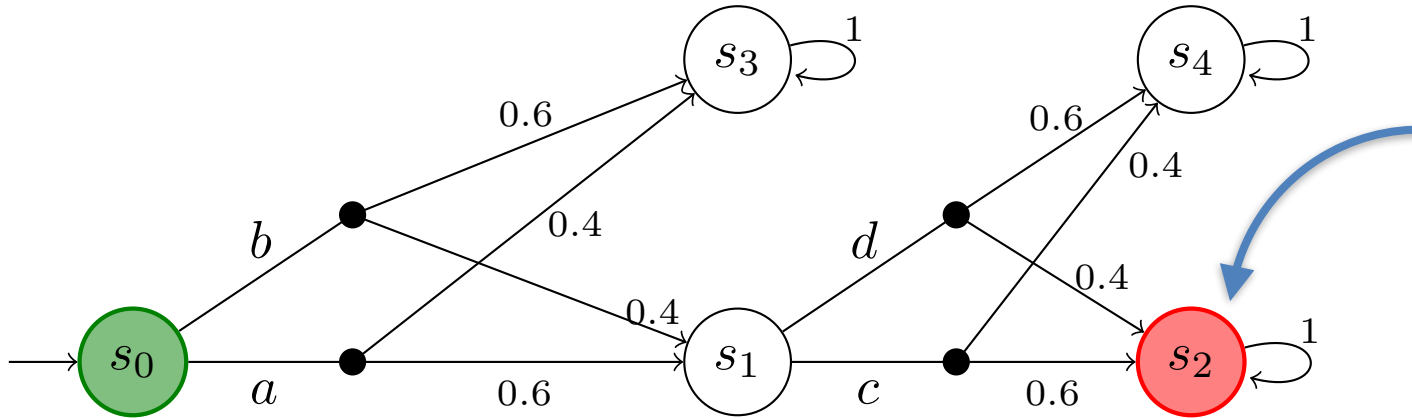
Klaus Dräger, Vojtech Forejt, Marta Z. Kwiatkowska, David Parker, Mateusz Ujma:  
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**Permissive Controller Synthesis for Probabilistic Systems.** TACAS 2014

Alexandre David, Peter Gjøøl Jensen, Kim Guldstrand Larsen, Marius Mikucionis, Jakob Haahr Taankvist:  
**Uppaal Stratego.** TACAS 2015

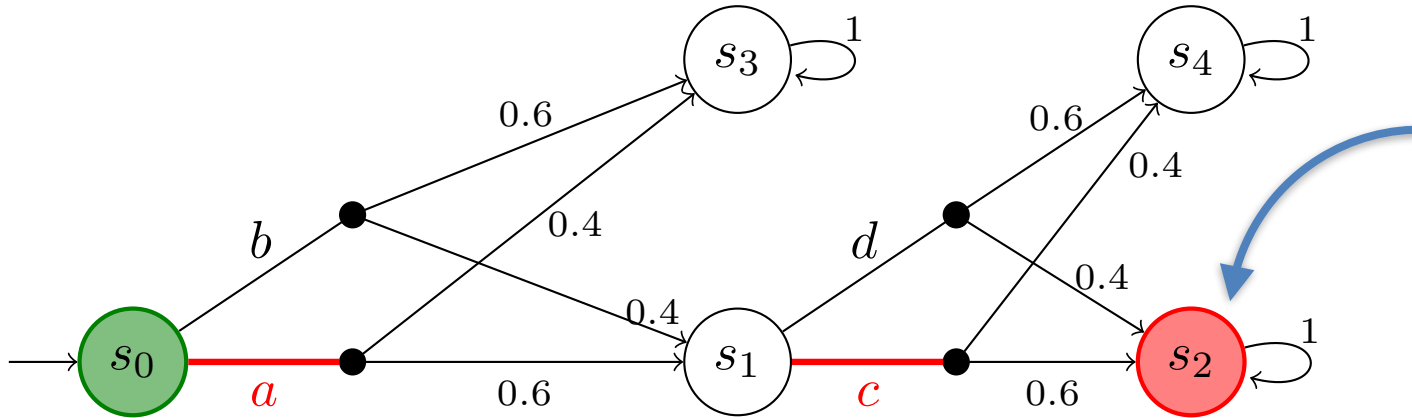
Kim G. Larsen, Marius Mikucionis, Marco Muniz, Jiri Srba and Jakob Haahr Taankvist:  
**Online and Compositional Learning of Controllers with Application to Floor Heating.** TACAS 2016

# Permissive Strategies



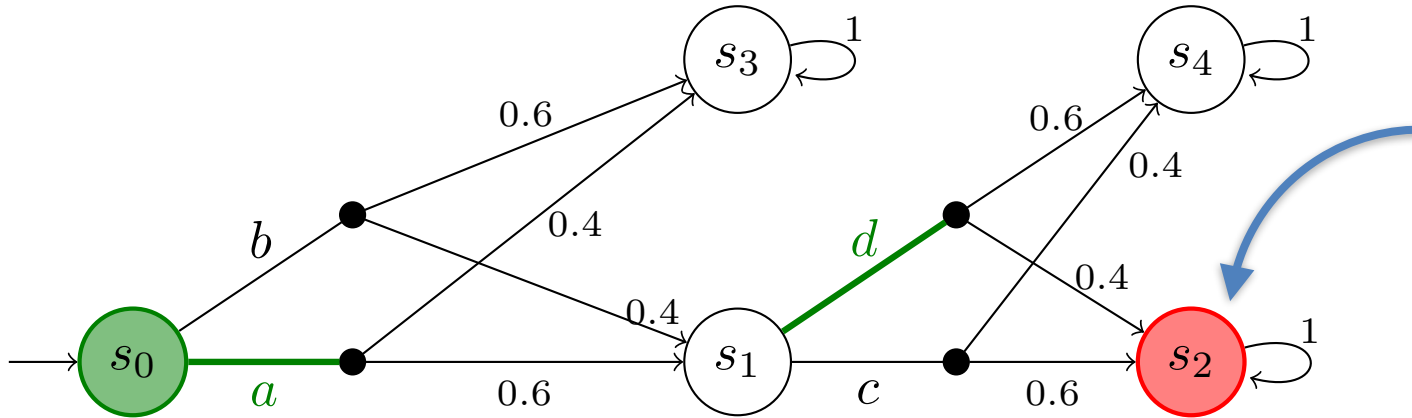
Probability of reaching shall be less than 0.3

# Permissive Strategies



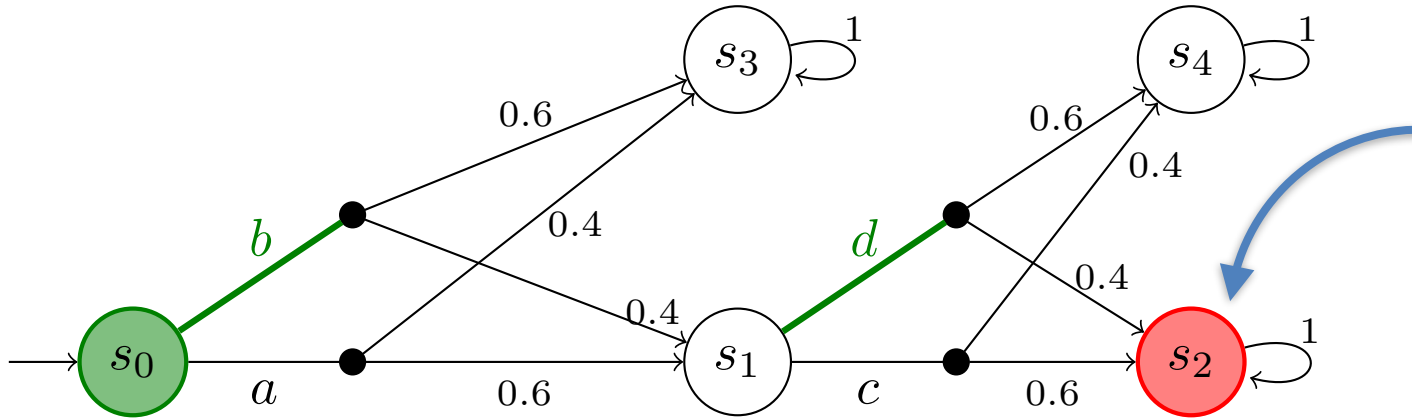
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# Permissive Strategies



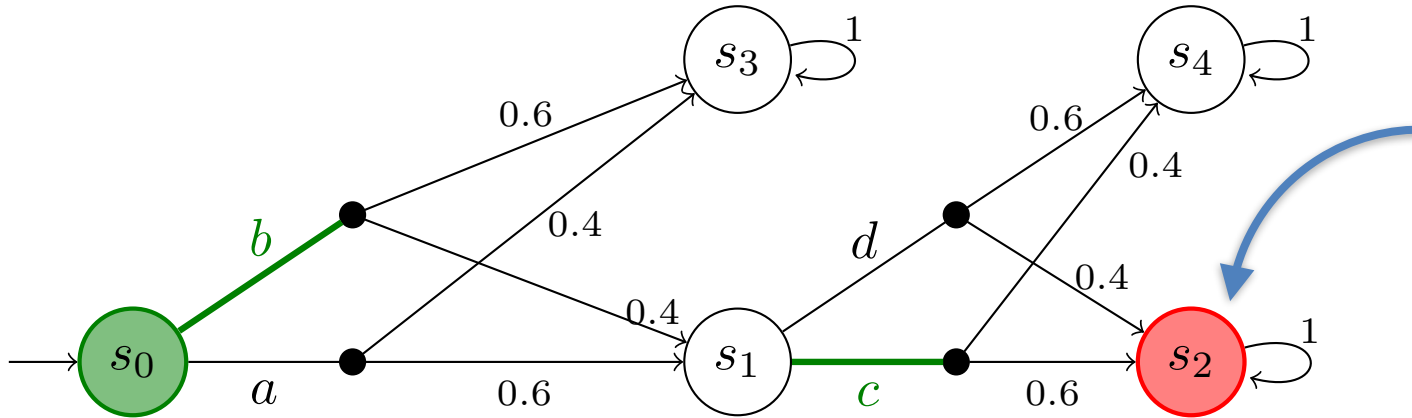
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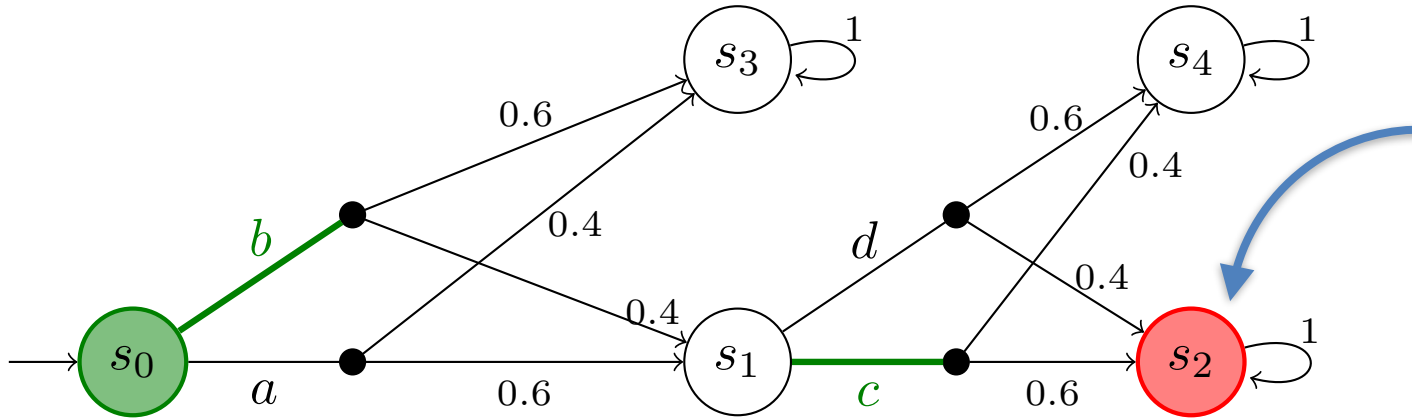
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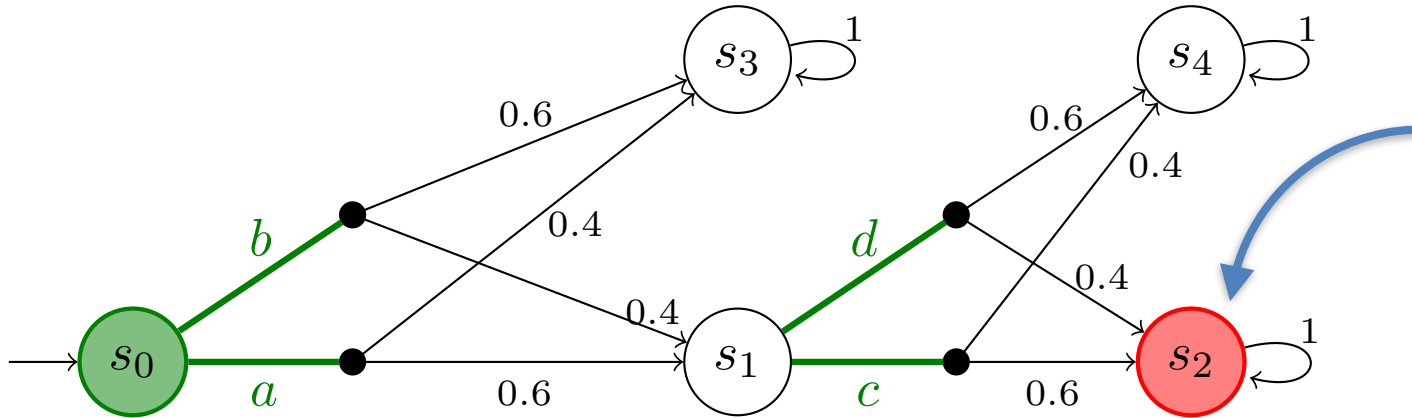
$s_0 \mapsto a, s_1 \mapsto d$

$s_0 \mapsto b, s_1 \mapsto c$  **safe**

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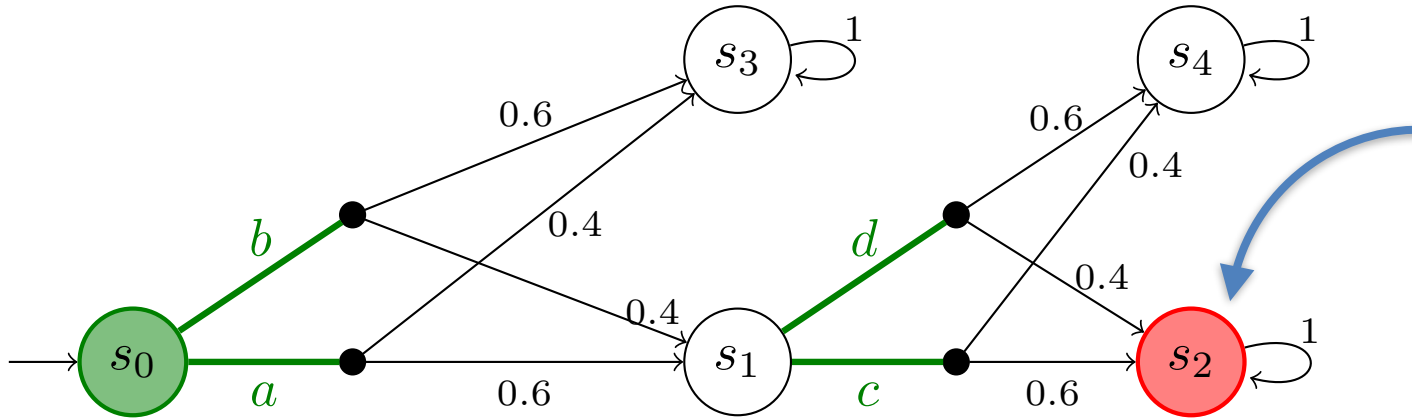
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Find maximal set of safe strategies

