Software Analysis Workbench (SAW)

A Tool Suite for Compositional Cryptographic Verification

Galois, Inc.
Presented by Joe Hendrix

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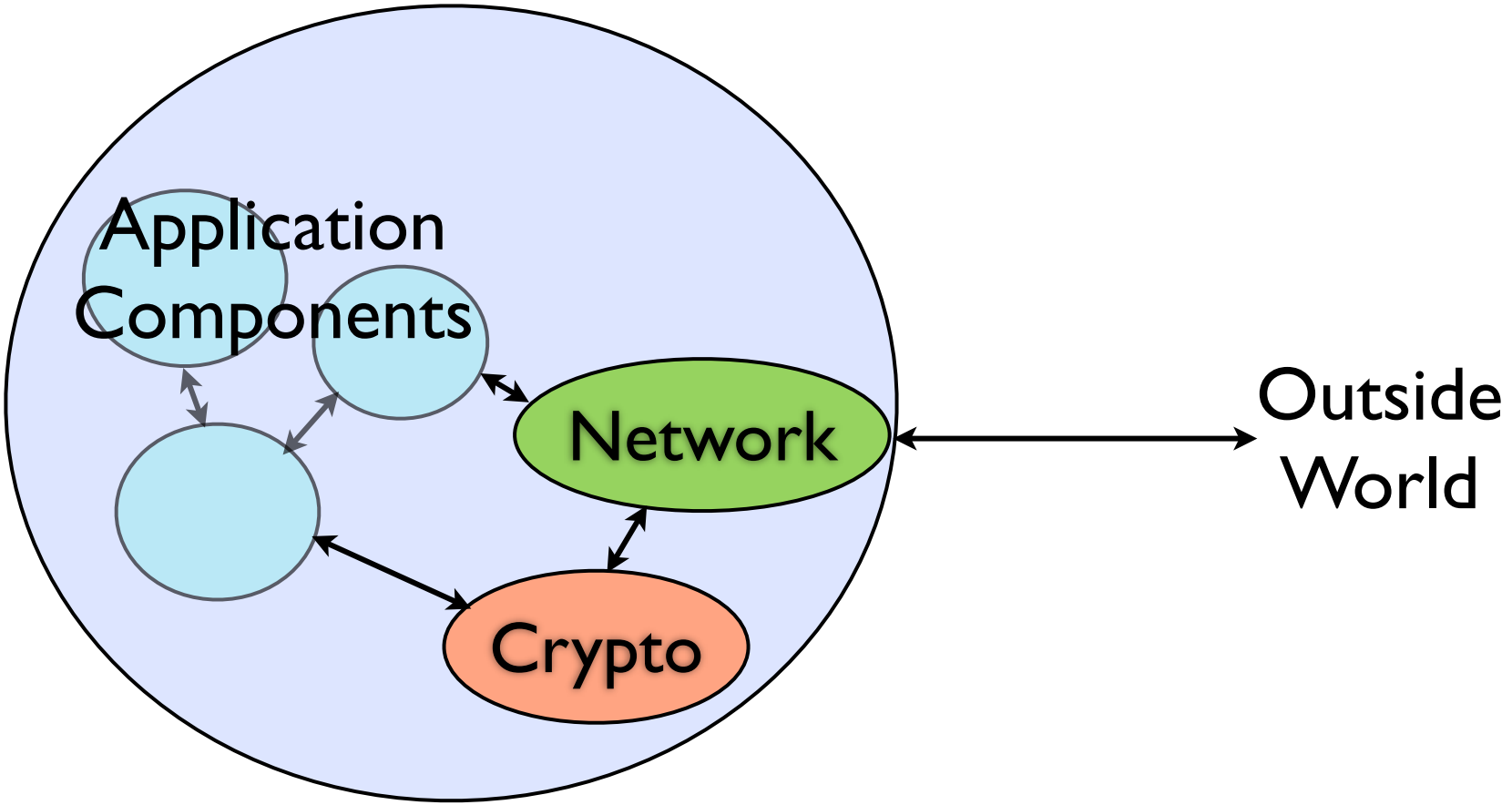
Joe Hurd

Cryptographic Verification

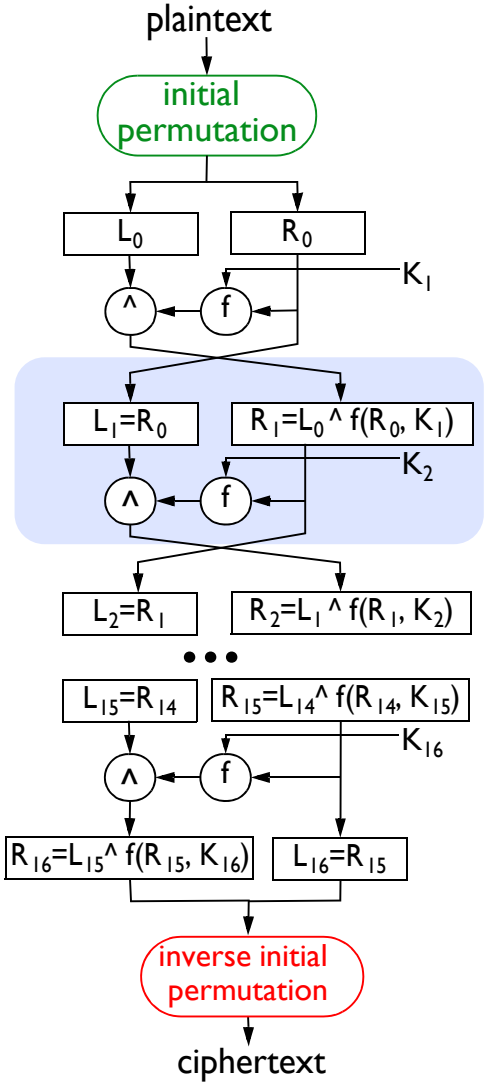


Cryptographic algorithms are a small, but critical component of networked systems.

Structure of a Block Cipher

```
des : ([64],[56]) -> [64];
des (pt, key) = permute (FP, last)
  where {
    pt' = permute(IP, pt);
    iv = [| round(lr, key, rnd)
         || rnd <- [0 .. 15]
         || lr <- [(split pt')] # iv
         |];
    last = join (swap (iv @ 15));
    swap [a b] = [b a];
  };
```

```
round : ([2][32], [56], [4]) -> [2][32];
round([l r], key, rnd) = [r (l^f(r, kx))]
  where {
    kx = expand(key, rnd);
    f(r,k) =
      permute(PP, SBox(k^permute(EP, r)));
  };
```



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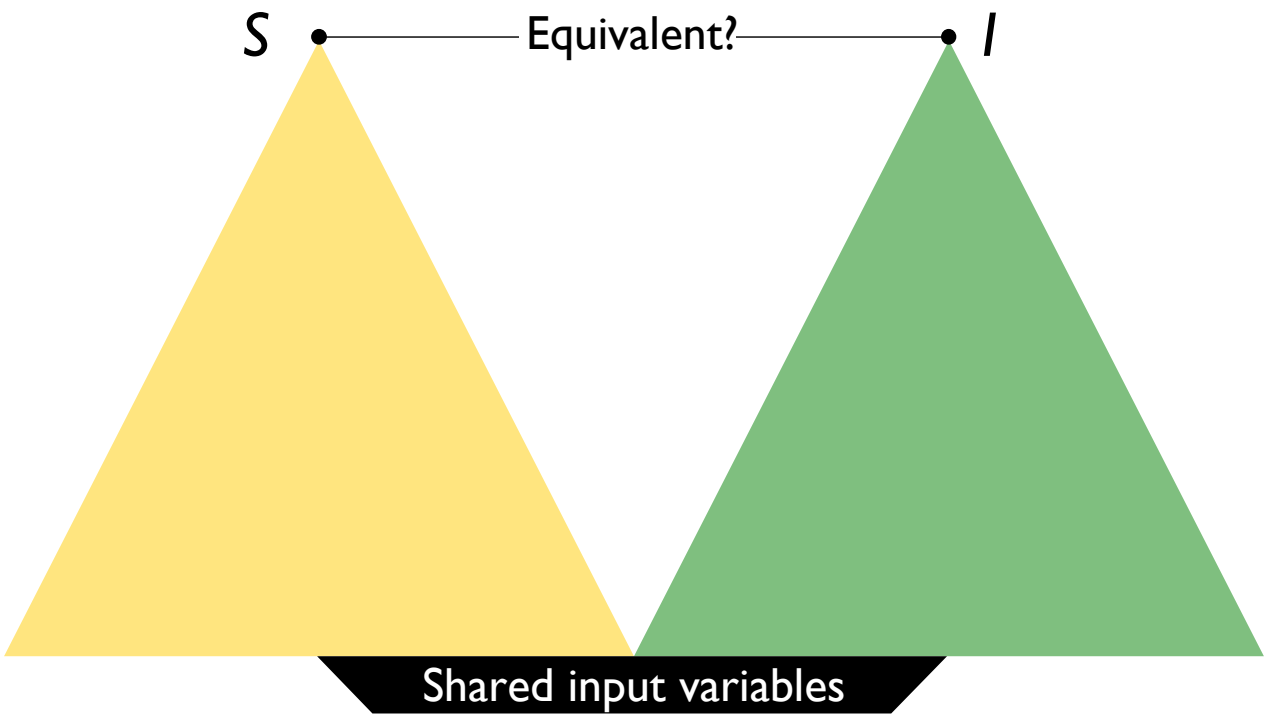
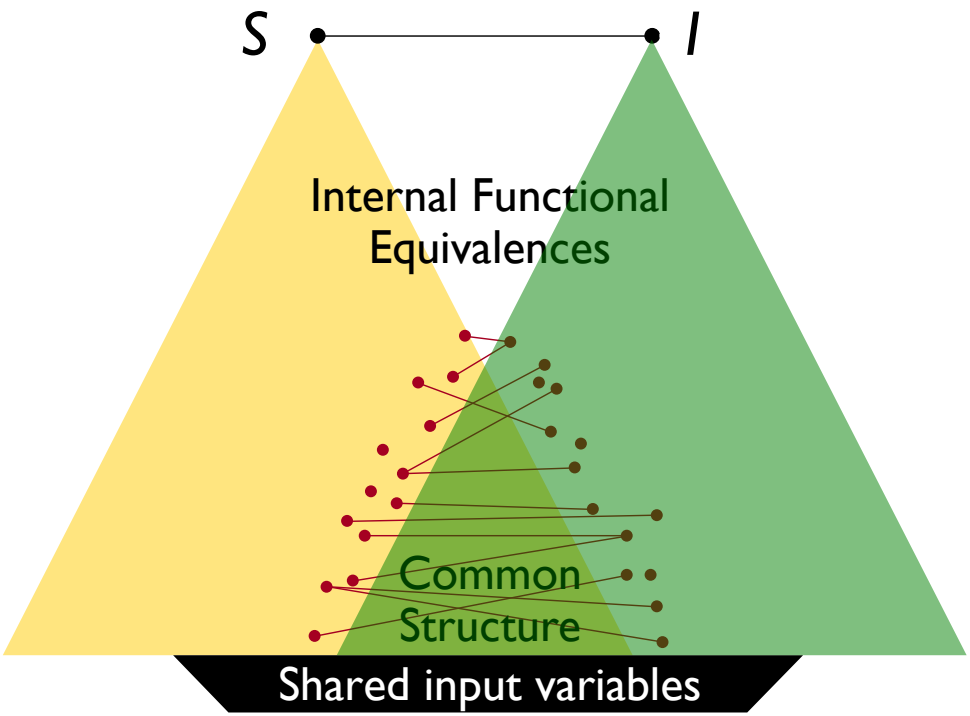
Equivalence Checking



