

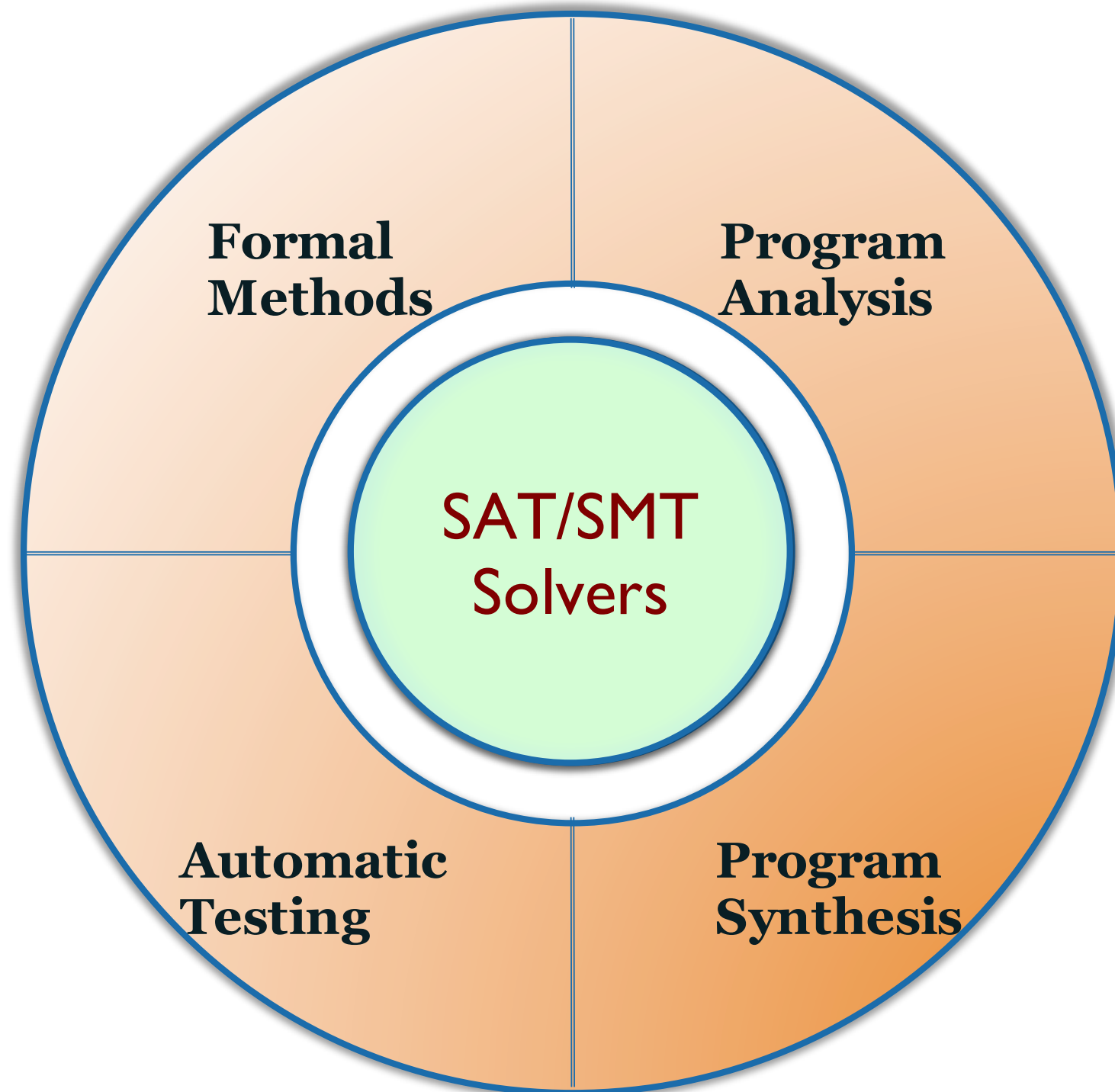
# String Solvers for Web Security

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# Software Engineering & SMT Solvers

## An Indispensable Tactic for Most Strategies



# Why a String Solver?

## Efficient Analysis of String Programs

- Strings are heavily used in Web applications
- Web applications plagued by string-related errors
- An SMT solver that natively reasons about strings can lower analysis burden