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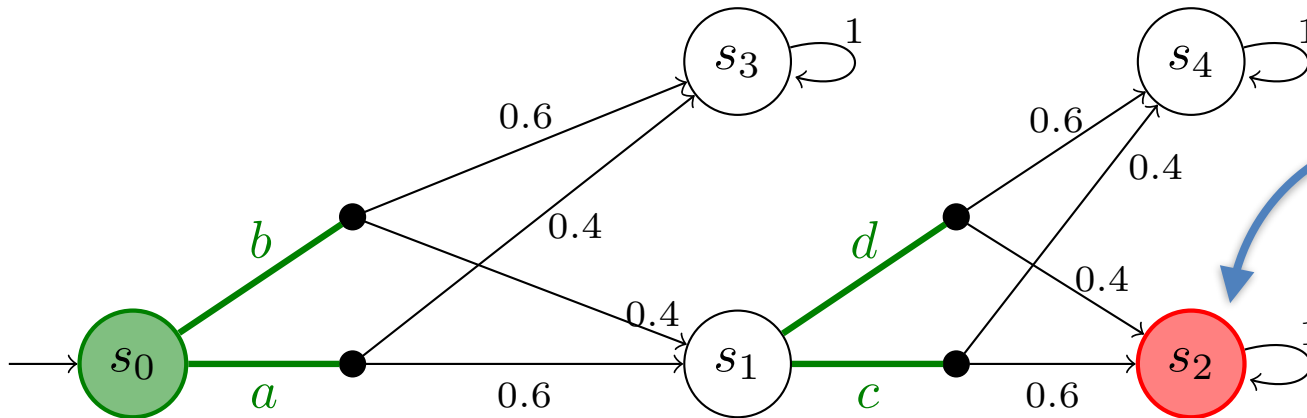
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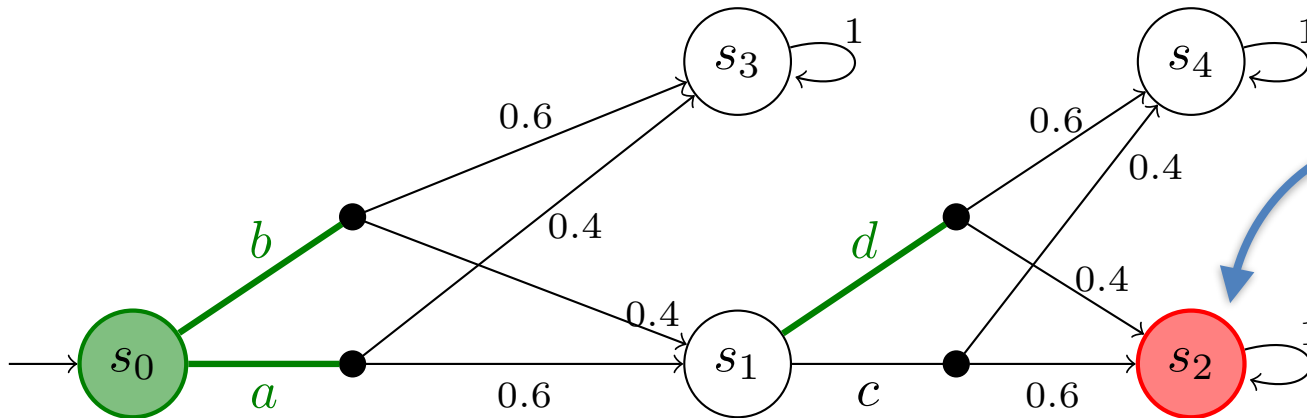
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exponentially many

# Permissive Strategies



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# SMT for permissive strategies

$$p_{s_I} \leq \lambda$$

probability smaller than threshold

$$\forall s \in S. \bigvee_{a \in Act(s)} y_{s,a}$$

at least one action per state

$$\forall s \in T. p_s = 1$$

probability of target states is one

$$\forall s \in S. \forall a \in Act(s). y_{s,a} \rightarrow p_s \geq \sum_{s' \in S} \mathcal{P}(s, a, s') \cdot p_{s'}$$

probability of each state is assigned the maximum under the permissive strategy

## Variables

$y_{s,a}$

action is chosen at state

$p_s$

probability of state

Solution induces  
safe permissive  
strategy

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$y_{s,a}$  action is chosen at state

$p_s$  probability of state

Solution induces  
safe permissive  
strategy

MILP  
quantify permissiveness

# Bounds on the Safe Optimum

Reinforcement learning explores the state space

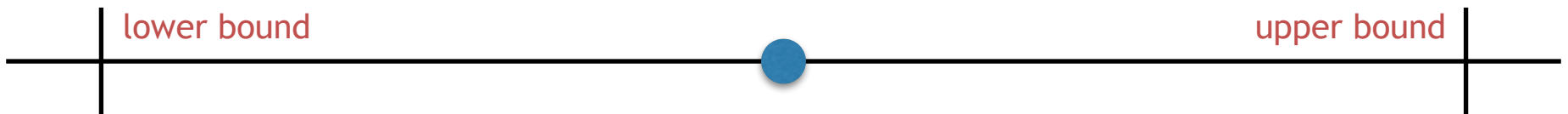
- (unknown) cost function  $\rho: S \rightarrow \mathbb{R}$  is refined and
- unknown cost values are instantiated by given lower bounds yielding  $\rho_l: S \rightarrow \mathbb{R}$



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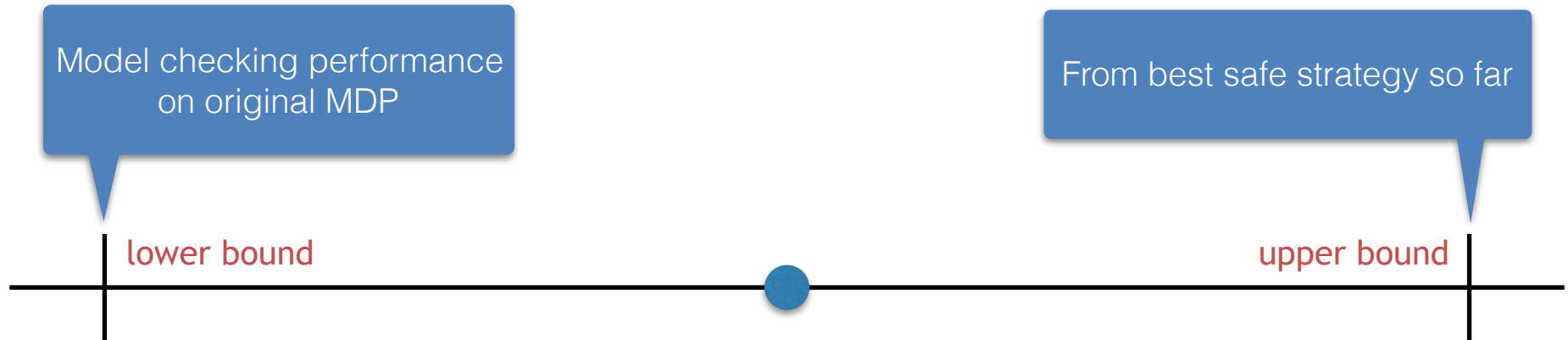
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Reinforcement learning explores the state space

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- unknown cost values are instantiated by given lower bounds yielding  $\rho_l: S \rightarrow \mathbb{R}$

Model checking performance  
on original MDP

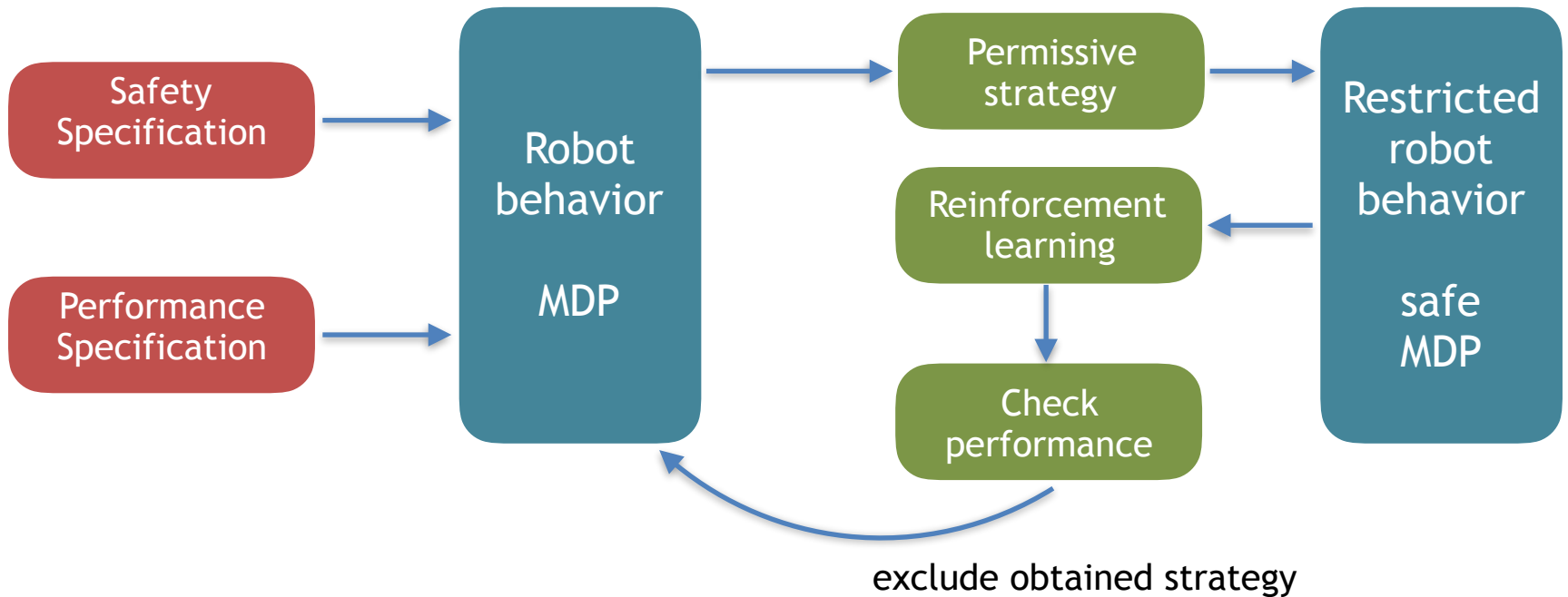
lower bound

From best safe strategy so far

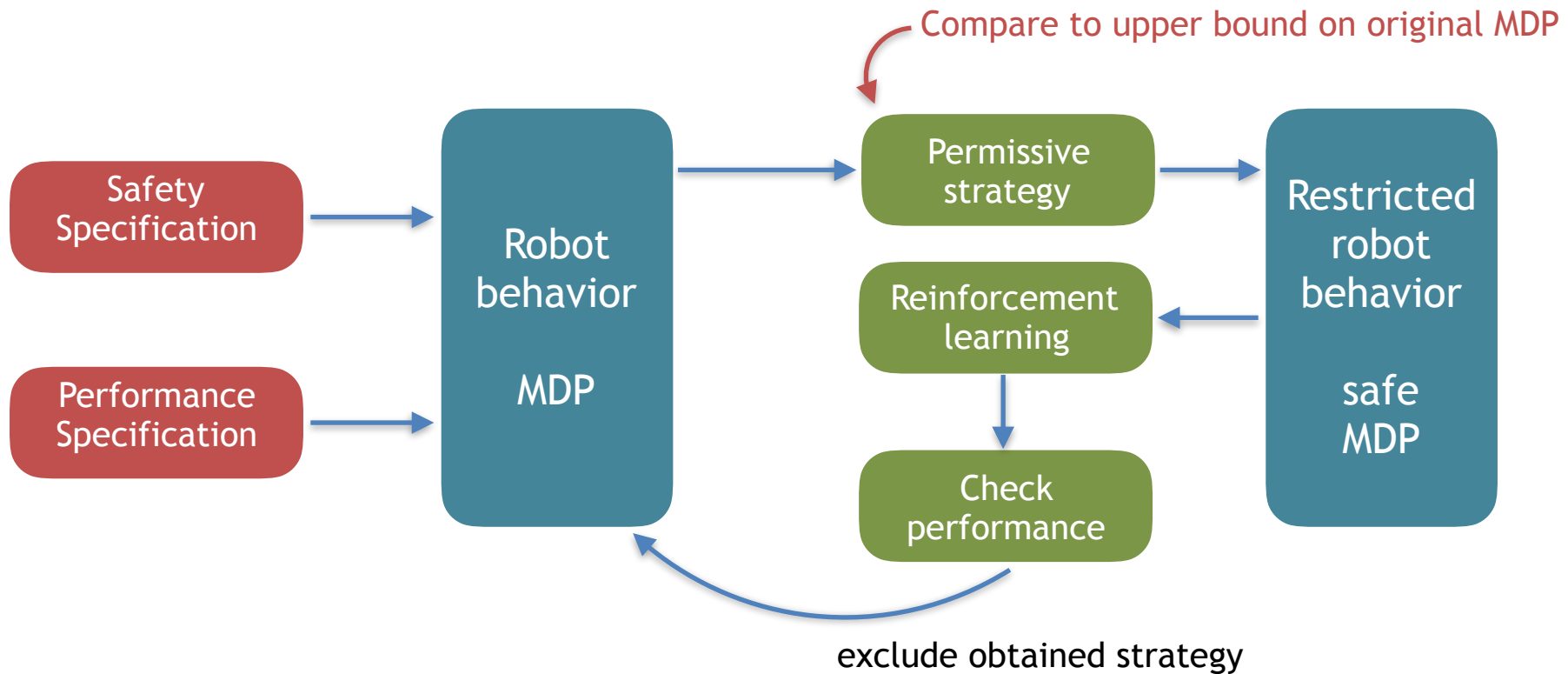
upper bound

Multi-objective model checking yields even tighter bounds.