From A. Biere, "SAT in Formal Hardware Verification"

Equivalence Checking



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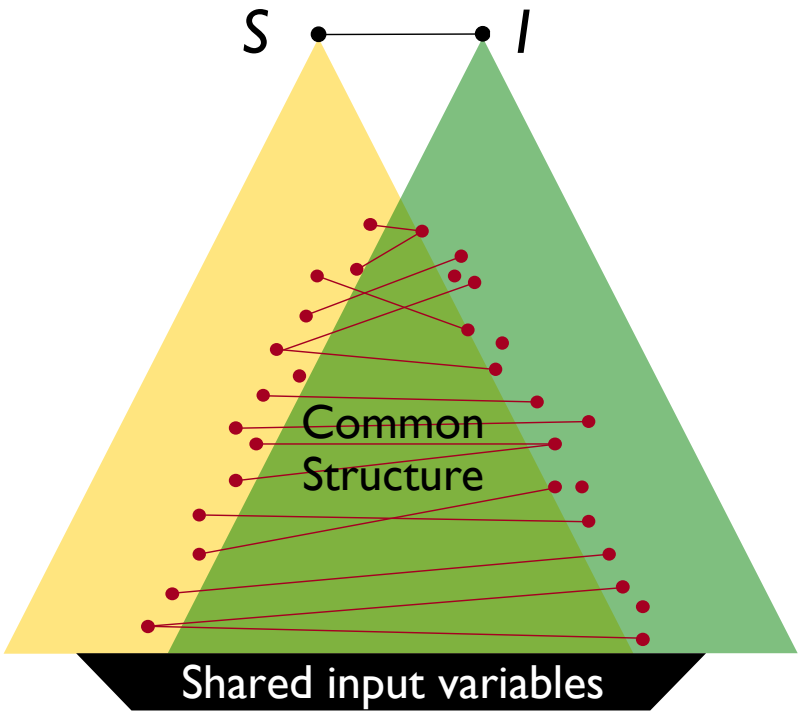
Equivalence Checking



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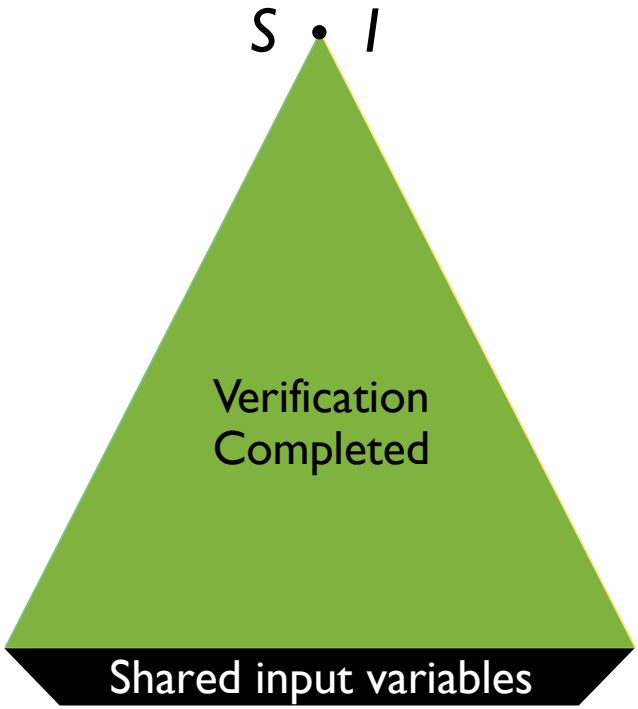
Equivalence Checking



From A. Biere, "SAT in Formal Hardware Verification"

Public Key Cryptography

RSA & Diffie Hillman

Elliptic Curve Cryptography (ECC)

Homomorphic Encryption?

Public key cryptography contain hard algorithms for automated verification, such as

- Large Word Multiplication
- Field Division
- Modular Exponentiation

Public Key Cryptography

RSA & Diffie Hillman

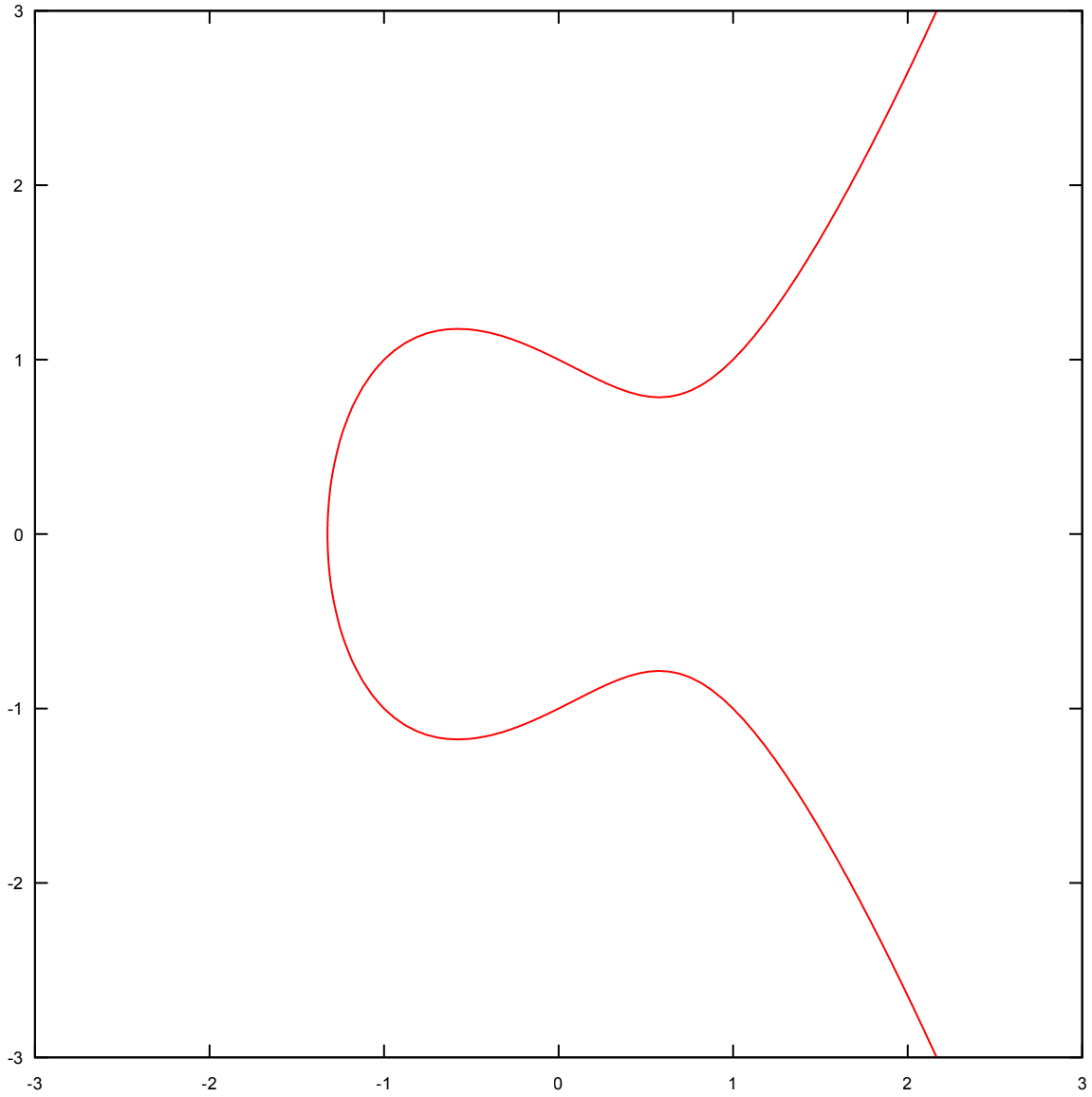
Elliptic Curve Cryptography (ECC)