<b>Programs that use strings</b>	<b><u>Errors attributable to insufficient string analysis</u></b>
<b><u>Traditional Applications</u></b> C/C++ programs (string operations)	<b><u>Memory-related Errors</u></b> Buffer overflow due overly long strings
<b><u>Web Applications</u></b> PHP JavaScript String manipulation by Server or client-side code	<b><u>Improper Sanitization</u></b> SQL injection XSS scripting JavaScript Eval with user/attacker-supplied strings

# Theory of Word Equations, Length and Membership

Symbol	String Sort	Number Sort
Constants	Finite-length strings defined over a finite alphabet $\Sigma$	Integers
Variables	Range over $\Sigma^*$	Range over integers
String functions	Concat: $\text{String} \times \text{String} \Rightarrow \text{String}$ Length: $\text{String} \Rightarrow \text{Integer}$	
Integer functions		Addition: $\text{Integer} \times \text{Integer} \Rightarrow \text{Integer}$
String predicates	Equality over string terms ( $= : \text{String} \times \text{String} \Rightarrow \text{Bool}$ ) membership in regular expressions/CFGs ( $\in : \text{String} \times \text{regular-expression} \Rightarrow \text{Bool}$ ) Contains predicate: (Contains: $\text{String} \times \text{String} \Rightarrow \text{Bool}$ )	
Integer predicates		Equality over integer terms ( $= : \text{Integer} \times \text{Integer} \Rightarrow \text{Bool}$ ) Inequality over integer terms ( $< : \text{Integer} \times \text{Integer} \Rightarrow \text{Bool}$ )

# Theory of Strings

## Example Constraints

- $X = \text{concat}(\text{"SELECT msg FROM msgs WHERE topicid = "}, v)$   
AND  
 $(X \in \text{SQL\_Grammar})$ 
  - $\text{input} \in \text{RegExp}([0-9]^+)$
- $X = \text{concat}(\text{str\_term1}, \text{str\_term2}, \text{"c"})[1:42]$   
AND  
 $X \text{ contains "abc"}$
- $Xa = aX, XabY = YbaX$

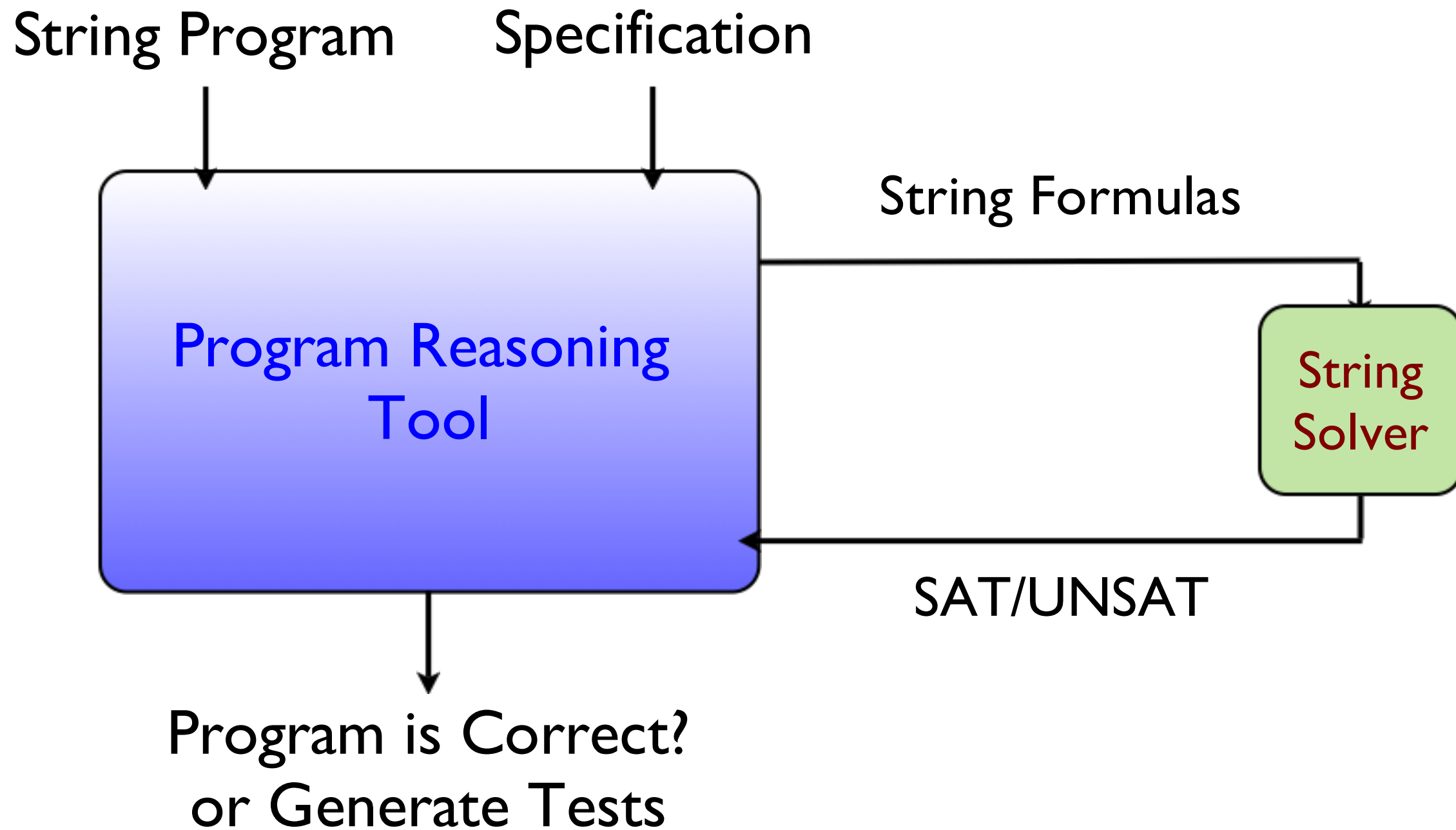
# Word Equations, Membership, and Length