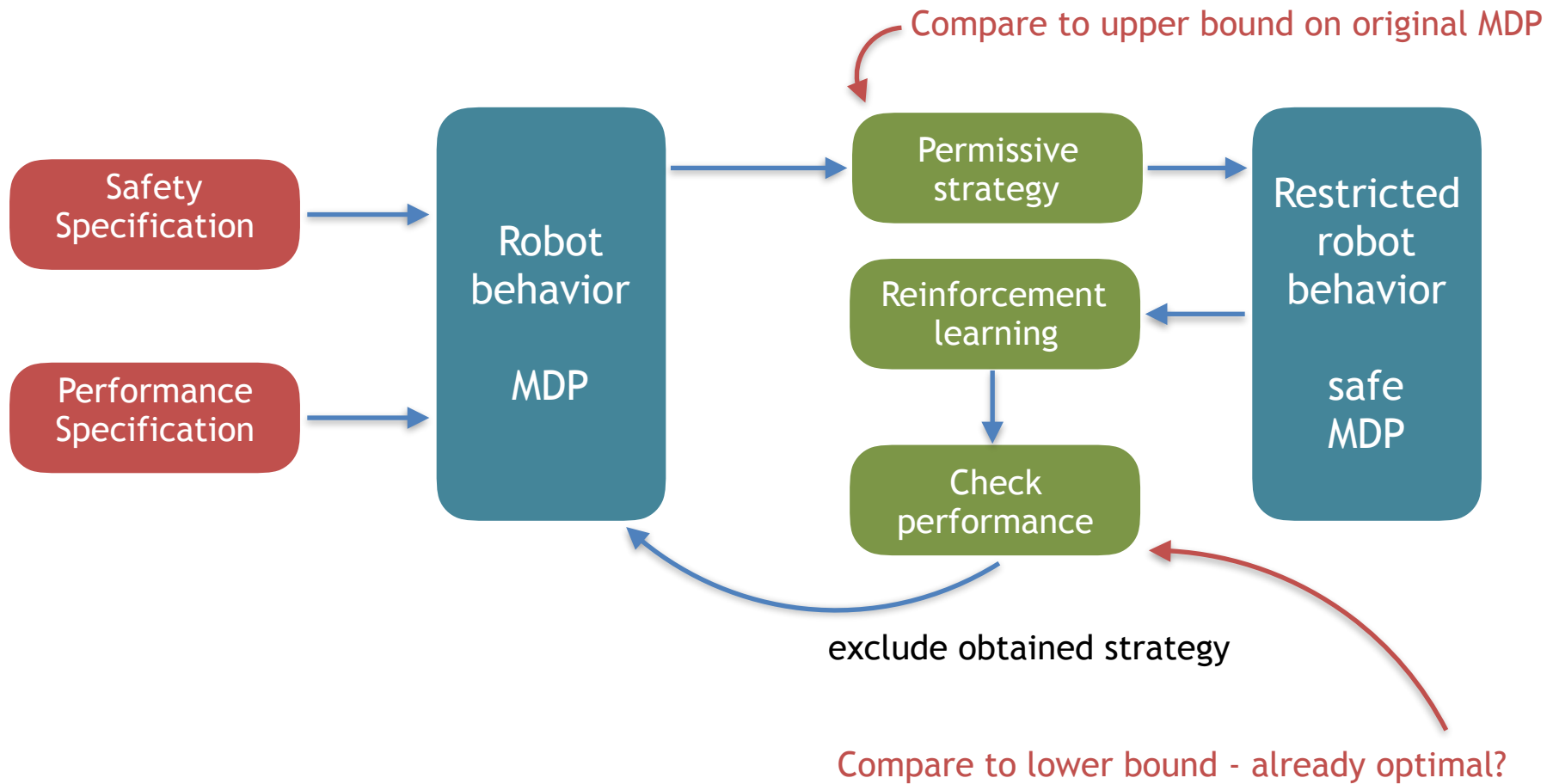
# Incorporating the Bounds



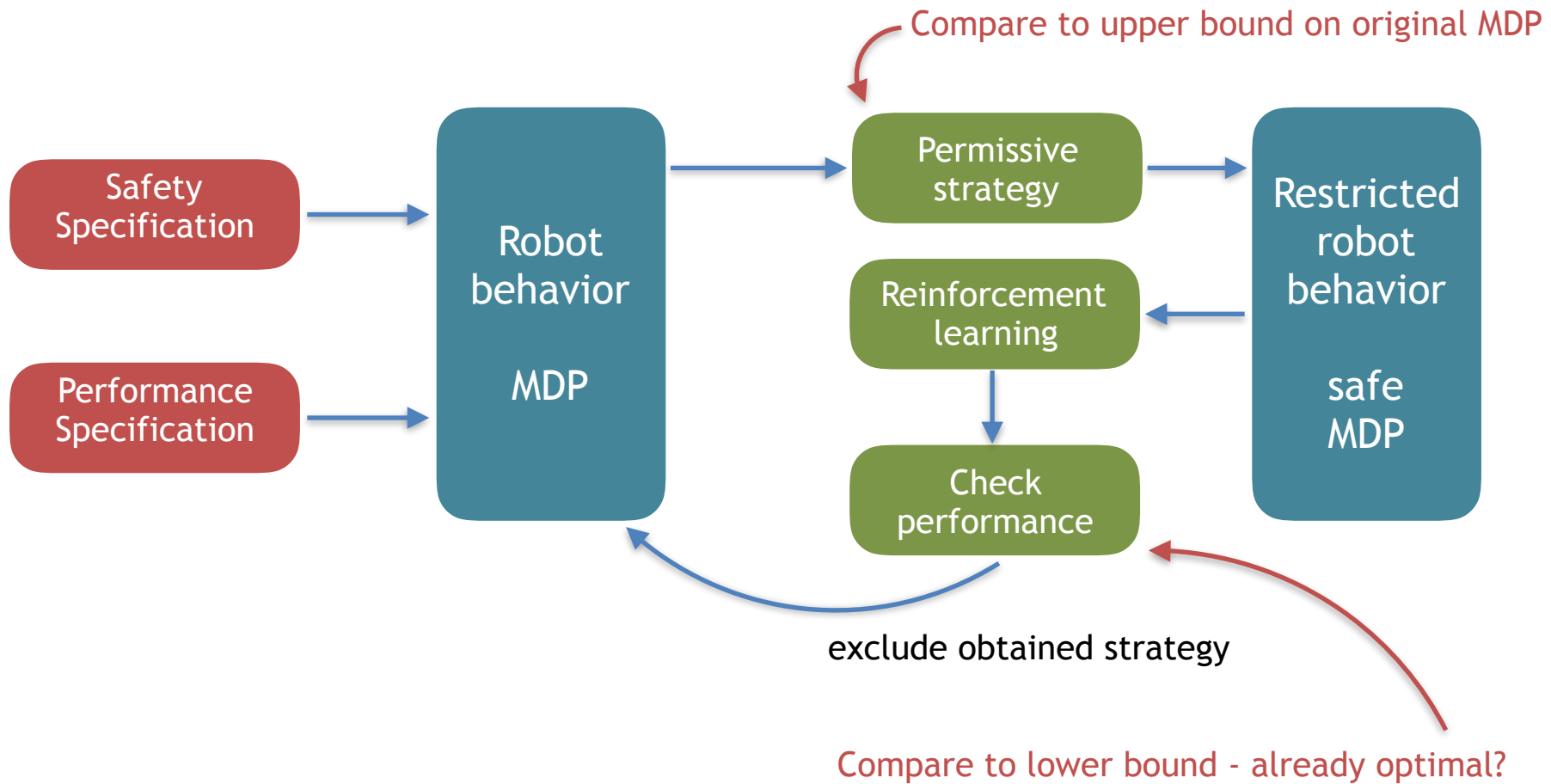
# Incorporating the Bounds



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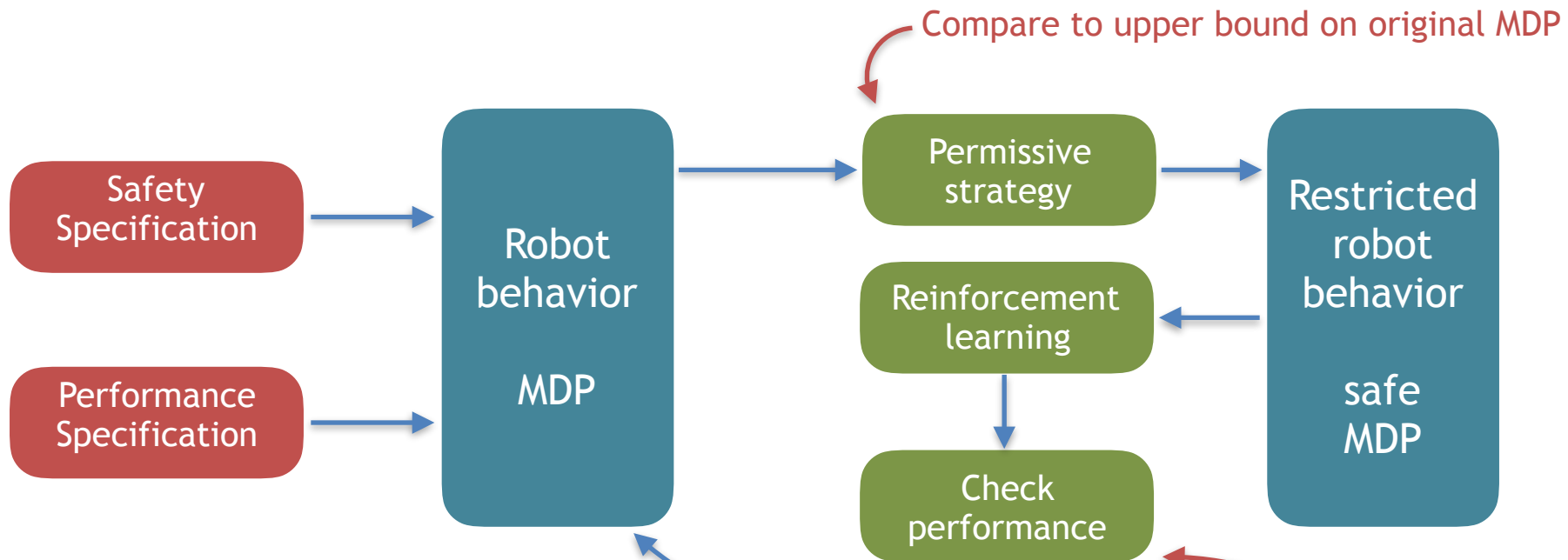


# Incorporating the Bounds



Iterating means tightening the bounds

# Incorporating the Bounds



## Exploitation vs. Exploration

- overly optimistic expected rewards
- initialize Q-learning (via **lower bounds**) to favor unexplored parts of the MDP

exclude obtained strategy

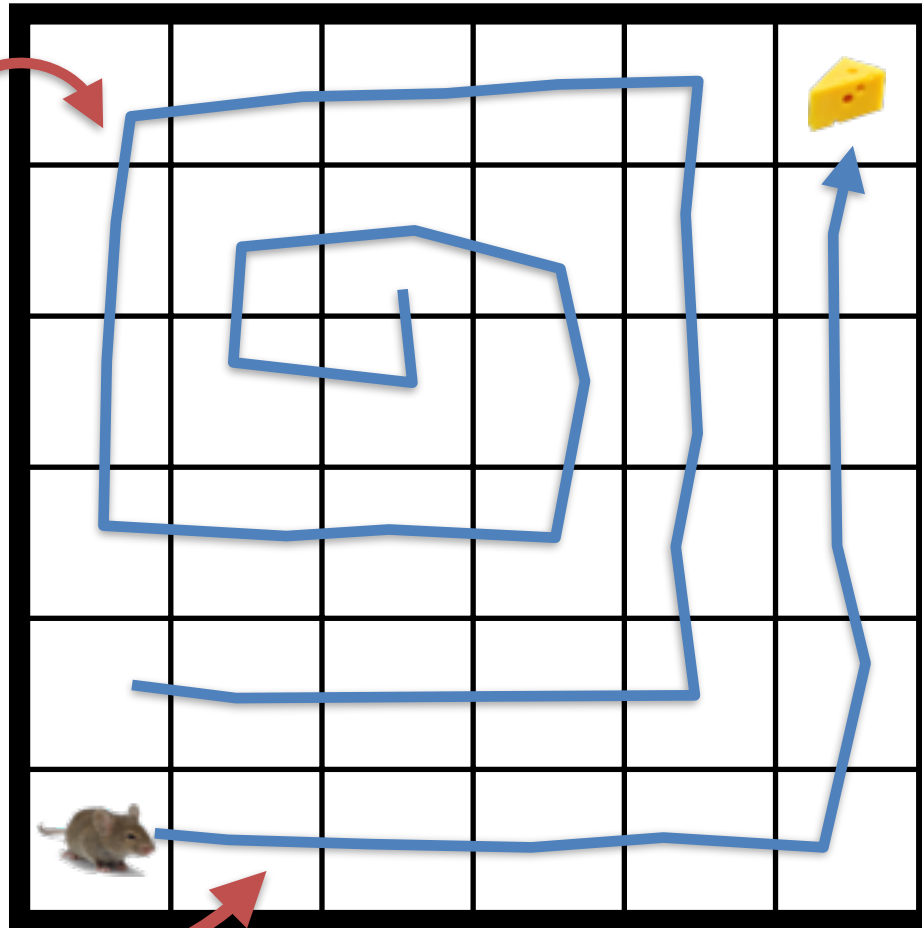
Compare to lower bound - already optimal?

Iterating means tightening the bounds



# About permissiveness

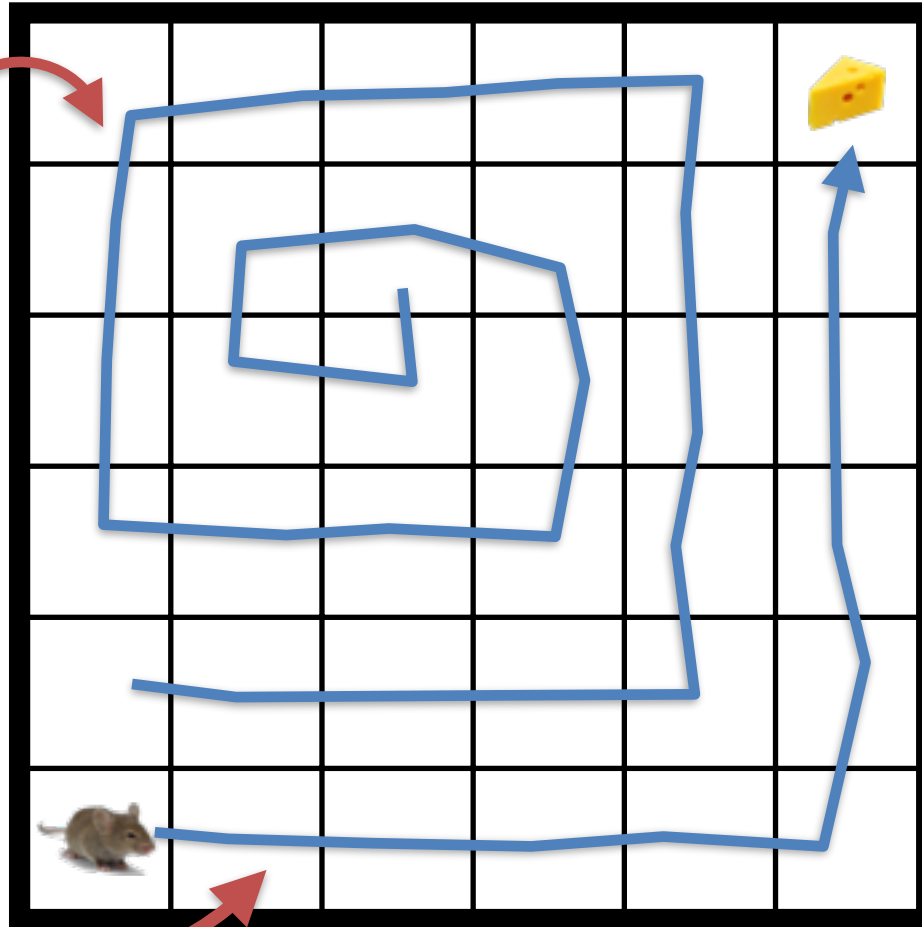
everything allowed  
except of reaching  
the cheese



single safe strategy

# About permissiveness

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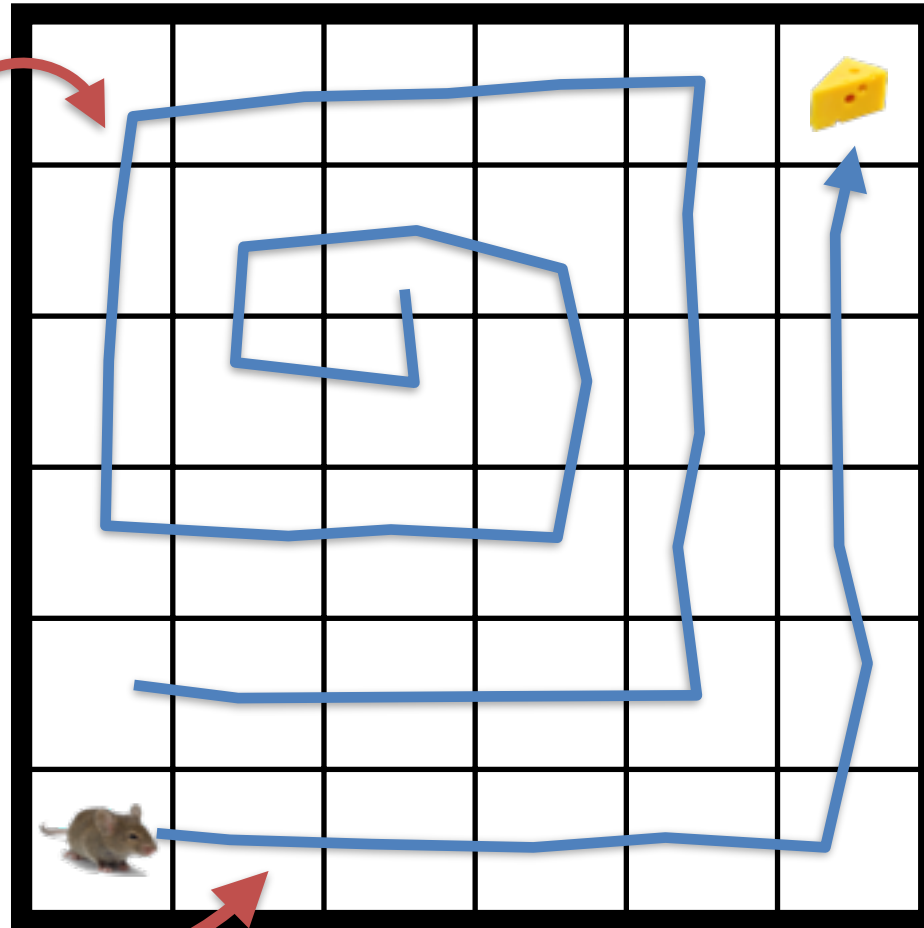