Focus of this talk

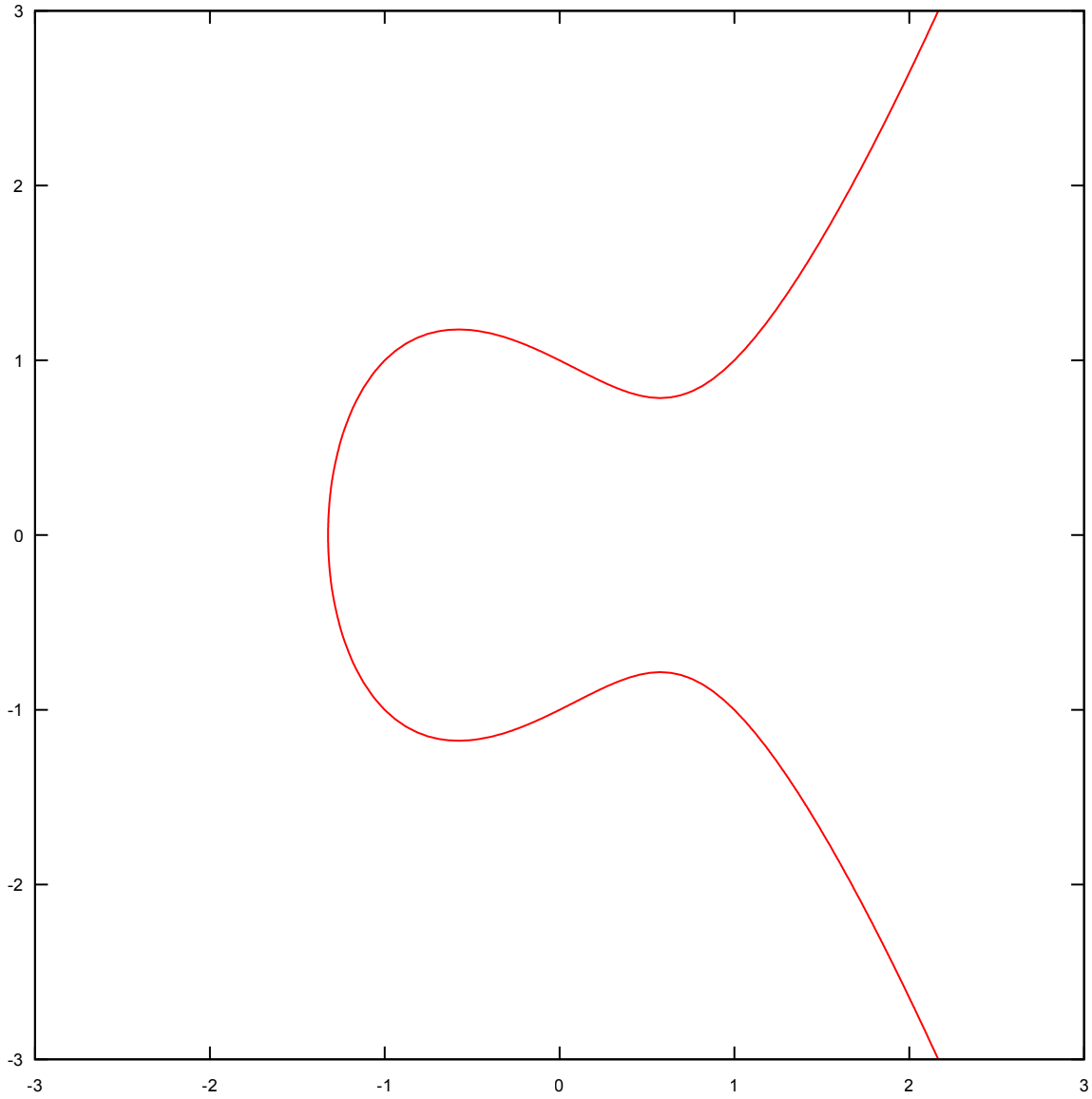
Homomorphic Encryption?

Public key cryptography contain hard algorithms for automated verification, such as

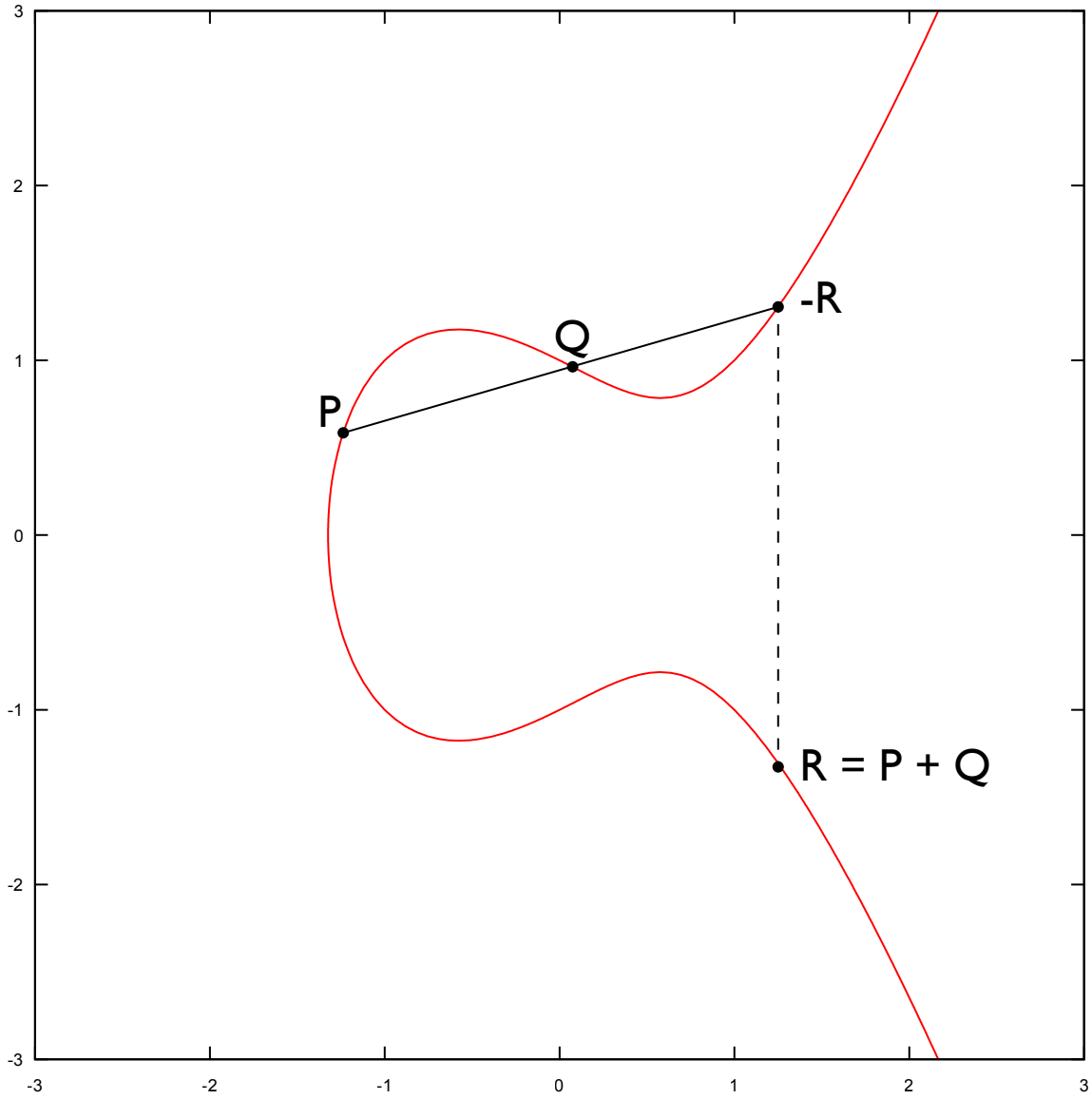
- Large Word Multiplication
- Field Division
- Modular Exponentiation