## What is Known

<b>Result</b>	<b>Person (Year)</b>	<b>Notes</b>
Undecidability of Quantified Word Equations	Quine (1946)	Multiplication reduced to concat
Undecidability of Quantified Word Equations with single alternation	Durnev (1996), G. (2012)	2-counter machines reduced to words with single quantifier alter.
Decidability (PSPACE) of QF Theory of Word Equations	Makanin (1977) Plandowski (1996, 2002/06)	Makanin result very difficult Simplified by Plandowski
Decidability (PSPACE-complete) of QF Theory of Word Equations + RE	Schultz (1992)	RE membership predicate
QF word equations + Length() (?)	Matiyasevich (1971)	Unsolved

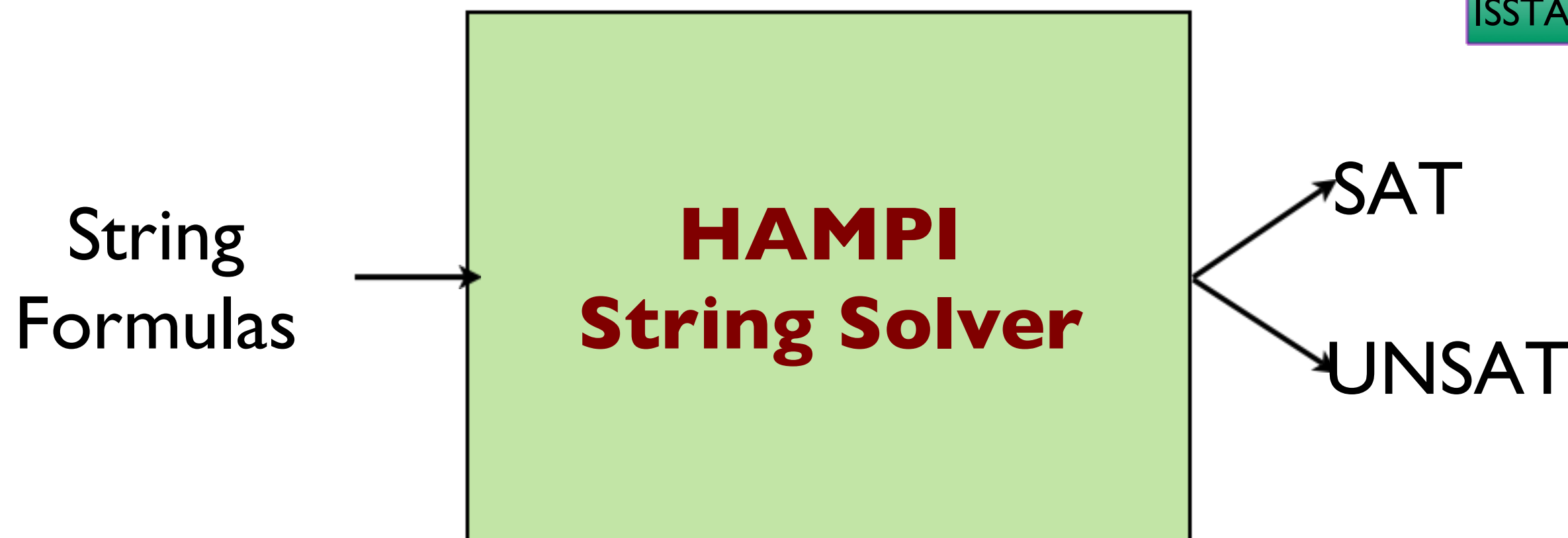
# String Solver Problem Statement

## Efficient Solver for Analysis of String Programs



# HAMPI String Solver

TOSEM 2012  
CAV 2011  
ISSTA 2009



- $X = \text{concat}(\text{"SELECT..."}, v)$  AND ( $X \in \text{SQL\_grammar}$ )
- JavaScript, PHP, ... string expressions
- NP-complete
- **ACM Distinguished Paper Award 2009**
- **Google Faculty Research Award 2011**



# Rest of the Talk

- **HAMPI string solver**
  - String equations and membership in regular expressions/CFGs
  - How HAMPI works
  - Experimental results
- **Theoretical results**
  - Undecidability of forallexists fragment of word equations
  - Conditional decidability results for word equations and length
  - Open theoretical problems
- **Z3-str**

# Theory of Strings

## The Hampi Language

<b>PHP/JavaScript/C++...</b>	<b>HAMPI: Theory of Strings</b>	<b>Notes</b>
Var a; \$a = 'name'	Var a : 1..20; a = 'name'	Bounded String Variables String Constants
string_expr." is "	concat(string_expr, " is ");	Concat Function
substr(string_expr,1,3)	string_expr[1:3]	Extract Function
assignments/strcmp a = string_expr; a /= string_expr;	equality a = string_expr; a /= string_expr;	Equality Predicate
Sanity check in regular expression RE Sanity check in context-free grammar CFG	string_expr in RE string_expr in SQL string_expr NOT in SQL	Membership Predicate
string_expr contains a sub_str string_expr does not contain a sub_str	string_expr contains sub_str string_expr NOT?contains sub_str	Contains Predicate (Substring Predicate)

# HAMPI Solver Motivating Example

## SQL Injection Vulnerabilities

Buggy Script



Malicious SQL Query

Unauthorized Database Results

Backend DataBase

```
SELECT m FROM messages WHERE id='1' OR 1 = 1
```

# HAMPI Solver Motivating Example

## SQL Injection Vulnerabilities

Should be: “^[0-9]+\$”