Quantifying  
permissiveness  
may not be  
beneficent

single safe strategy

# About permissiveness

everything allowed  
except of reaching  
the cheese



single safe strategy

Quantifying  
permissiveness  
may not be  
beneficent

Enforce  
reachability of  
target states

very costly

number of  
deployments vs  
costly  
computations

# Experimental Results

Benchmark		states	trans.	branch.	$\lambda$	opt.	$i$	$t$	lower	upper
Janitor	5,5	625	1125	3545	0.1	<b>88.6</b>	1	813	84	<b>88.6</b>
							2	2578	84	<b>88.6</b>
FolLine	30,15	455	1265	3693	0.01	<b>716.0</b>	1	41	715.4	<b>717.1</b>
							3	85	715.62	<b>716.83</b>
							13	306	715.9	<b>716.5</b>
	40,15	625	1775	5223	0.12	<b>966.0</b>	1	304	964.8	<b>968.2</b>
							3	420	965.4	<b>967.2</b>
							8	738	965.6	<b>966.7</b>
ComExp	6,6,6	823	2603	3726	0.08	<b>54.5</b>	1	5	0.3	<b>113.3</b>
							2	26	0.3	<b>74.9</b>
							3	105	0.3	<b>57.3</b>
	8,8,6	1495	4859	6953	0.12	<b>72.9</b>	1	15	0.42	<b>163.1</b>
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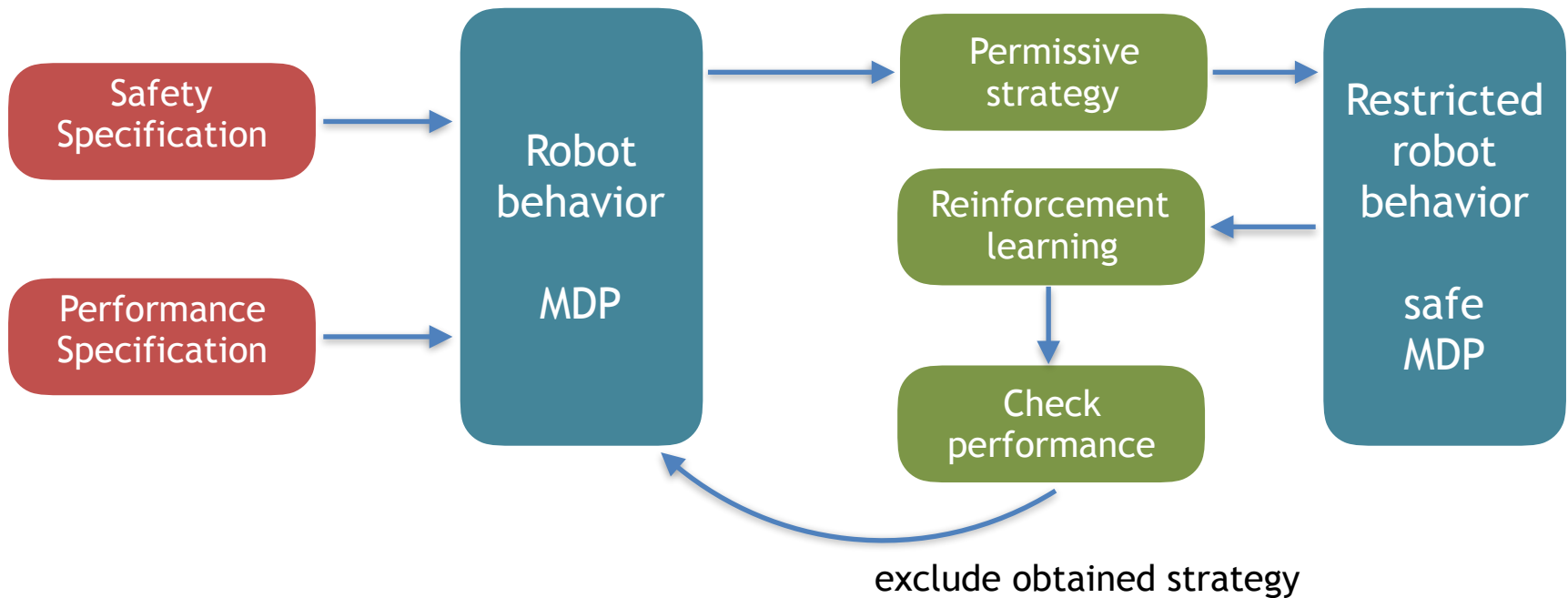
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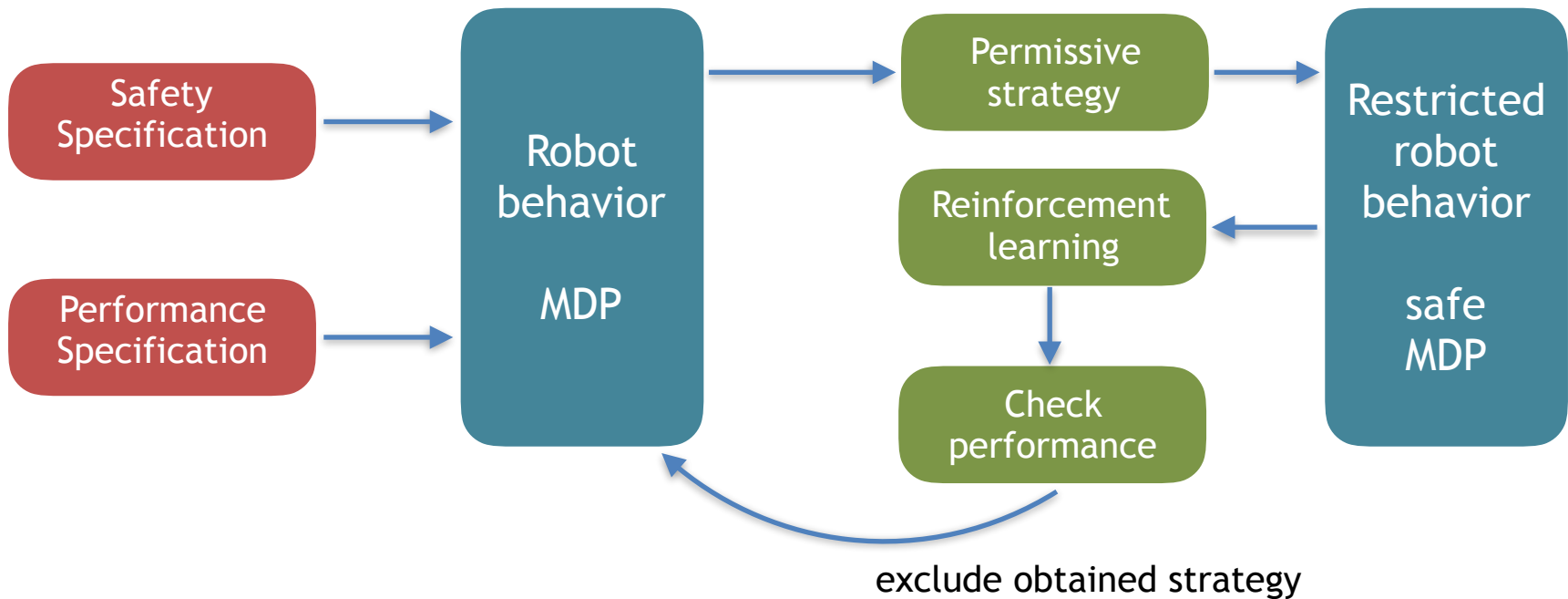
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# Results