$$y^2 = x^3 + ax + b$$



$$y^2 = x^3 - x + 1$$



$$y^2 = x^3 - x + 1$$

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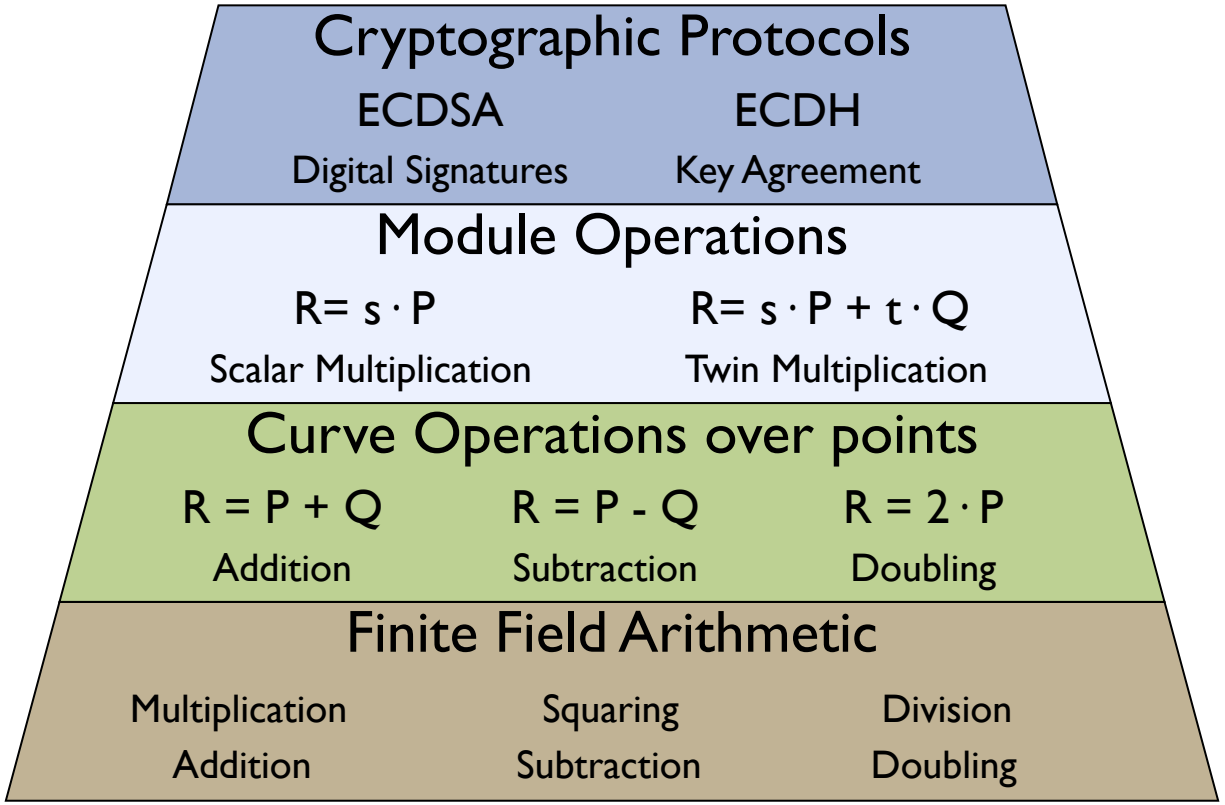
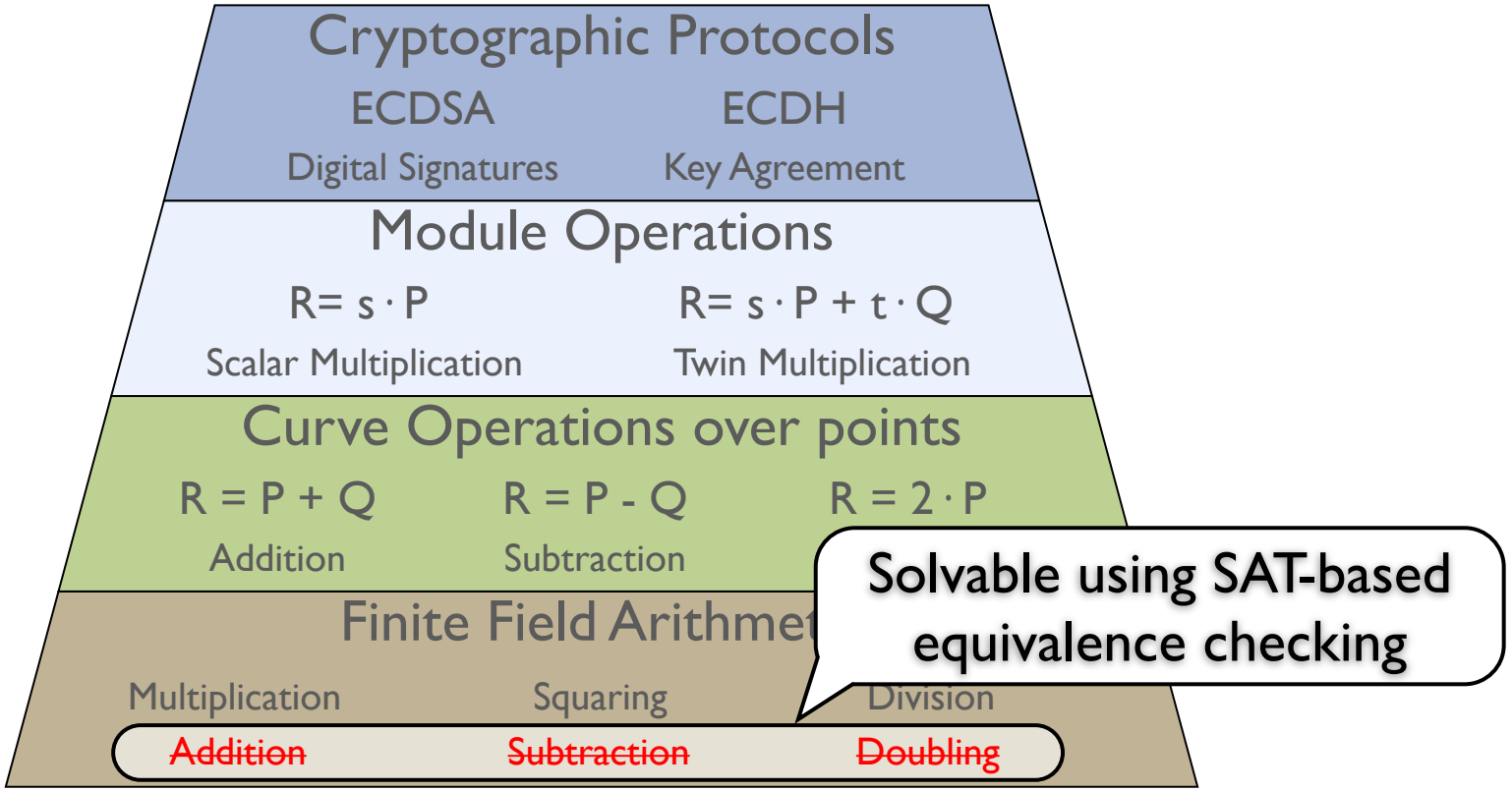
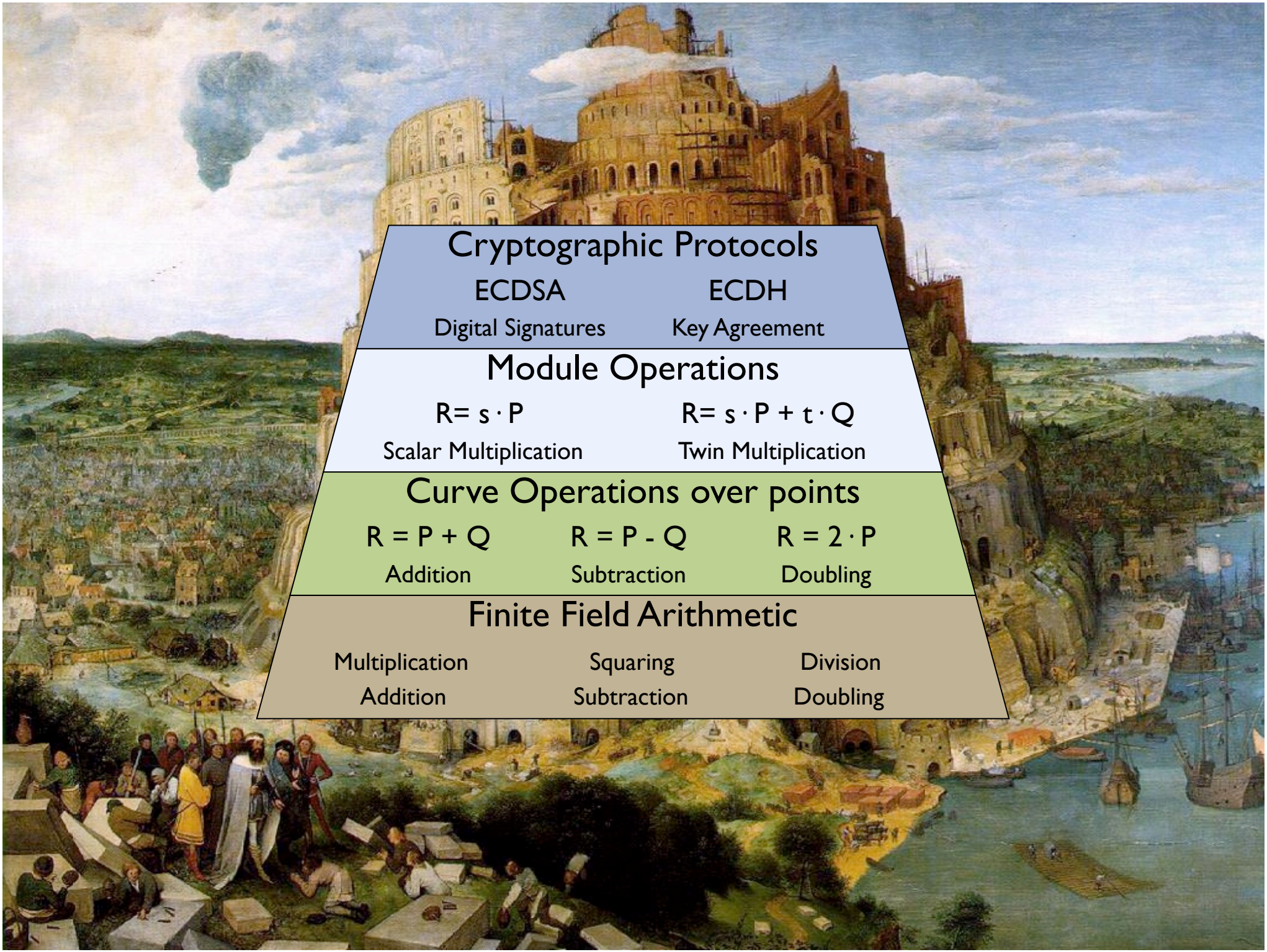
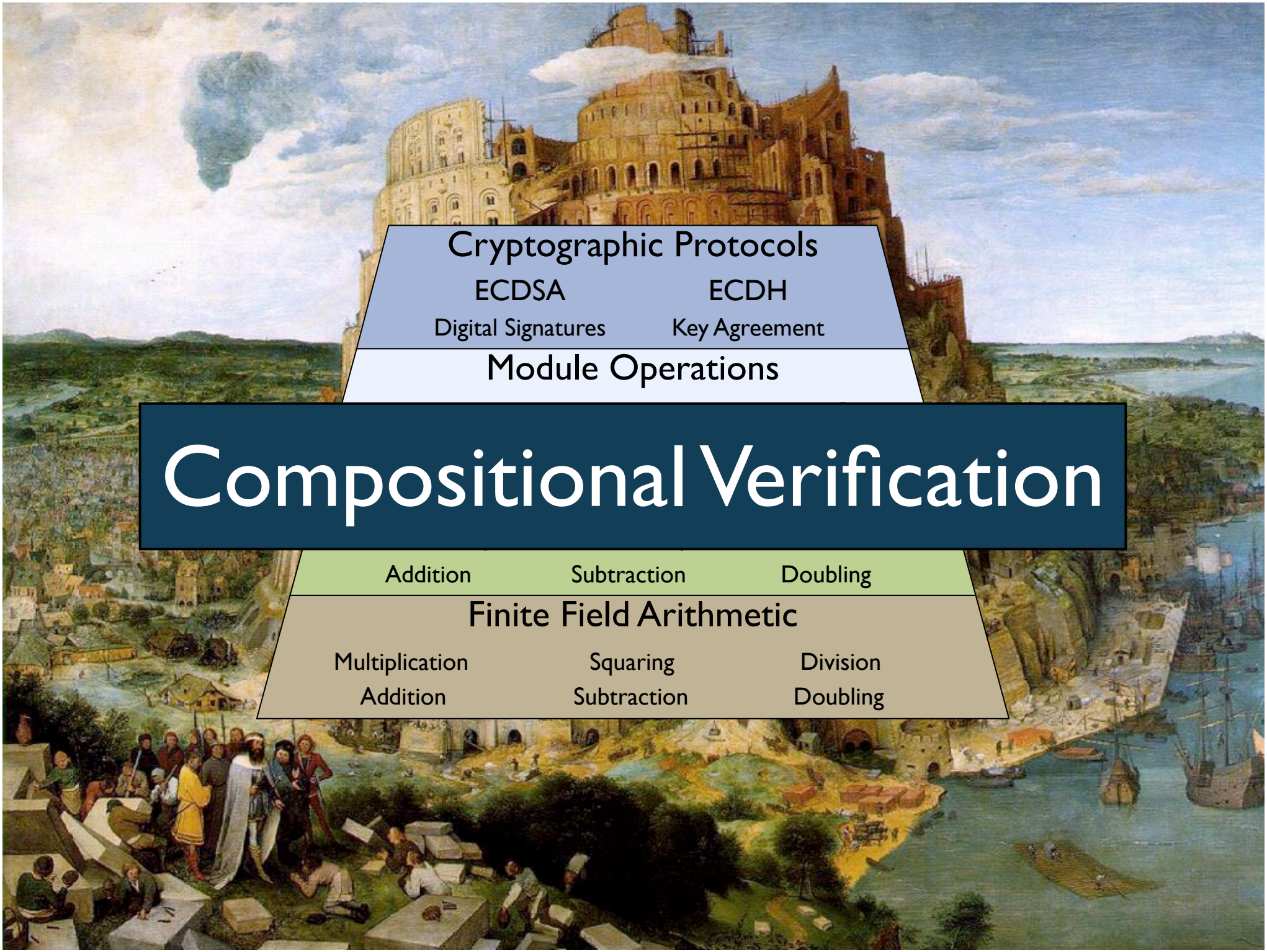
Elliptic Curve Crypto (ECC)