Buggy Script

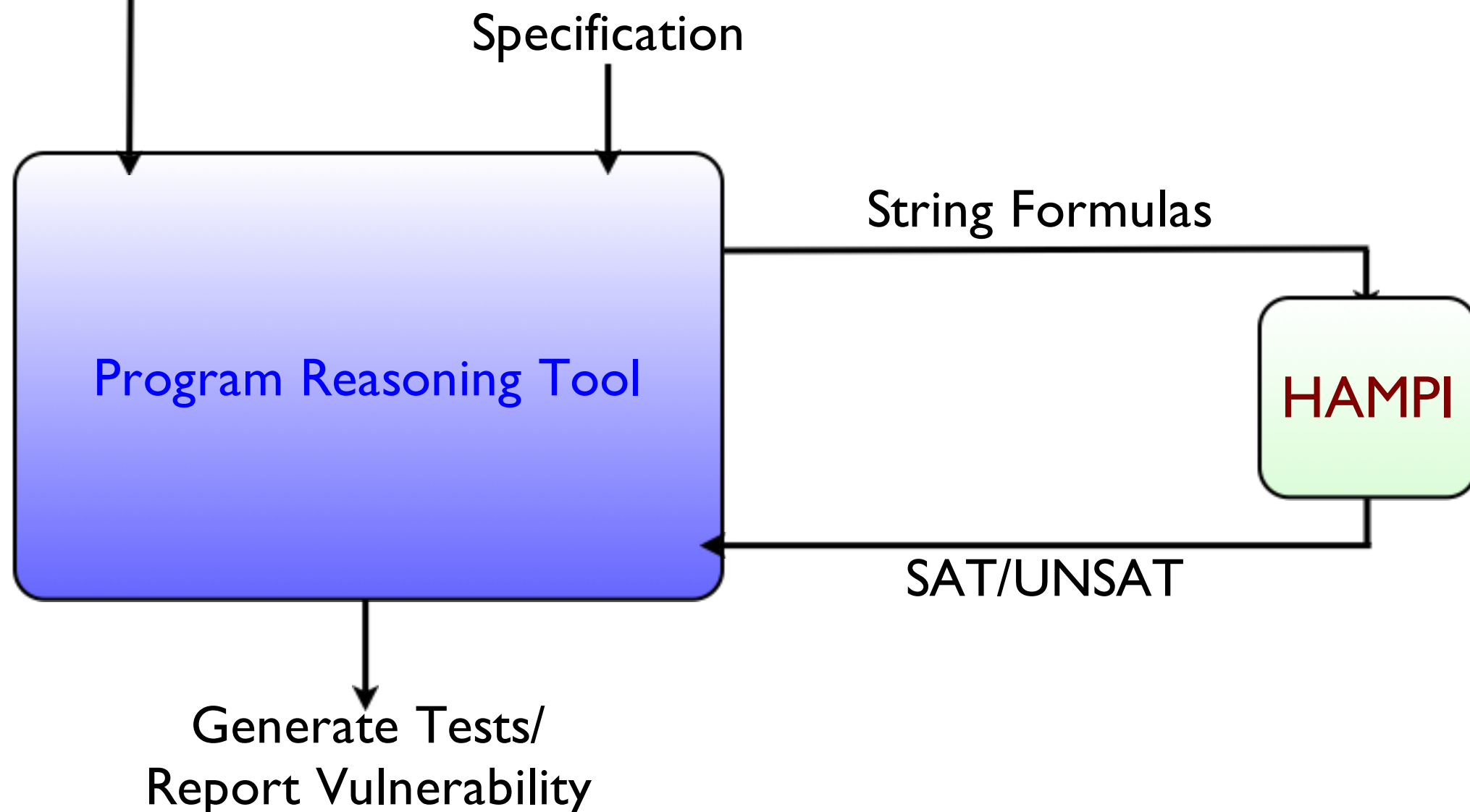
```
if (input in regexp(“[0-9]+”))  
  query := “SELECT m FROM messages WHERE id=’ ” + input + “ ’ ”)
```

- **input** passes validation (regular expression check)
- **query** is syntactically-valid SQL
- **query** can potentially contain an attack substring (e.g., `1' OR '1' = '1`)

# HAMPI Solver Motivating Example

## SQL Injection Vulnerabilities

```
if (input in regexp("[0-9]+"))  
  query := "SELECT m FROM messages WHERE id=' " + input + "'")
```



# Expressing the Problem in HAMPI

## SQL Injection Vulnerabilities

Input String



```
Var v : 12;
```

SQL Grammar



```
cfg SqlSmall := "SELECT " [a-z]+ " FROM " [a-z]+ " WHERE " Cond;
```

```
cfg Cond := Val "=" Val | Cond " OR " Cond;
```

```
cfg Val := [a-z]+ | "'" [a-z0-9]* "'" | [0-9]+;
```

SQL Query



```
val q := concat("SELECT msg FROM messages WHERE topicid=", v, "");
```

```
assert v in [0-9]+;
```