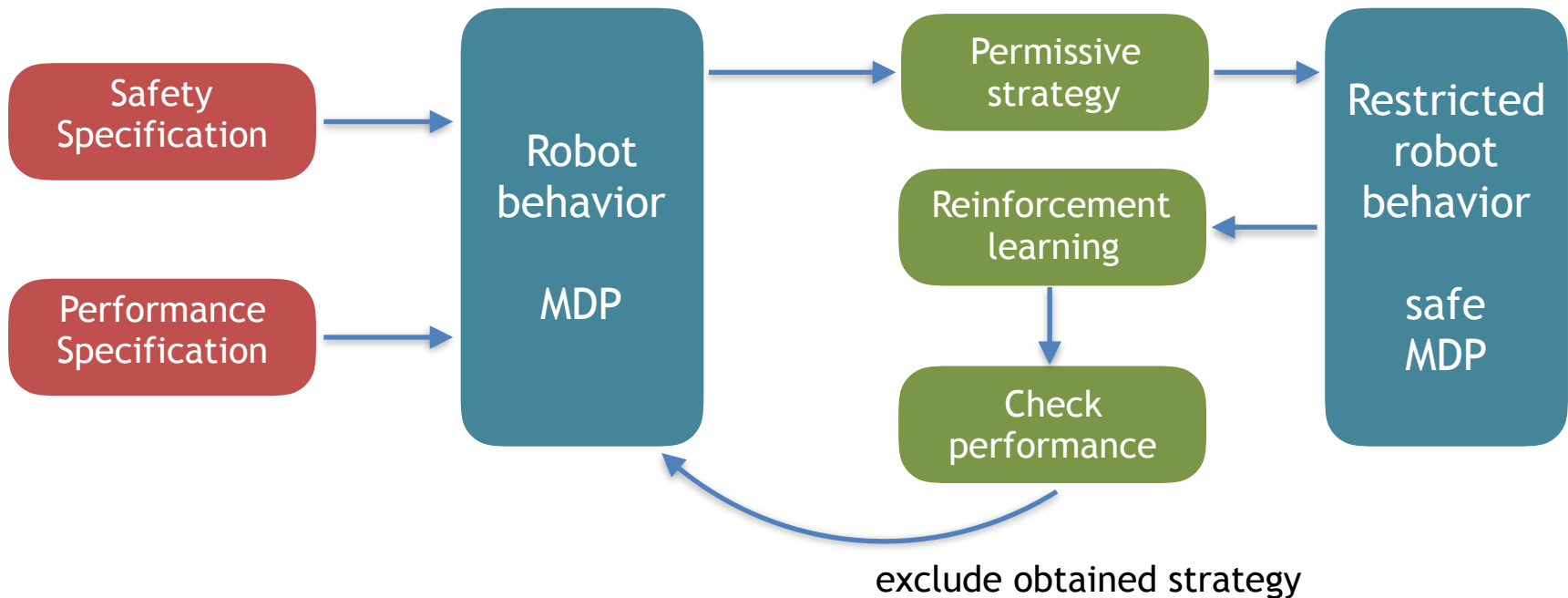


# Results



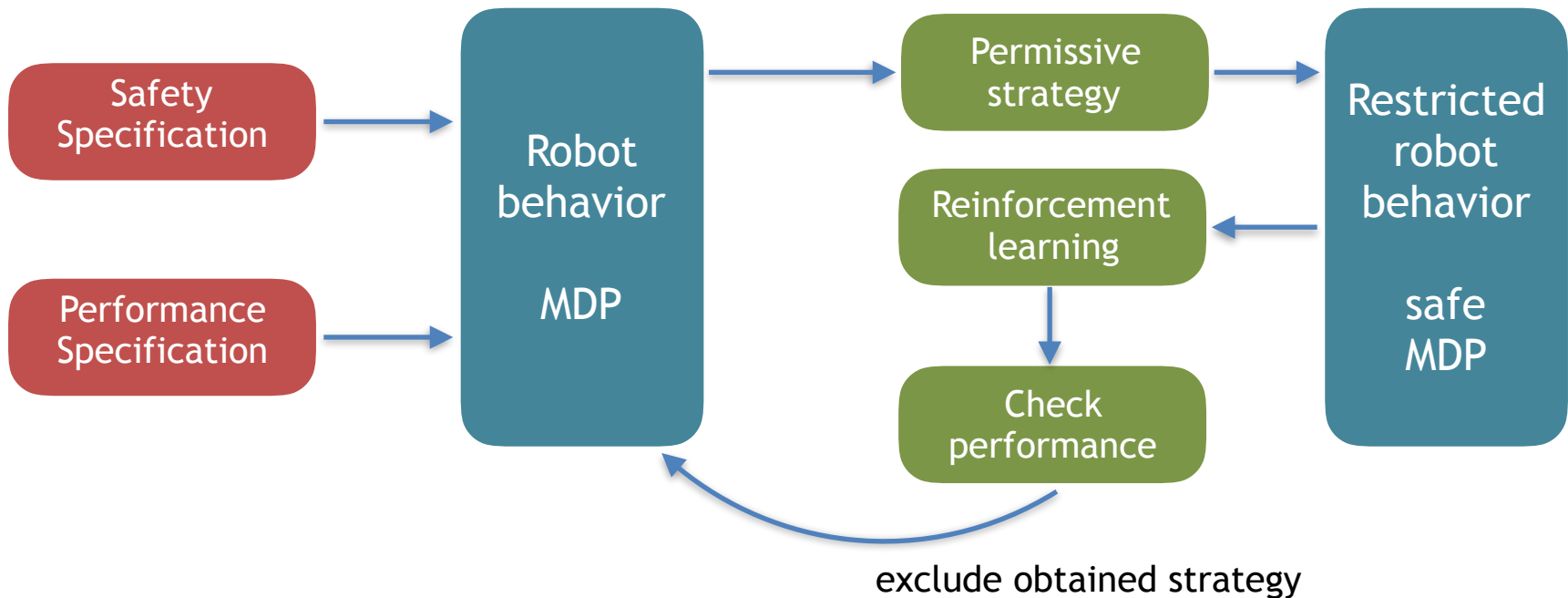
- Soundness: SMT encoding is correct

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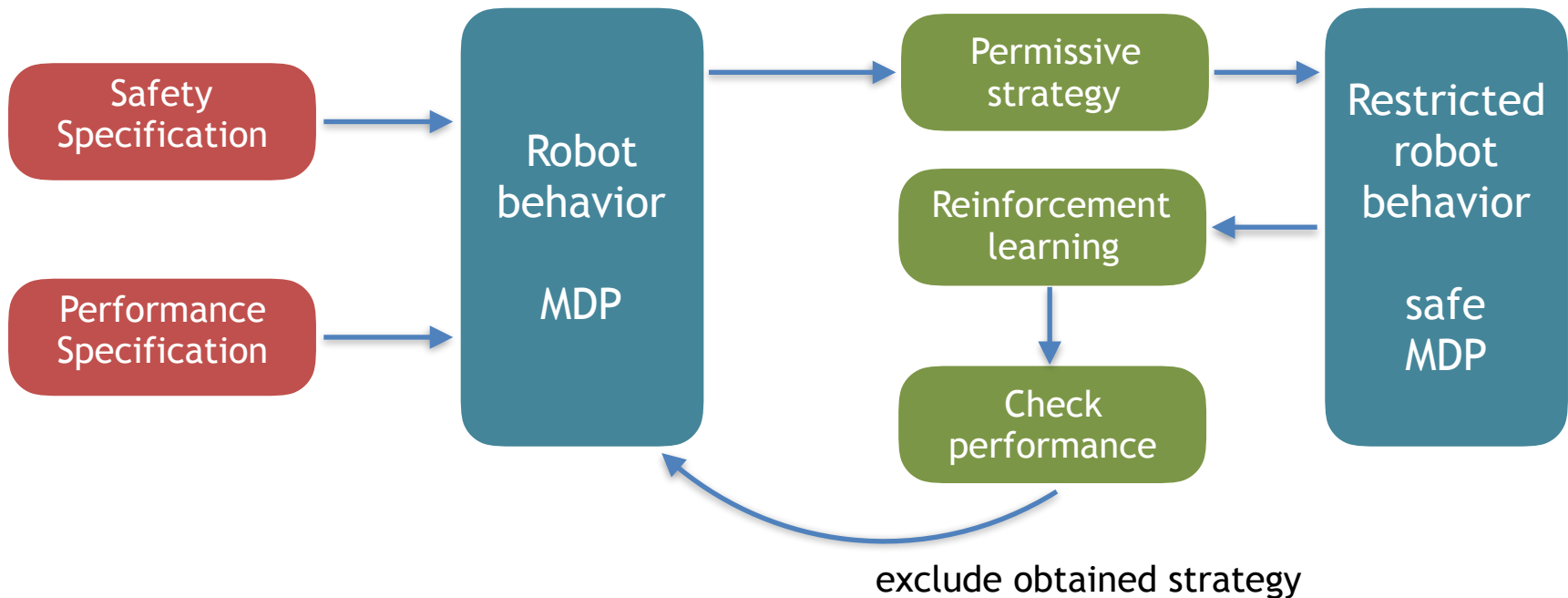
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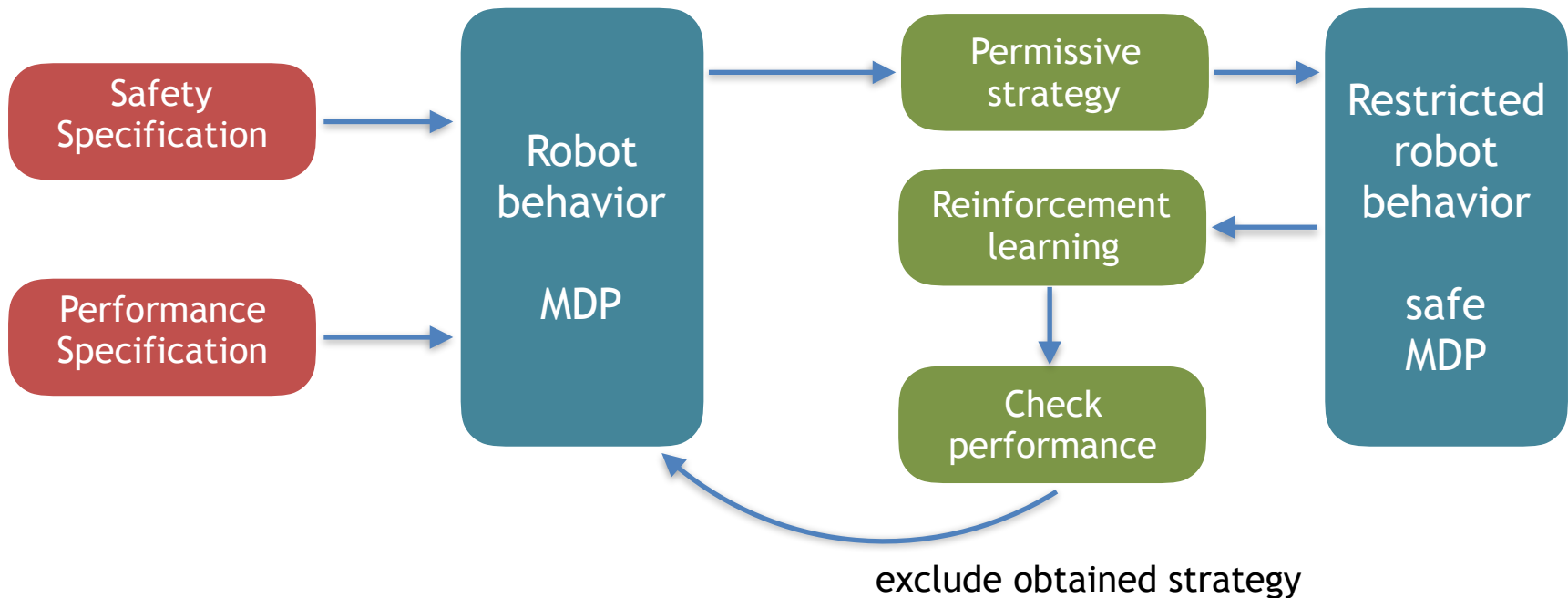
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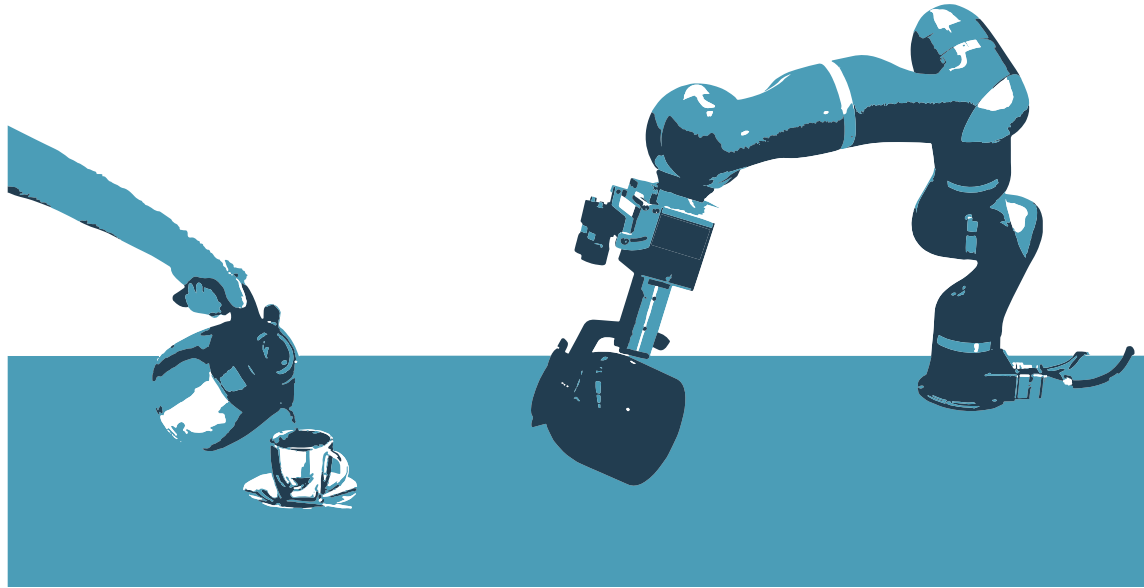
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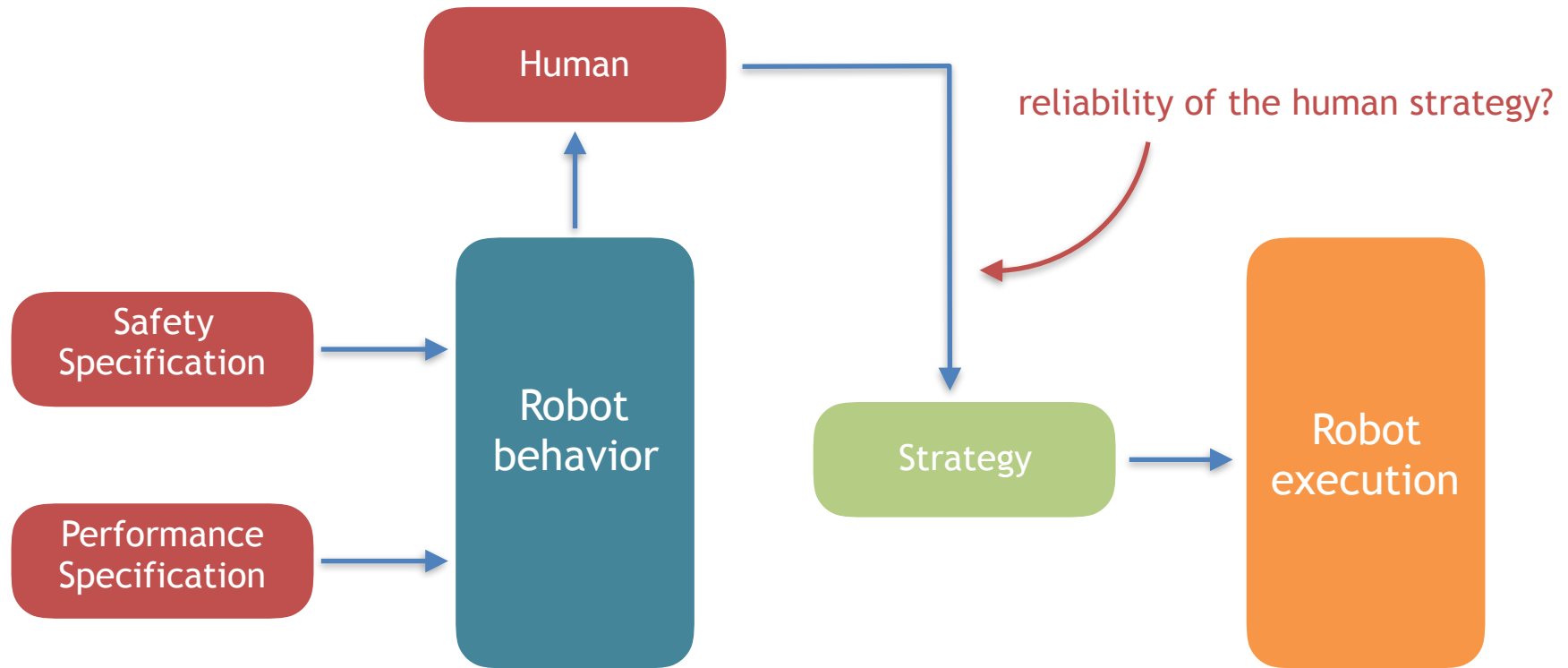
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- No efficient representation for ‘maximally’ permissive strategy
- Utilizing bounds: Significant speedup for costly computation
- Extension to randomized schedulers (non-linear)