Elliptic Curve Crypto (ECC)




Cryptographic Protocols
ECDSA
Digital Signatures
ECDH
Key Agreement
Module Operations
R= s · P
Scalar Multiplication
R= s · P + t · Q
Twin Multiplication
Curve Operations over points
R = P + Q
Addition
R = P - Q
Subtraction
R = 2 · P
Doubling
Finite Field Arithmetic
Multiplication
Addition
Squaring
Subtraction
Division
Doubling


Cryptographic Protocols
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Compositional Verification
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NIST P384 Curve

ECC is a family of algorithms, with many choices...

- NIST P384 is a standardized curve that is part of NSA Suite B.

Symmetric Key Size (bits)	Elliptic Curve Key Size (bits)	RSA Key Size (bits)
128	256	3072
192	384	7680
256	521	15360

NIST Recommended Key Sizes

NIST P384 Curve

- Prime field P_{384}

$$P_{384} = 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1$$

- Curve Equation: $y^2 = x^3 - 3x + b$

b = b3312fa7 e23ee7e4 988e056b e3f82d19 181d9c6e fe814112
0314088f 5013875a c656398d 8a2ed19d 2a85c8ed d3ec2aef

Implementing P384

Cryptol Specification	Java Implementation
Clarity	Performance
Declarative	Imperative
384bit Integers	Arrays of 32bit Integers
Higher-order Functions	Object Oriented

Field addition in Cryptol

```
/* Returns x + y (mod p384_prime). */
p384_add(x,y) = prime_field_add(x, y, p384_prime);

p384_prime : [384];
p384_prime = 2 ** 384 - 2 ** 128 - 2 ** 96 + 2 ** 32 - 1;

prime_field_add : {n} (fin n) => ([n],[n],[n]) -> [n];
prime_field_add(x,y,p) = mod(uext(x) + uext(y), p)
  where {
    /* Unsigned word extension. */
    uext : {n} (fin n) => [n] -> [n+1];
    uext(x) = x # zero;
    /* Modular reduction on input. */
    mod : {n} (fin n) => ([n+1],[n]) -> [n];
    mod(x,p) = take(width(p), x % uext(p));
  };
```

Field addition in Cryptol

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p384_pr
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```

Definition of field addition for P384

Field addition in Cryptol

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prime_fie
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```

Definition of field prime.

Field addition in Cryptol

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p384_prime : [384];
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    uext(x) = x # zero;
    /* Modular reduction on input. */
    mod :
    mod(x
  };
```

Extend precision to avoid overflow

Field addition in Cryptol

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/* Returns x + y (mod p384_prime). */
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    mod(x,p) = take(width(p), x % uext(p));
  };
```

Perform modular reduction to original precision.

Field addition in Java

```
/** Assigns z = x + y (mod field_prime). */
public void field_add(int[] z, int[] x, int[] y) {
  if (add(z, x, y) != 0 || leq(field_prime, z)) decFieldPrime(z);
}

int[] field_prime = { -1, 0, 0, -1, -2, -1, -1, -1, -1, -1, -1, -1 };

static final long LONG_MASK = 0xFFFFFFFFL;

/** Assigns z = x + y and returns carry. */
protected int add(int[] z, int[] x, int[] y) {
  long c = 0;
  for (int i = 0; i != z.length; ++i) {
    c += (x[i] & LONG_MASK) + (y[i] & LONG_MASK);
    z[i] = (int) c; c = c >> 32;
  }
  return (int) c;
}

static boolean leq(int[] x, int[] y) { ... }
protected int decFieldPrime(int[] x) { ... }
```

Field addition in Java

In-place modification of result

```
/** Assigns z = x + y (mod field_prime). */
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static final long

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Field prime is an int array

Field addition in Java

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```

Mask used for unsigned conversion to long

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Addition loop with explicit carry

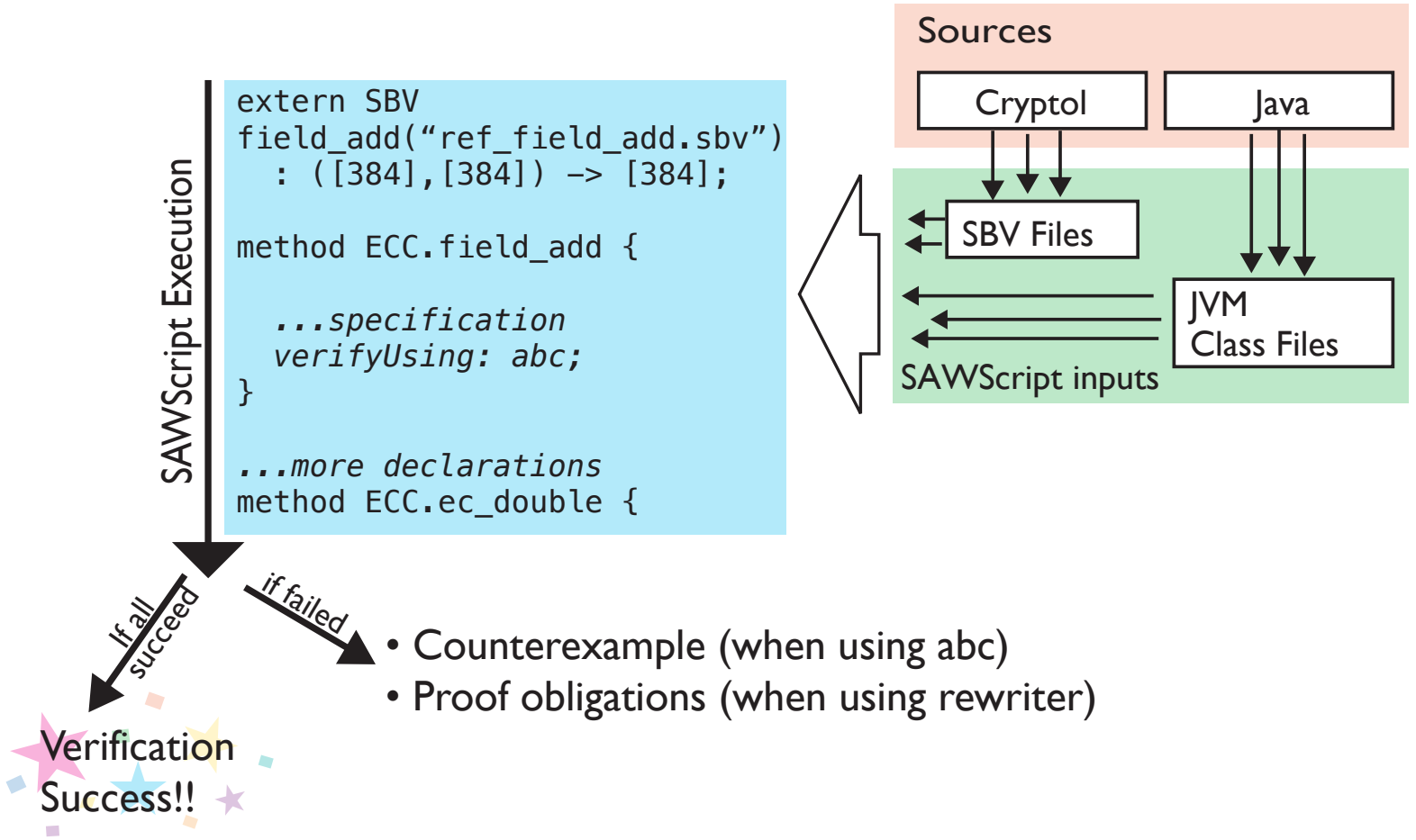
Software Analysis Workbench

- Allows Java methods to be specified using SAWScript, a bit-precise specification language relating Java and Cryptol.
- Specifications are sequentially processed by SAWScript verification tool.

```
sawScript –j ecc.jar:classes.jar ecc.saw
```

SAWScript Workflow

sawScript -j ecc.jar:classes.jar ecc.saw



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SAWScript Workflow

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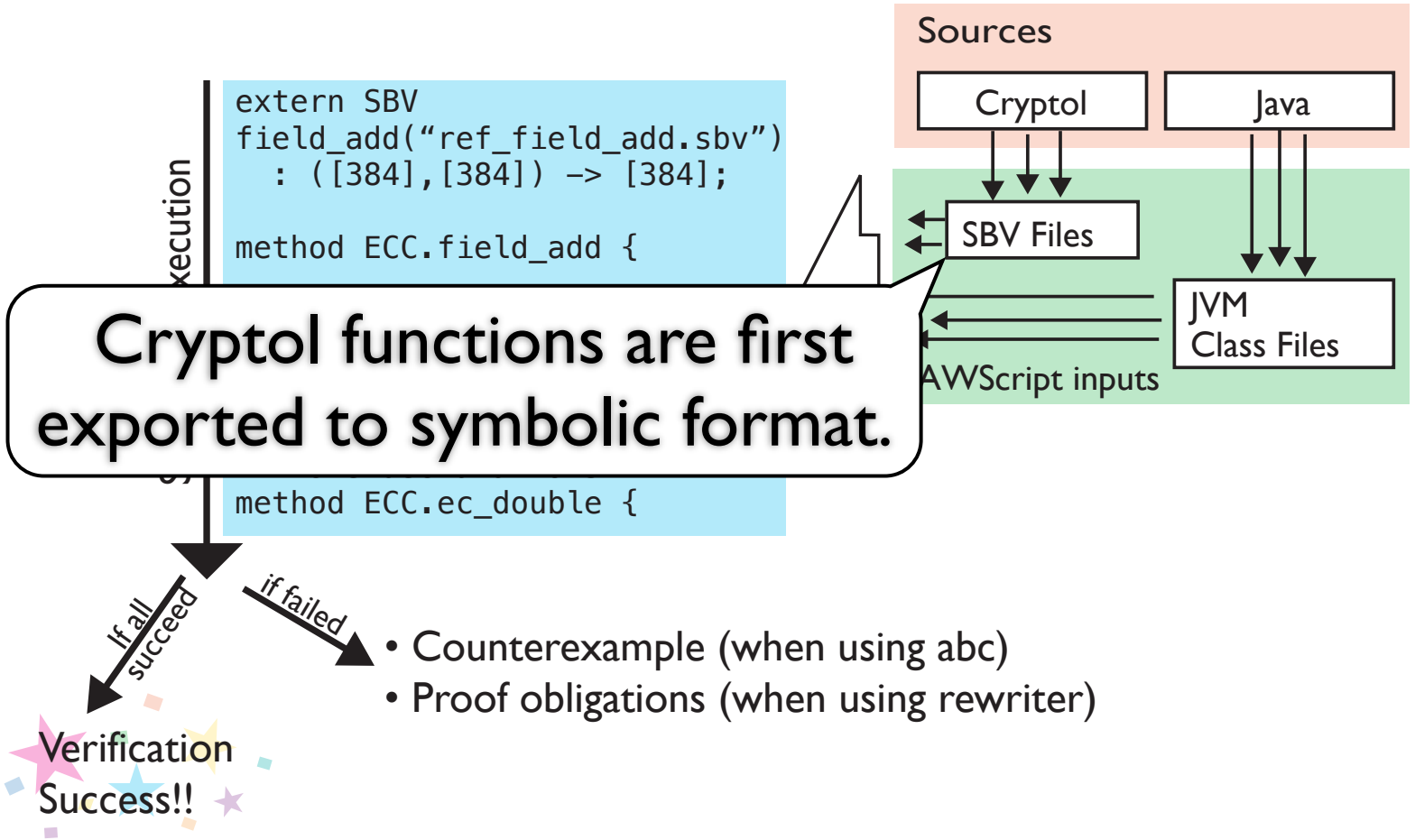

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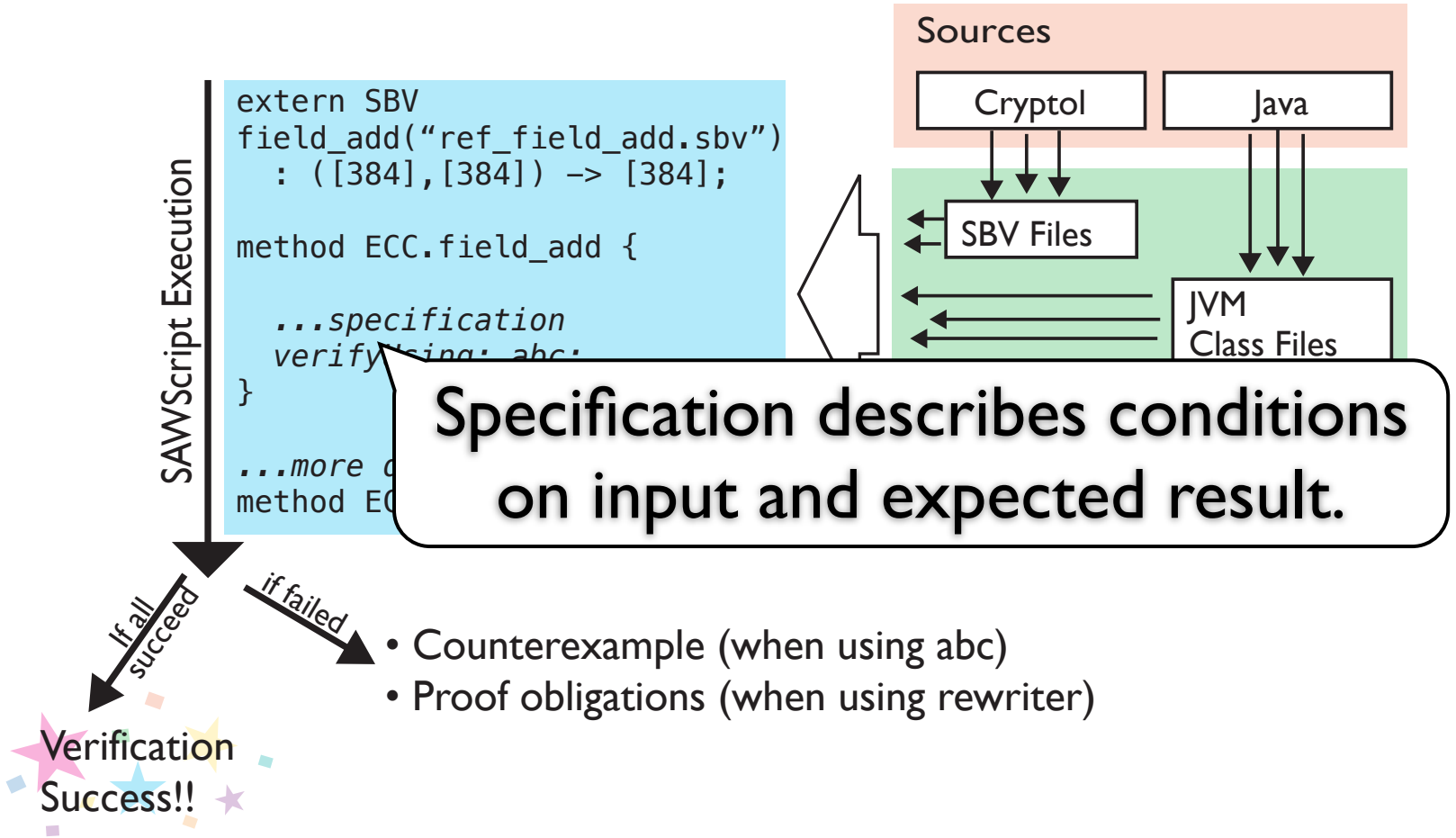

galois