# Synthesis of Shared Control Protocols



**Nils Jansen and Ufuk Topcu**

# Shared Control

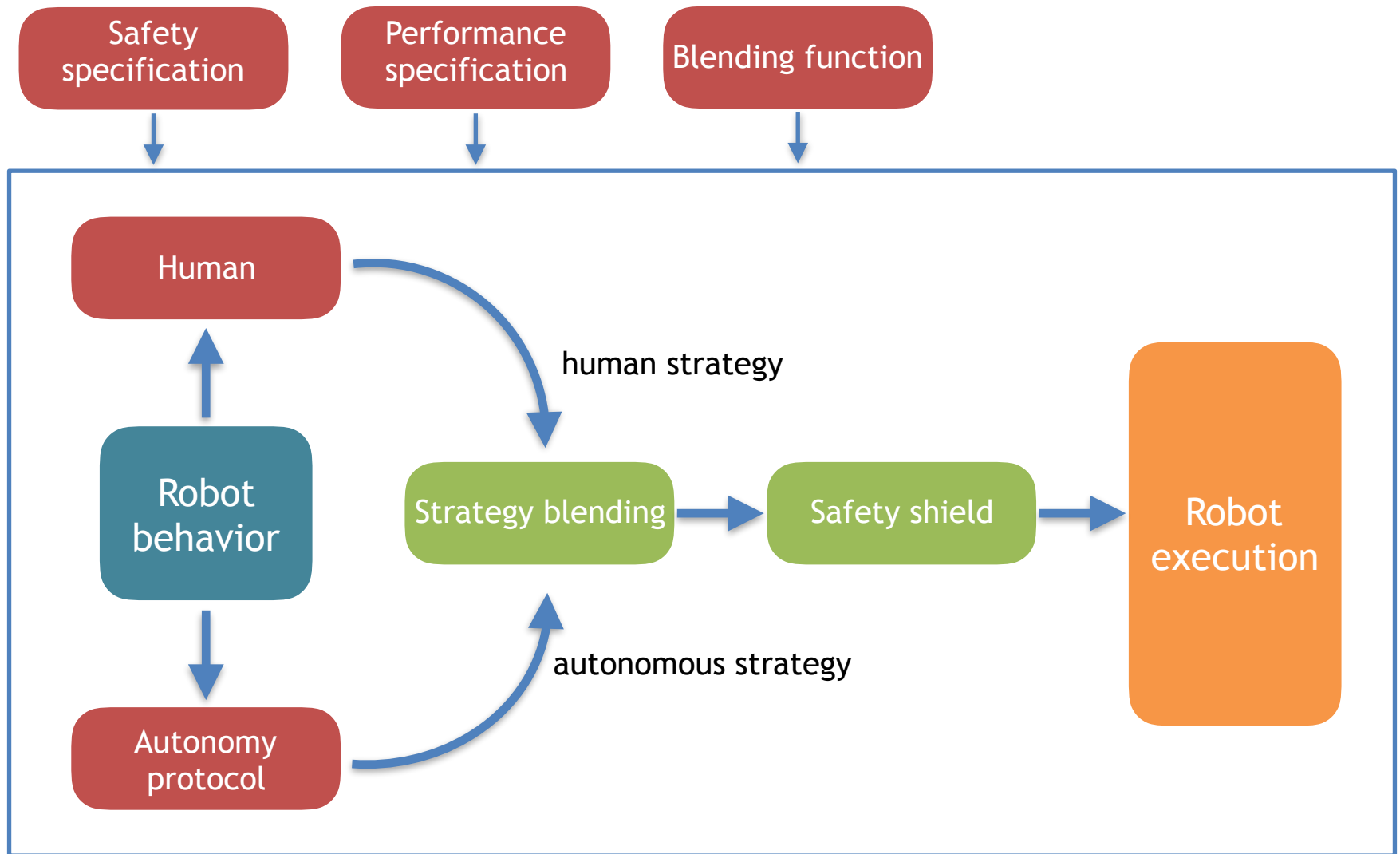


# Brain-actuated Wheelchair

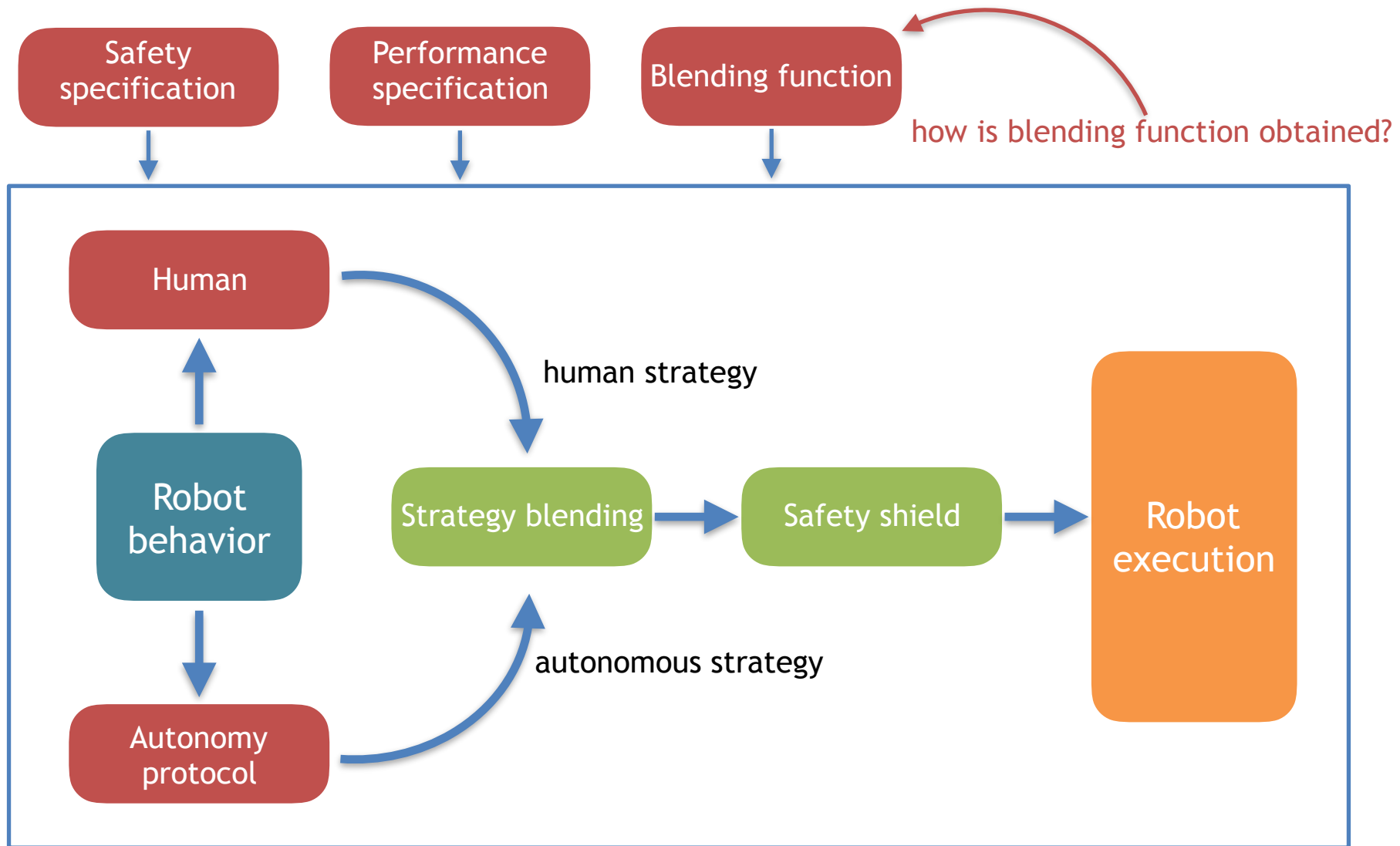




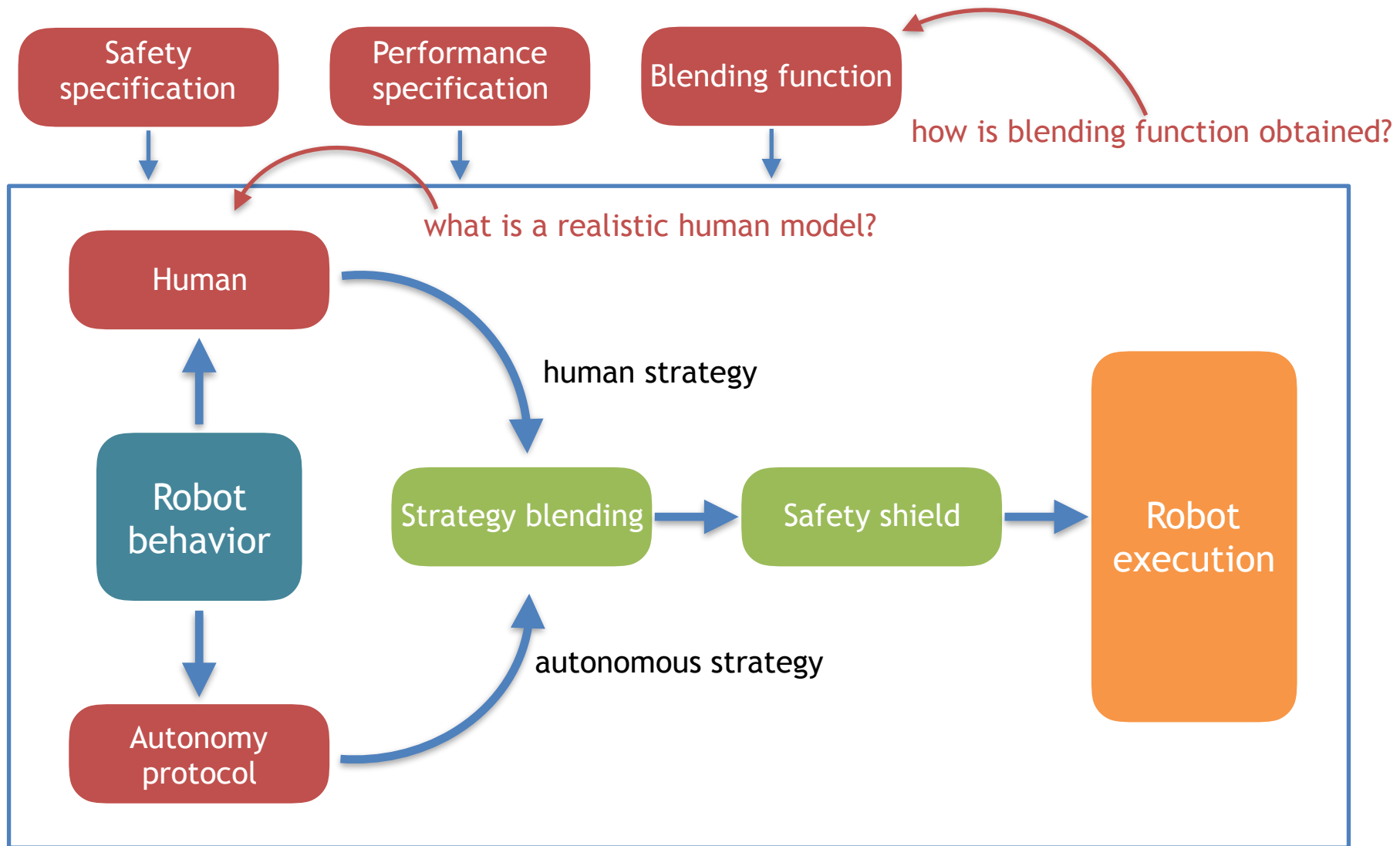
# Shared Control Protocol



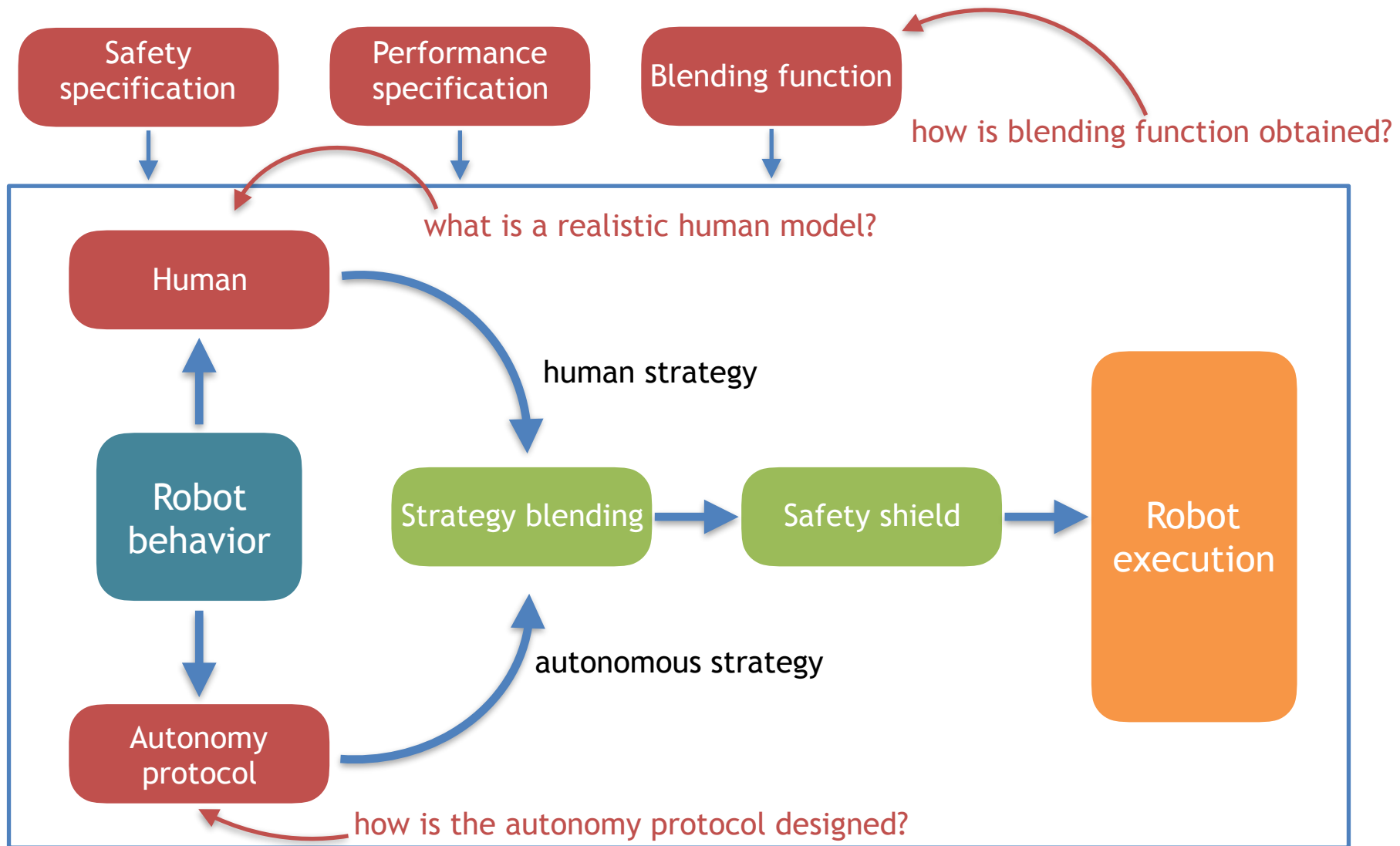
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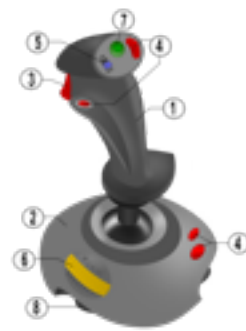
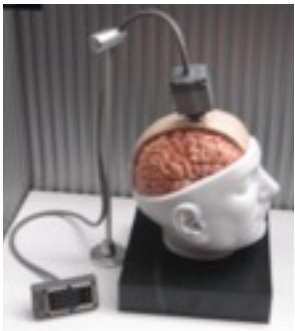
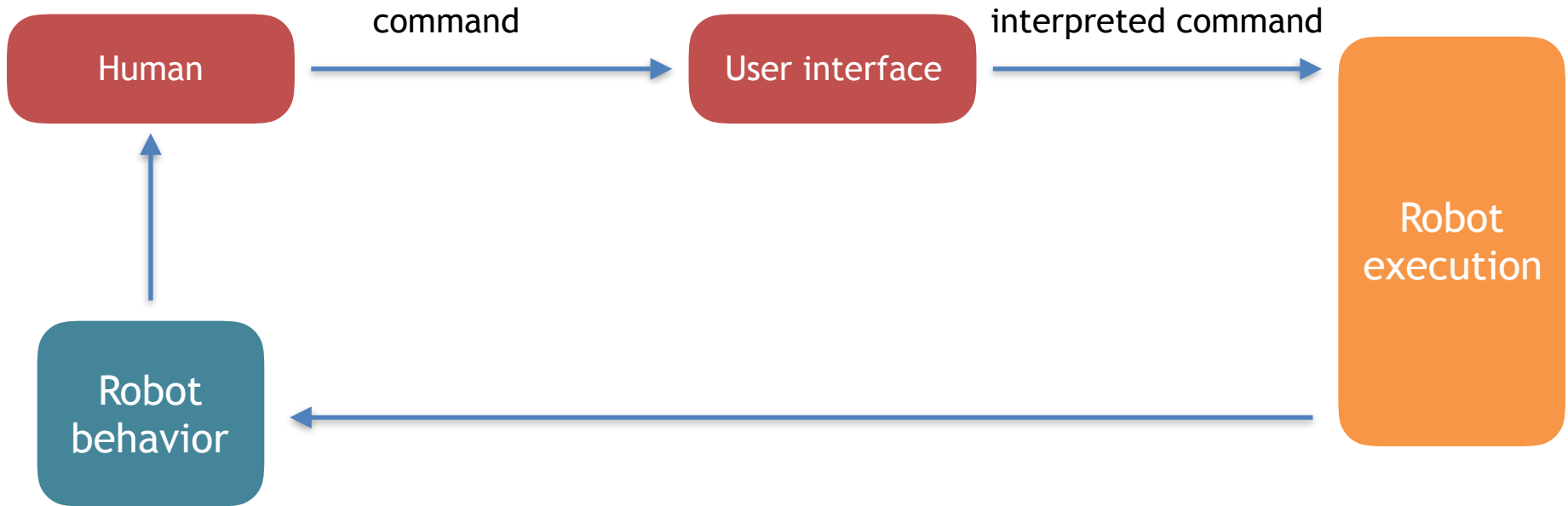
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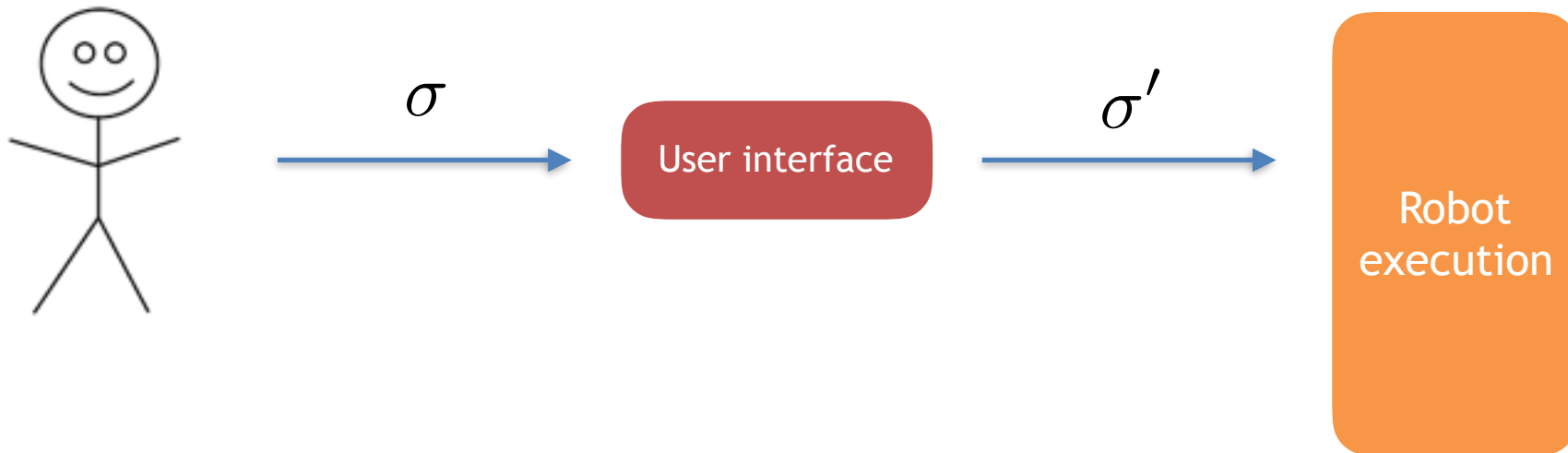
# Human Model



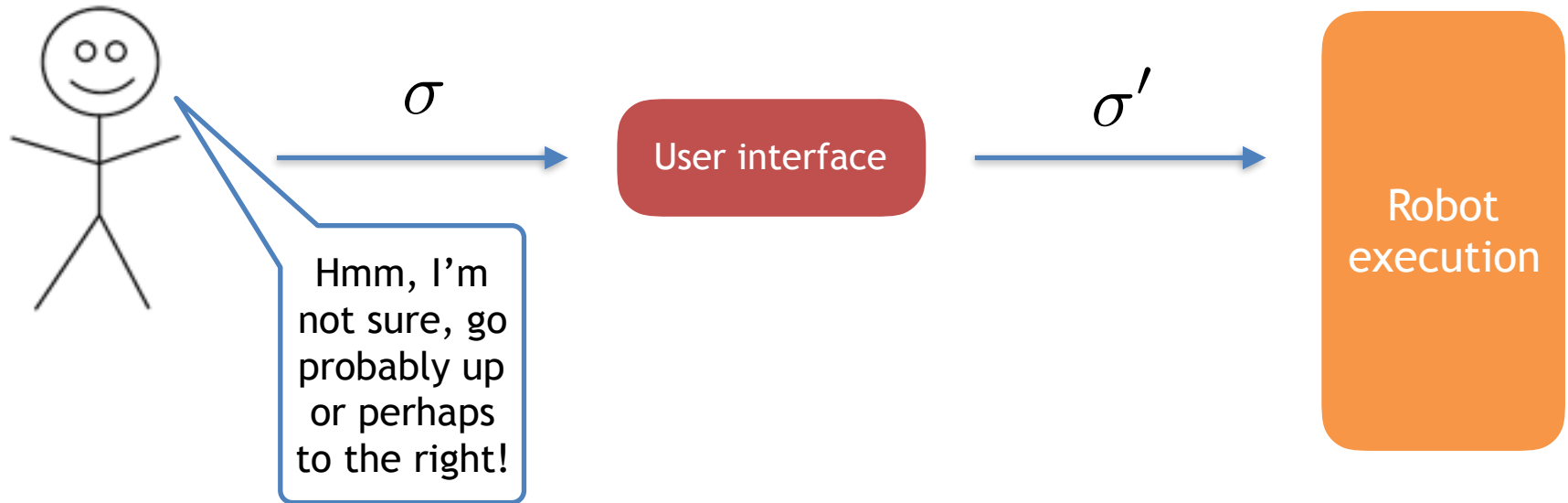
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# Human Model - Randomization