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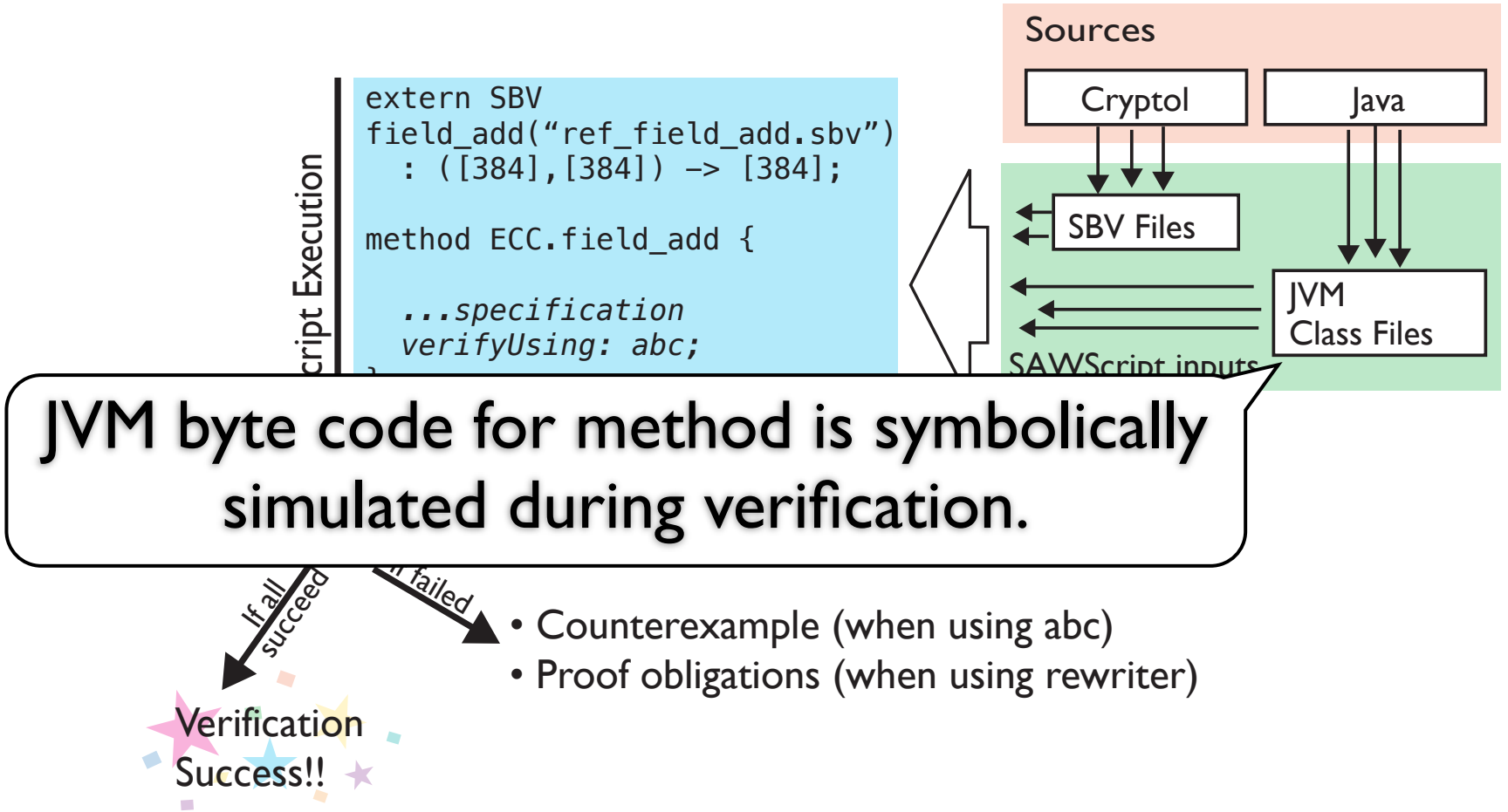

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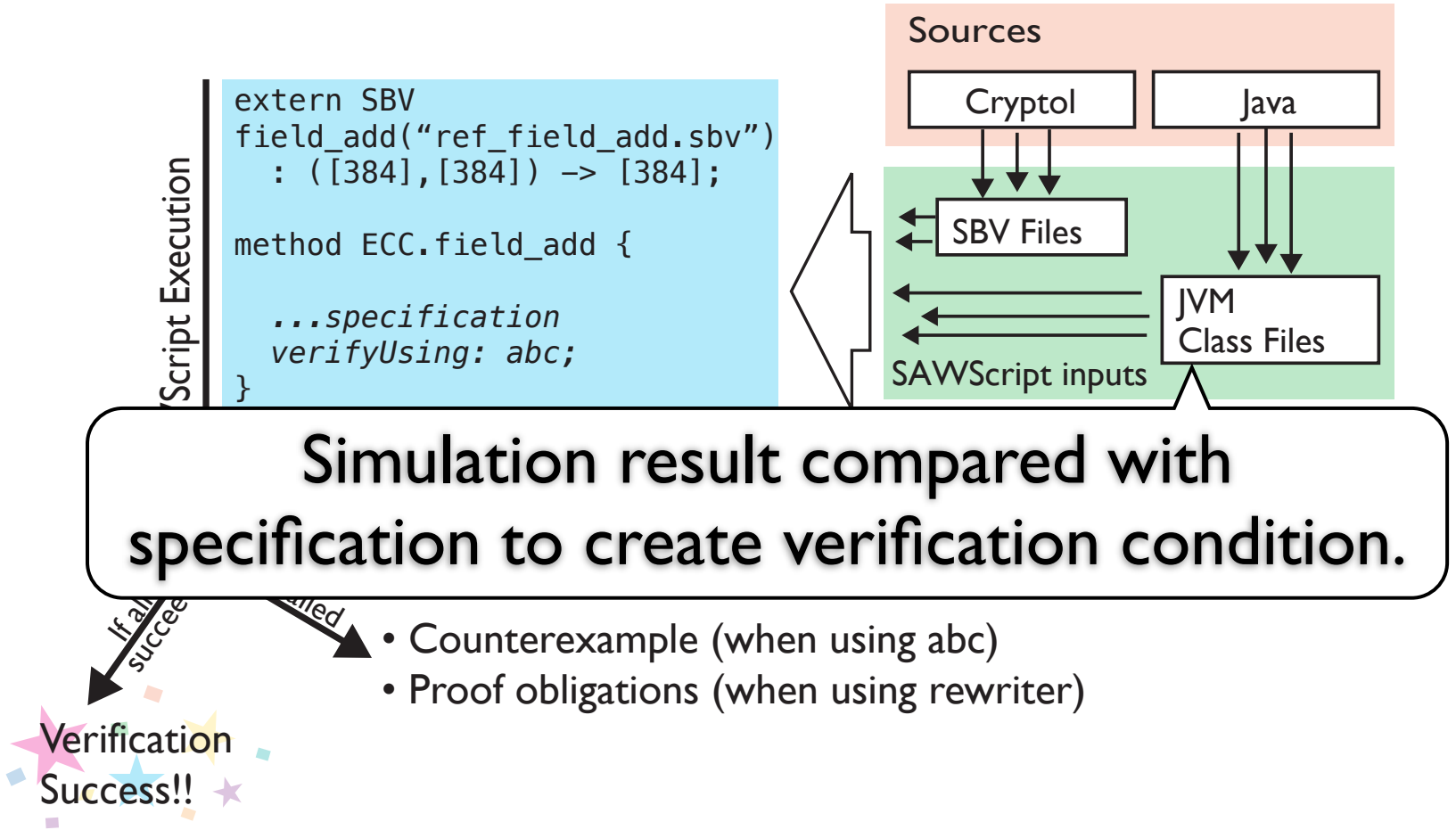

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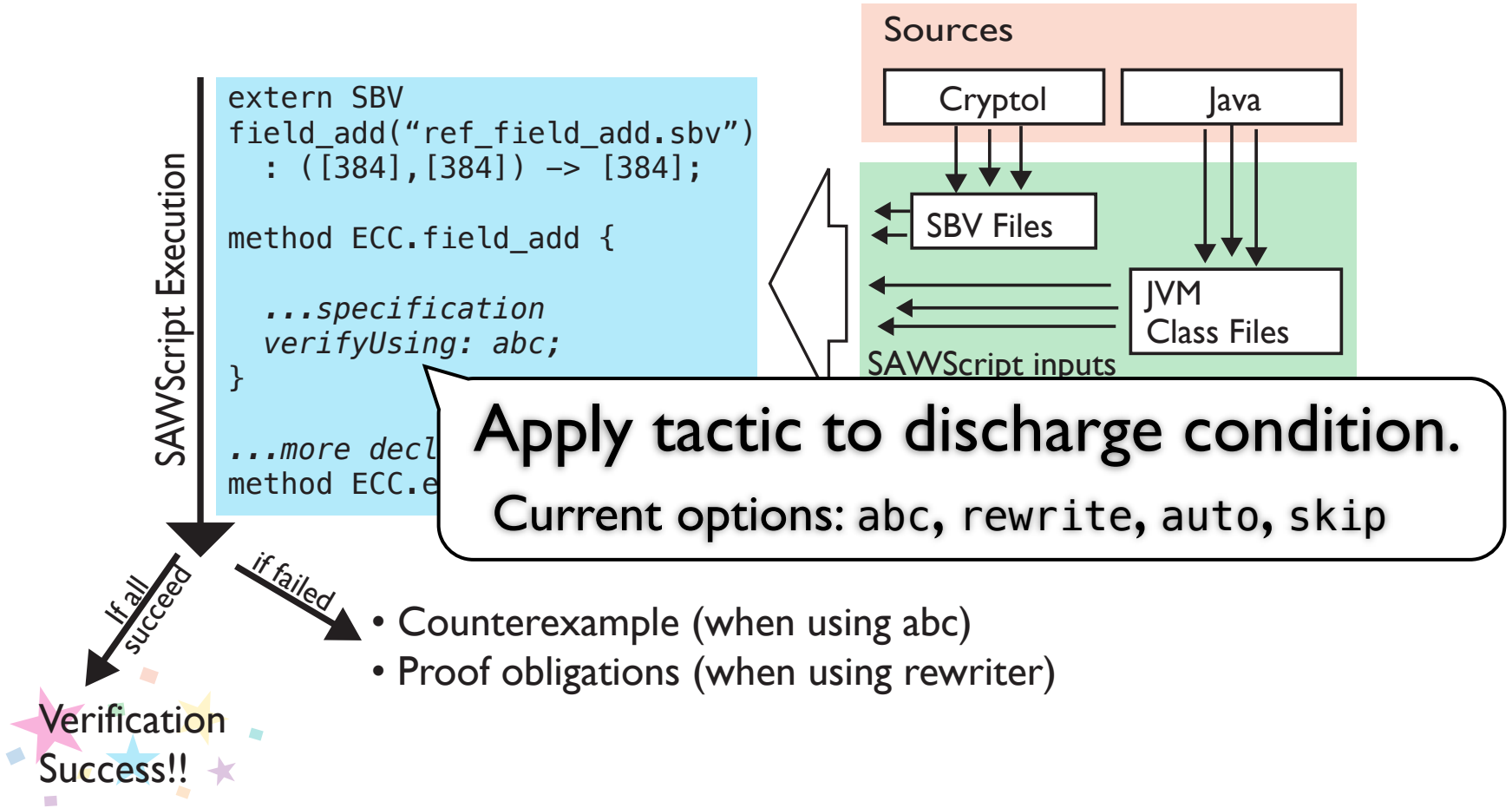

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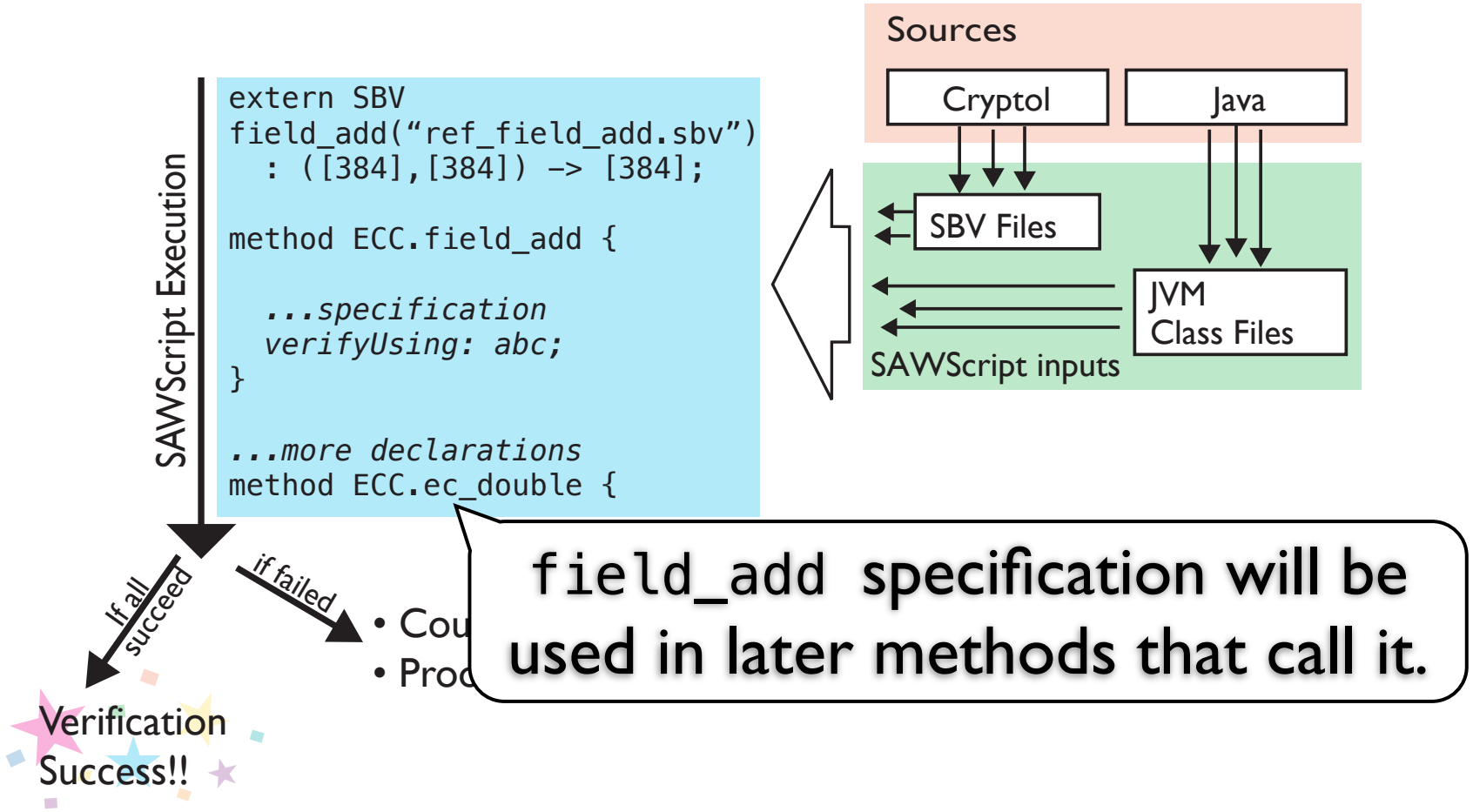

galois

field_add Specification