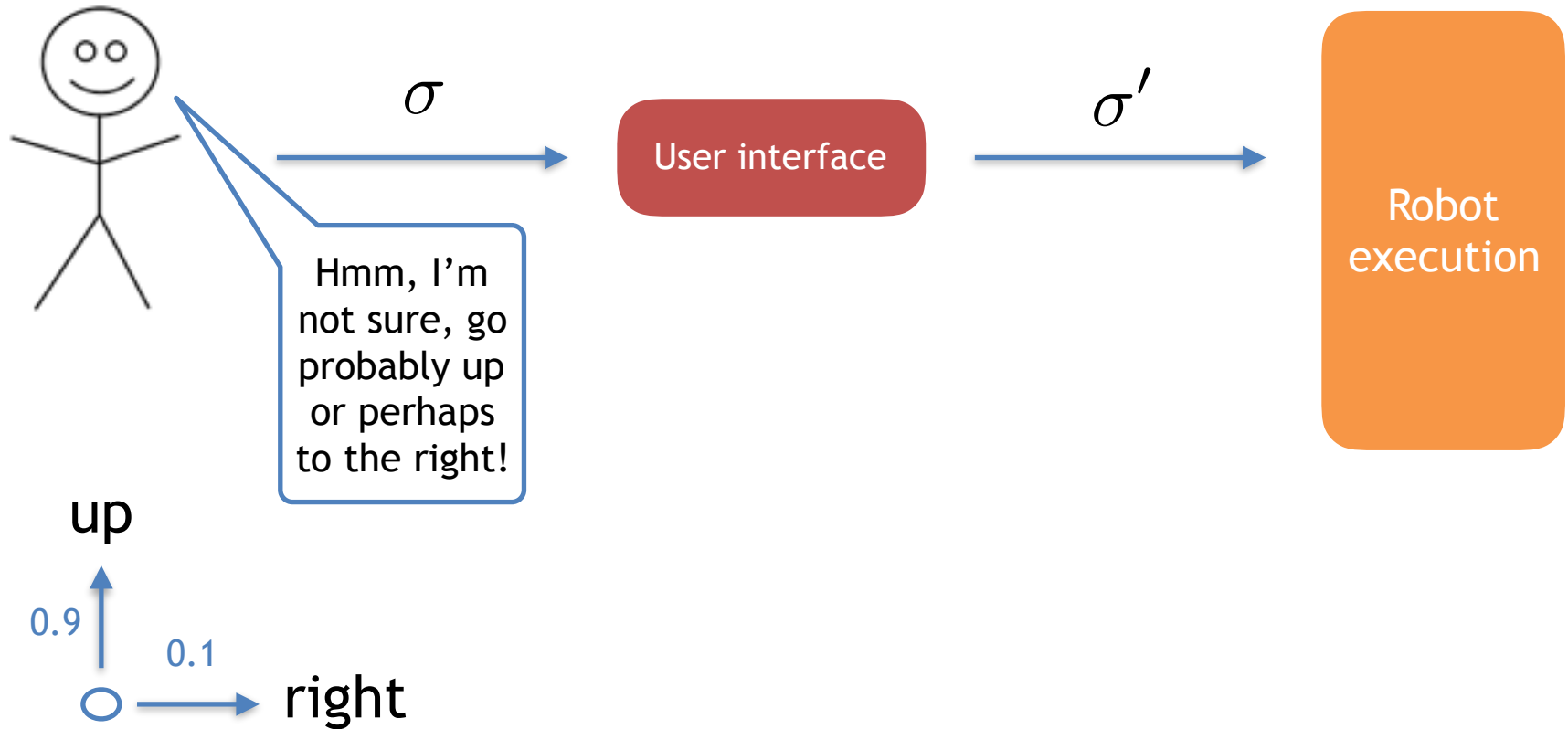


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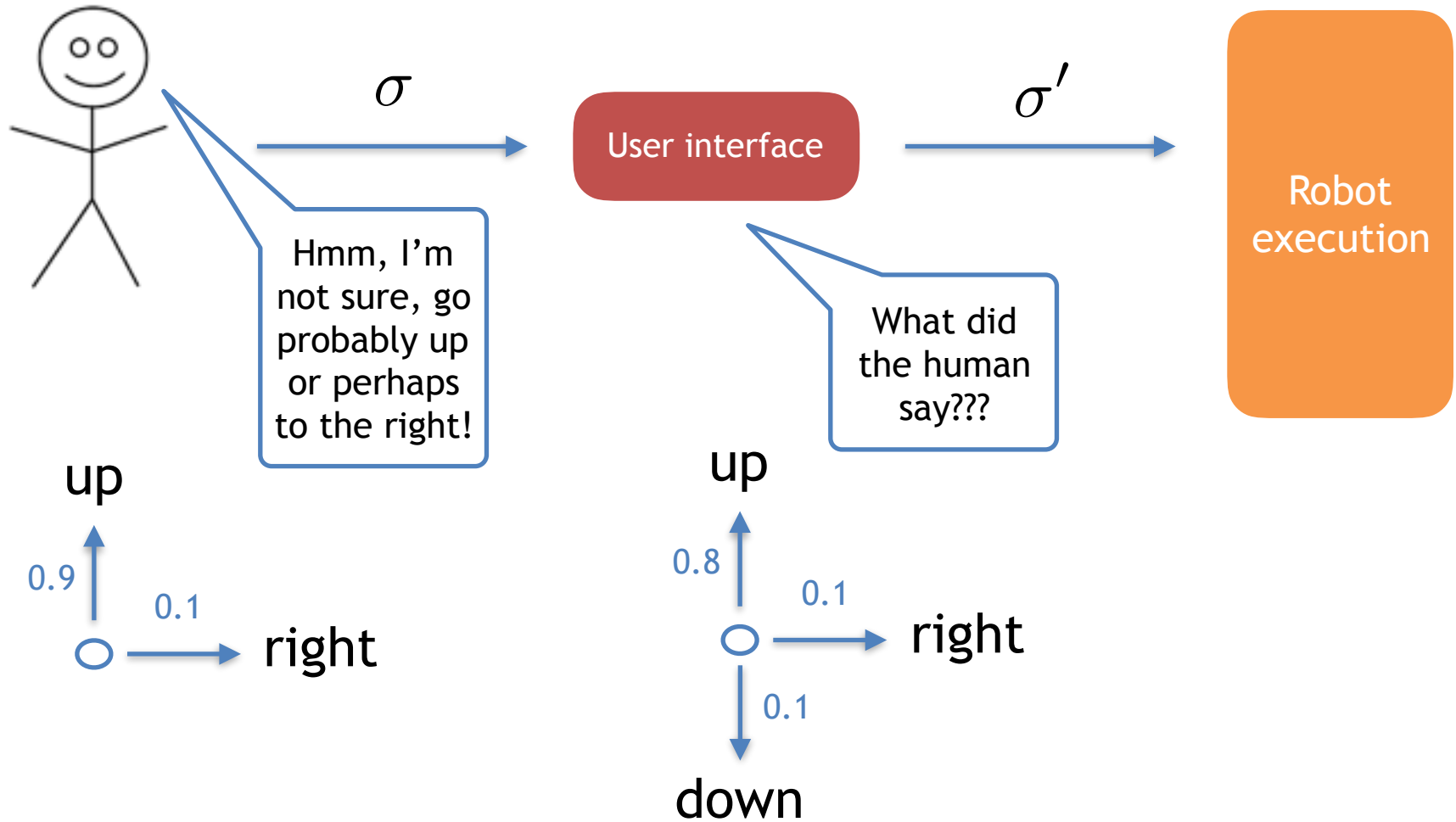




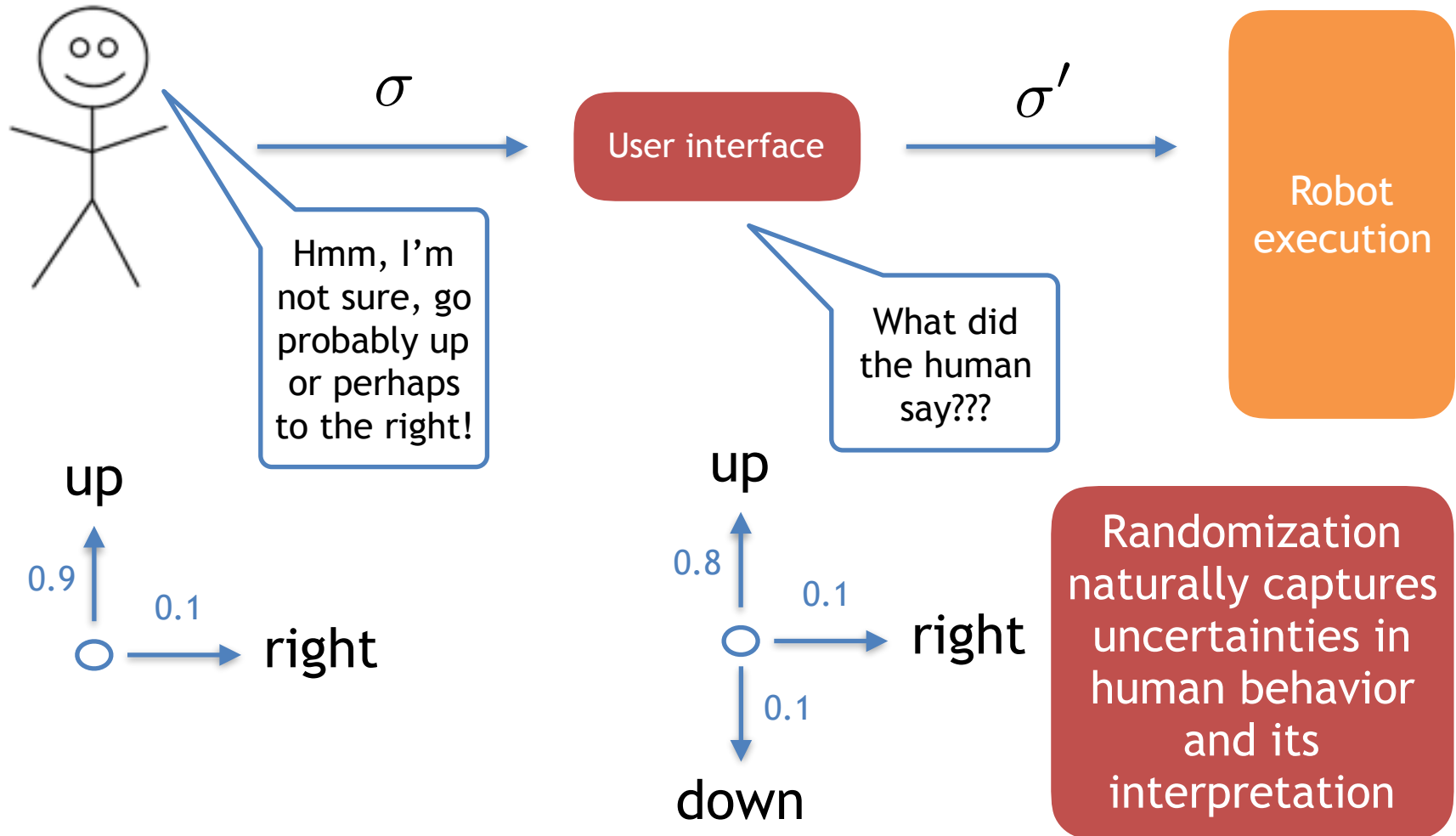
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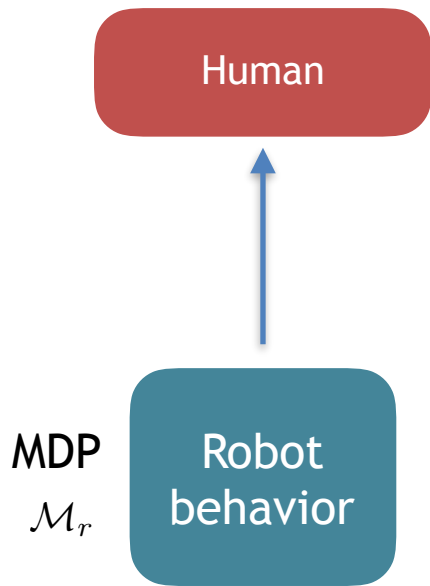
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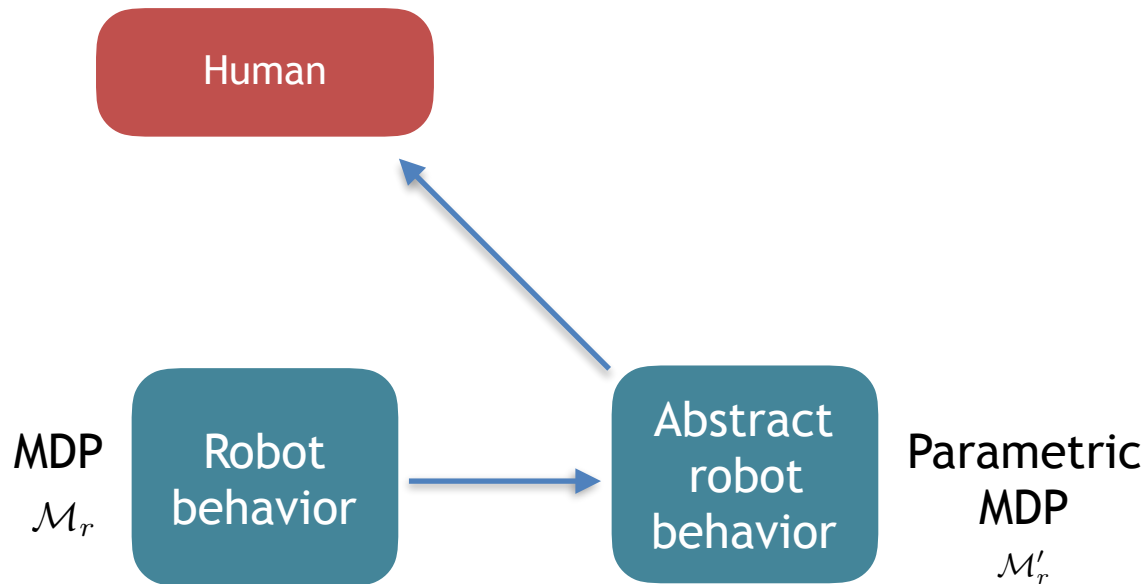
# Human Model - Randomization