```
extern SBV p384_add("sbv/ref_p384_add.sbv") : ([384],[384]) -> [384];

let field_const = <| 2^384 - 2^128 - 2^96 + 2^32 - 1 |> : [384];

method com.galois.ecc.P384ECC64.field_add {

  var args[0], args[1], args[2] : int[12];

  mayAlias { args[0], args[1], args[2] };

  const this.field_prime := split(field_const) : [12][32];

  ensures args[0] :=
    split(p384_add(join(fromJava(args[1])),
                   join(fromJava(args[2])))) : [12][32];

  verifyUsing: abc;
};
```

field_add Specification

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let field_const

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Import Cryptol field addition.

field_add Specification

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method com.galo
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  mayAlias { args[0], args[1], args[2] };

  const this.field_prime := split(field_const) : [12][32];

  ensures args[0] :=
    split(p384_add(join(fromJava(args[1])),
                   join(fromJava(args[2])))) : [12][32];

  verifyUsing: abc;
};
```

Declare constant for field prime

field_add Specification

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Identify Java method

field_add Specification

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```

Specify bit-precise types

field_add Specification

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  const this.field_p
                                                    [12][32]

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  verifyUsing: abc;
};
```

Specify potential aliasing

field_add Specification

```
extern SBV p384_add("sbv/ref_p384_add.sbv") : ([384],[384]) -> [384];

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method com.galois.ecc.P384ECC64.field_add {

  var args[0], args[1], args[2] : int[12];

  mayAlias { args[0], args[1], args[2] };

  const this.field_prime := split(field_const) : [12][32];

  ensures args[0] :=
    split(p384_add(joi
                   joi

  verifyUsing: abc;
};
```

Specify Constants

field_add Specification

```
extern SBV p384_add("sbv/ref_p384_add.sbv") : ([384],[384]) -> [384];

let field_const = <| 2^384 - 2^128 - 2^96 + 2^32 - 1 |> : [384];

method com.galois.ecc.P384ECC64.field_add {

  var args[0], args[1], args[2] : int[12];

  mayAlias { args[0], args[1], args[2] };

  const this.field_prime := split(field_const) : [12][32];

  ensures args[0] :=
    split(p384_add(join(fromJava(args[1])),
                   join(fromJava(args[2])))) : [12][32];

  verifyUsing: ab
};
```

Specify Postcondition

field_add Specification

```
extern SBV p384_add("sbv/ref_p384_add.sbv") : ([384],[384]) -> [384];

let field_const = <| 2^384 - 2^128 - 2^96 + 2^32 - 1 |> : [384];

method com.galois.ecc.P384ECC64.field_add {

  var args[0], args[1], args[2] : int[12];

  mayAlias { args[0], args[1], args[2] };

  const this.field_prime := split(field_const) : [12][32];

  ensures args[0] :=
    split(p384_add(join(fromJava(args[1])),
                   join(fromJava(args[2])))) : [12][32];

  verifyUsing: abc;
};
```

Select Verification Method

Bit-Precise Rewriting

- Rewriting is a general-purpose tactic in many theorem provers.
- SAWScript's rewrite engine has been designed to support Cryptol's type system.
- Multiple rewrite rules are compiled into a single automaton for efficiency as in many theorem provers and rewrite engines.

Results so far

- Verified 13 out of 18 Java methods in Java ECC implementation.
- Identified one error in modular reduction:

NISTCurve.java (line 964):

```
d = (z[ 0] & LONG_MASK) + of;
z[ 0] = (int) d; d >>= 32;
d = (z[ 1] & LONG_MASK) - of;
z[ 1] = (int) d; d >>= 32;
d += (z[ 2] & LONG_MASK);
```

Results so far

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- Identified one error in modular reduction:

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```
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z[ 0] = (int) d; d >>= 32;
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Results so far

- Verified 13 out of 18 Java methods in Java ECC implementation.
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Bug only occurs if this addition overflows.

```
d = (z[ 0] & LONG_MASK) + of;
z[ 0] = (int) d; d >>= 32;
d += (z[ 1] & LONG_MASK) - of;
z[ 1] = (int) d; d >>= 32;
d += (z[ 2] & LONG_MASK);
```

Results so far

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z[ 1] = (int) d; d >>= 32;
```

`of` is guaranteed to be less than 4.

Next Steps

- Add inductive assertions to SAWScript.
  - Field division uses extended gcd algorithm that is not symbolically terminating.
  - Inductive assertions needed to handle division.
- Complete remaining proofs for example ECC implementation.

Next Steps

- General purpose rewriting as a proof tactic is powerful, but labor intensive.
- Decision procedures for common theories:
  - Integrate SMT Solving.
  - Decision procedures for fields and Abelian groups.
  - Computer algebra techniques such as Gröbner Basis seem promising.

Summary

- Need for automated verification tools that can handle public key cryptographic implementations.
- Tool must support compositional verification, and a variety of proof tools for discharging obligations.
- Have infrastructure in place, and more work remains.