# Human Model - Abstraction



# Human Model - Abstraction

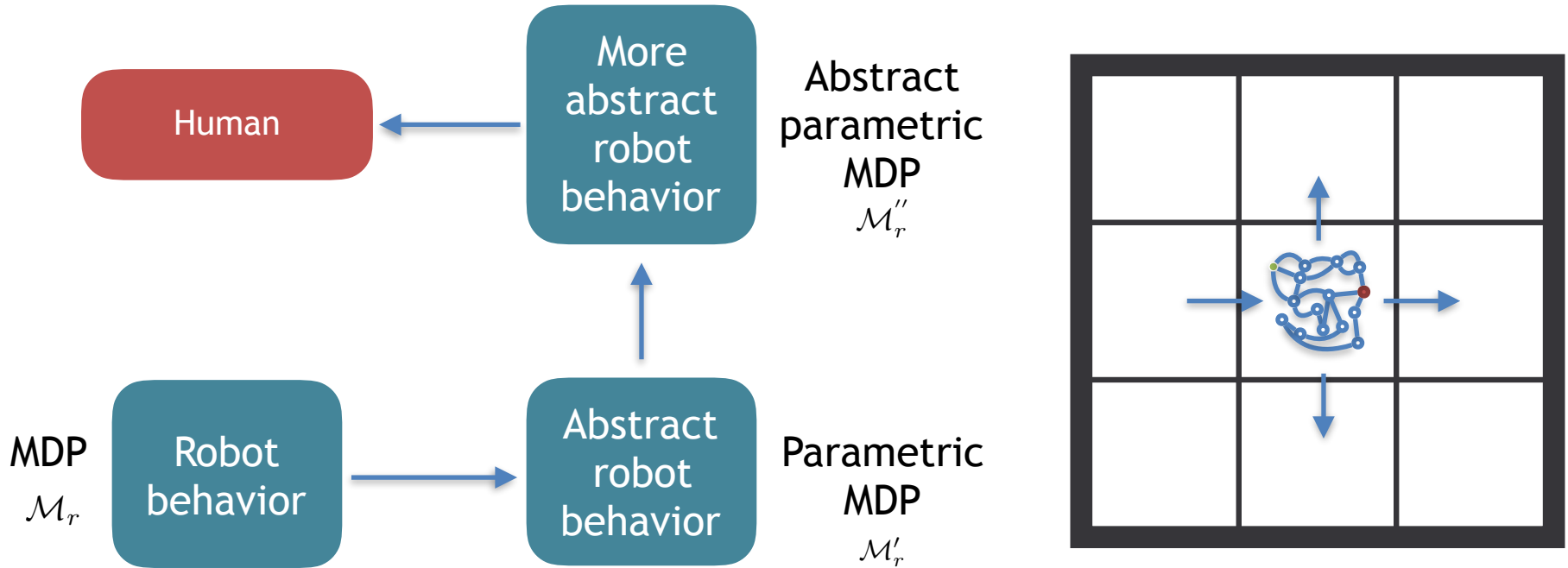


Abstract concrete  
probabilities by parameters

$$p_{high} \text{ if } \mathcal{P}_h(s, \alpha, s') > y$$

$$p_{low} \text{ if } \mathcal{P}_h(s, \alpha, s') \leq y$$

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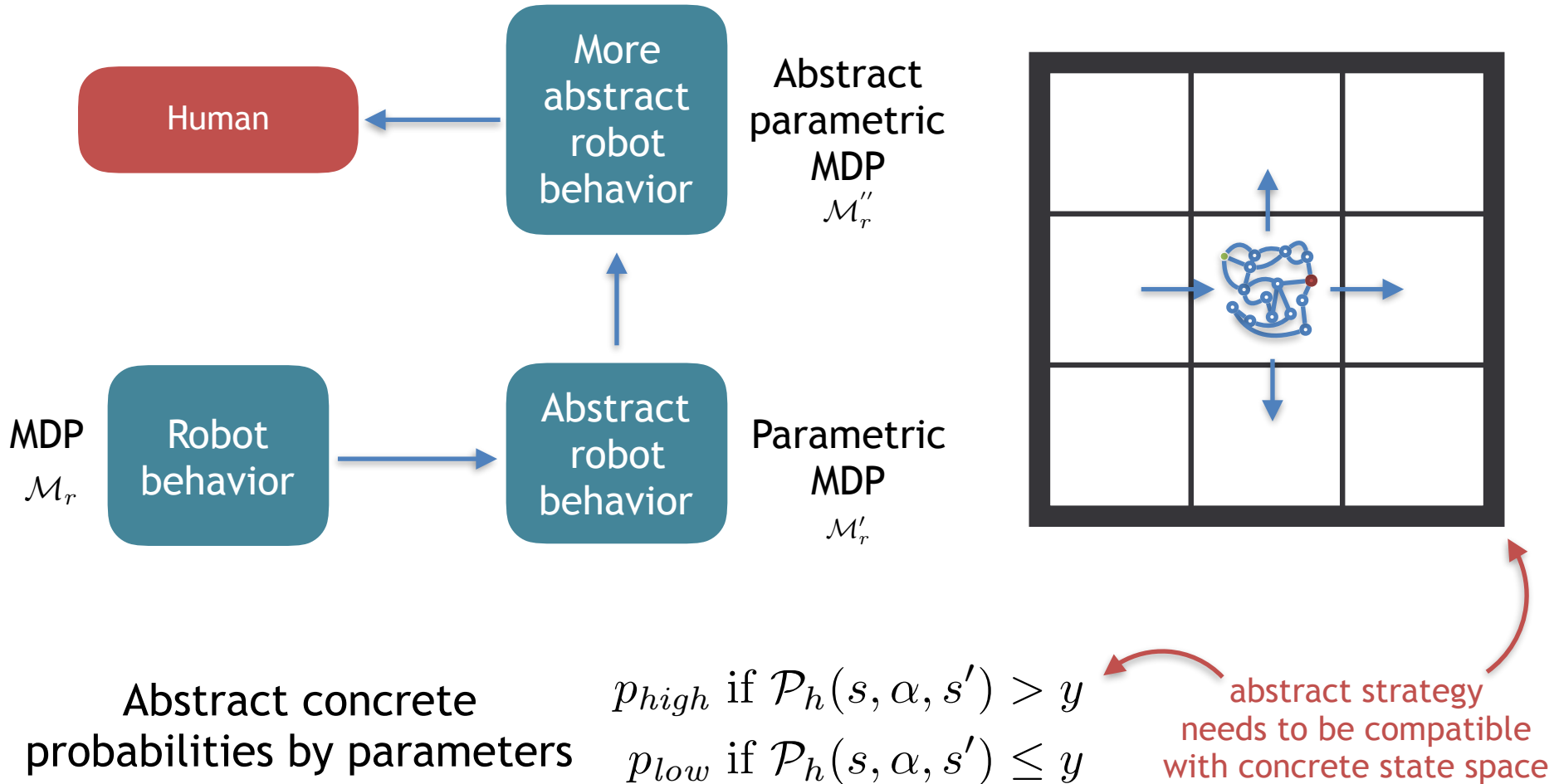


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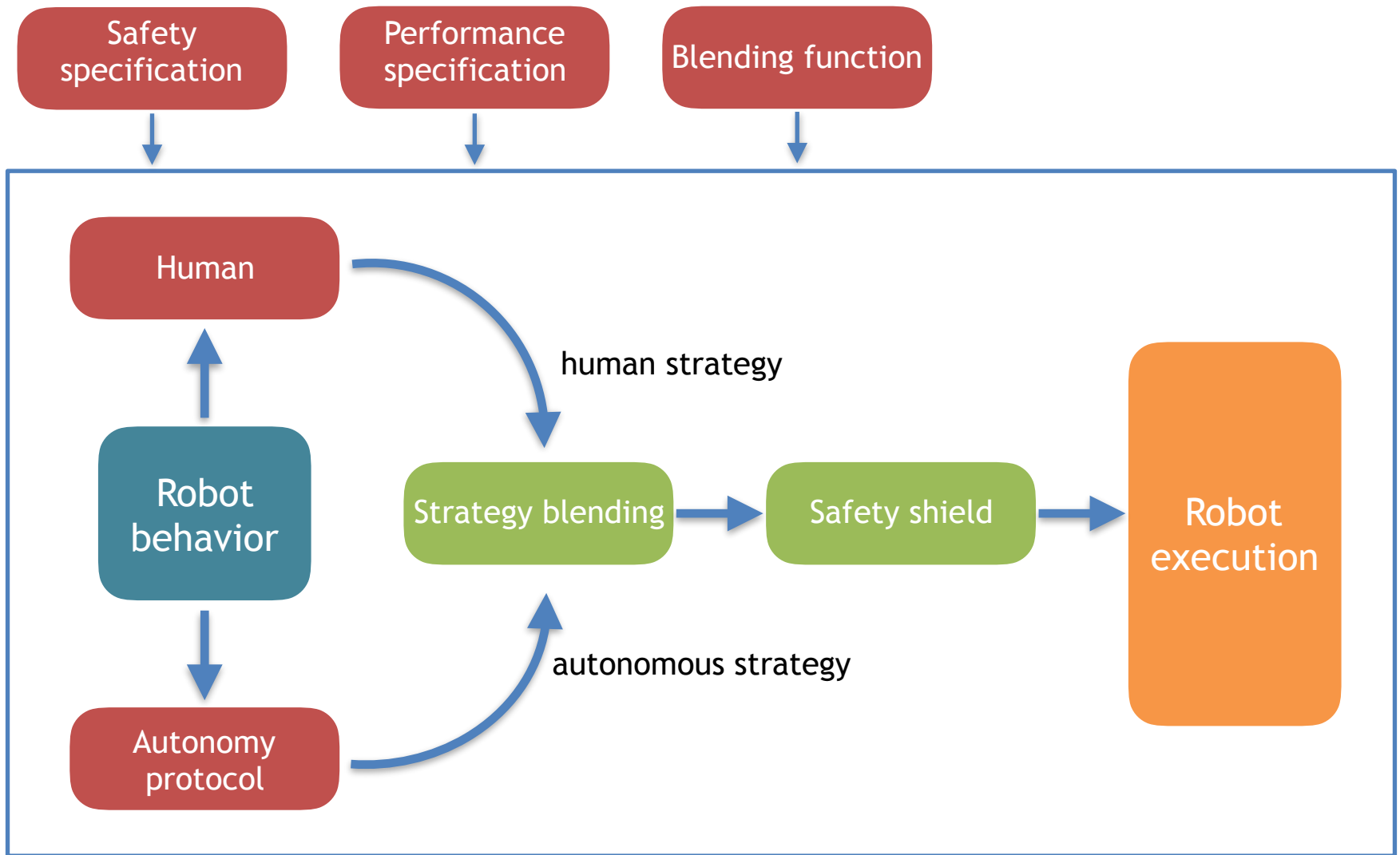
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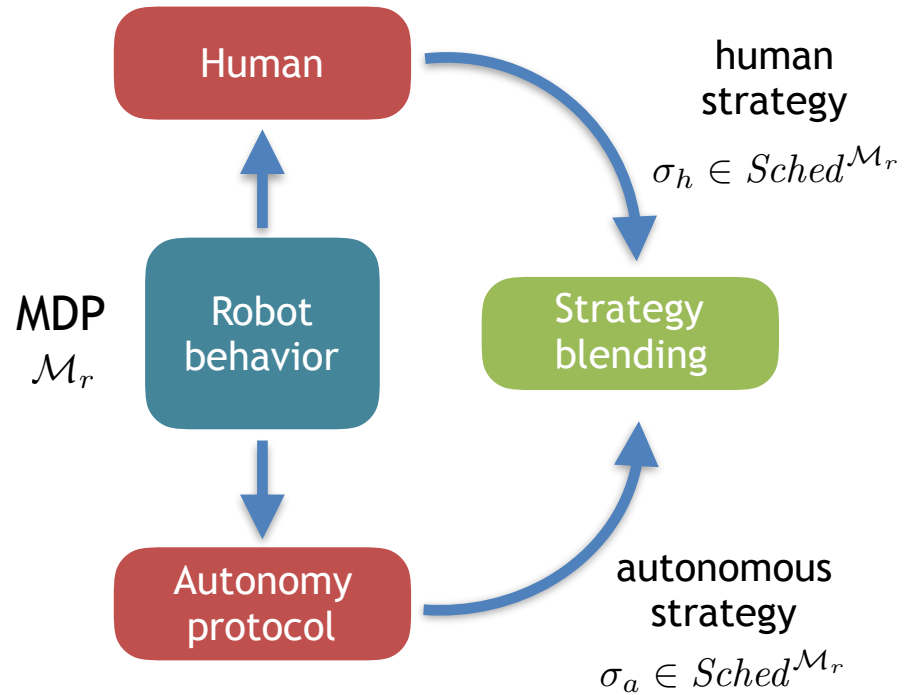


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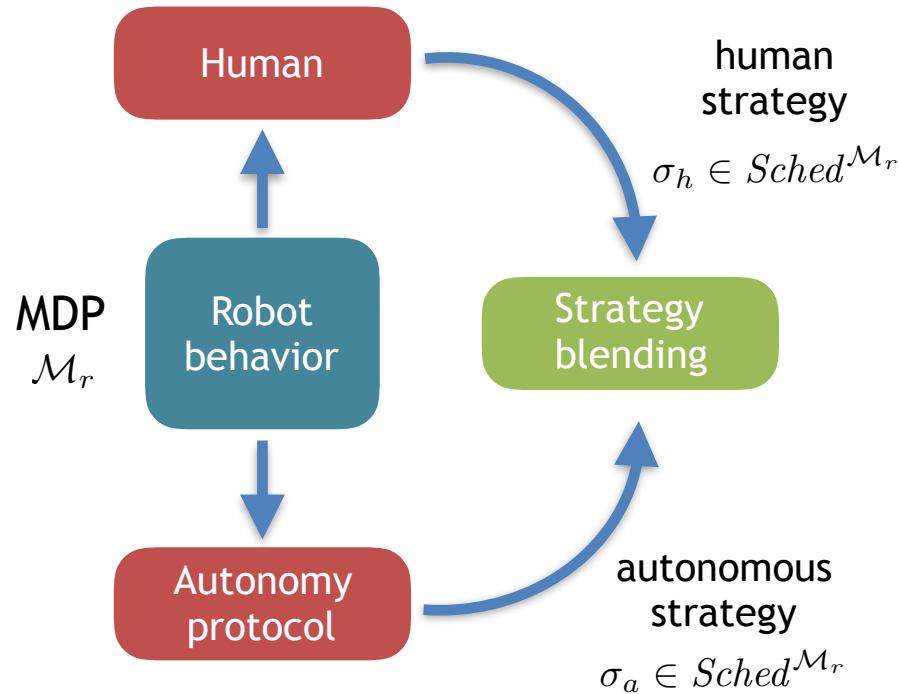




# Strategy Blending



# Strategy Blending

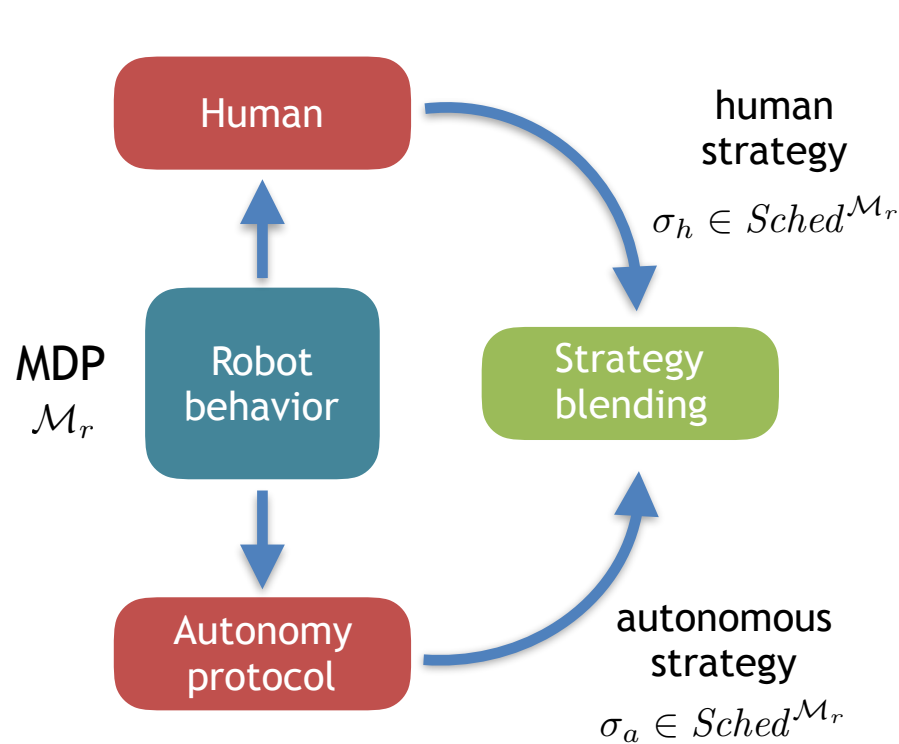


Blending  
function

$$f_a : S \rightarrow [0, 1]$$

- confidence in human
- level of abstraction for human
- accuracy of autonomy protocol

# Strategy Blending



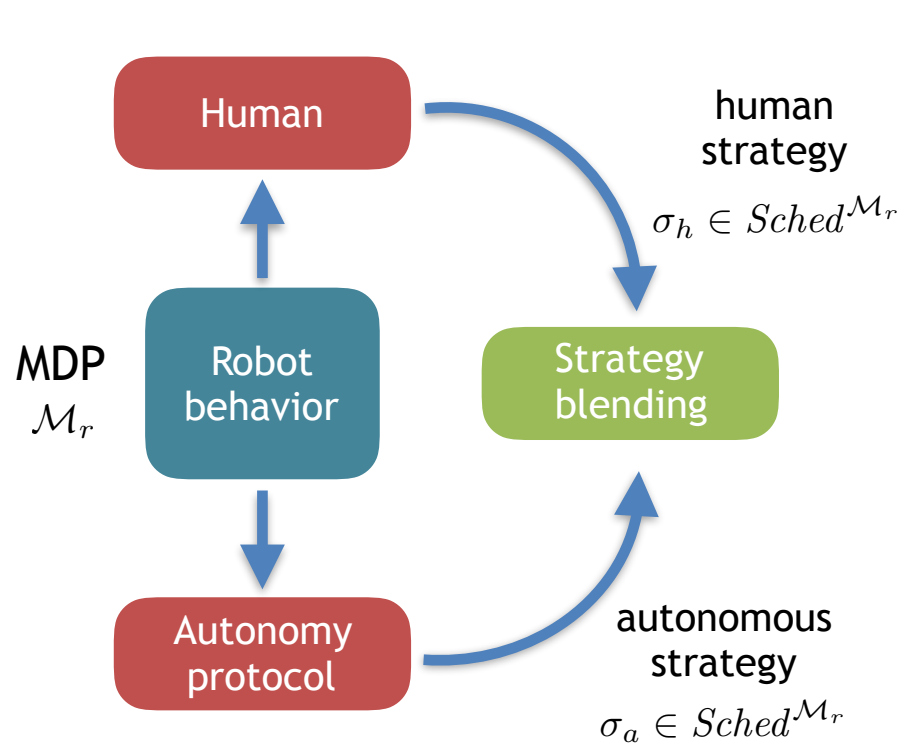
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$$\sigma_{ah}(s, \alpha) = f_a(s) \cdot \sigma_h(s, \alpha) + (1 - f_a(s)) \cdot \sigma_a(s, \alpha)$$

# Strategy Blending



Blending function

$$f_a : S \rightarrow [0, 1]$$

- confidence in human
- level of abstraction for human
- accuracy of autonomy protocol

up

right

without randomization  
blending not well-defined

$$\sigma_{ah}(s, \alpha) = f_a(s) \cdot \sigma_h(s, \alpha) + (1 - f_a(s)) \cdot \sigma_a(s, \alpha)$$

# Design of Autonomy Protocol

Compute autonomous strategy such that

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# Design of Autonomy Protocol

Compute autonomous strategy such that

- blended strategy deviates minimally from human strategy

minimal additive perturbation of human strategy

- safety and performance specs are satisfied

model checking

- if not feasible, obtain new blending function

minimal deviation from given blending function or safety shield

# Design of Autonomy Protocol

minimize  $\|(\delta^{s\alpha} \mid s \in S, \alpha \in Act)\|$

such that

$$p_{s_I} \leq \lambda$$

$$\forall s \in T. \quad p_s = 1$$

$$\forall s \in S. \quad \sum_{\alpha \in Act} \sigma_a^{s,\alpha} = \sum_{\alpha \in Act} \sigma_{ah}^{s,\alpha} = 1$$

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non-linear programming

# Current/Future Work

- avoid non-linear program
- model repair
- convex form
- include permissiveness into autonomy
- start conducting real case studies

**Thank you for your attention!**