```
assert q in SqlSmall;
```

SQLI attack conditions



```
assert q contains "OR '1'='1'";
```

“q is a valid SQL query”

“q contains an attack vector”

# Hampi Key Conceptual Idea

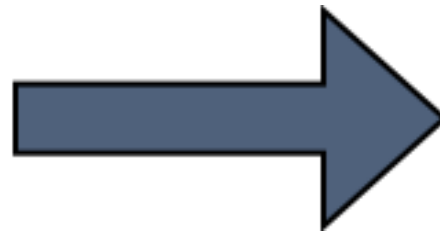
## Bounding, expressiveness and efficiency

$L_i$	Complexity of $\emptyset = L_1 \cap \dots \cap L_n$	Current Solvers
Context-free	Undecidable	n/a
Regular	PSPACE-complete	Quantified Boolean Logic
Bounded	NP-complete	SAT Efficient in practice

# Hampi Key Idea: Bounded Logics

## Testing, Analysis, Vulnerability Detection,...

- Finding SAT assignment is key
- Short assignments are sufficient



- Bounding is sufficient
- Bounded logics easier to decide



# HAMPI Solver Motivating Example

## SQL Injection Vulnerabilities

Input String



```
Var v : 12;
```

SQL Grammar



```
cfg SqlSmall := "SELECT " [a-z]+ " FROM " [a-z]+ " WHERE " Cond;
```

```
cfg Cond := Val "=" Val | Cond " OR " Cond;
```

```
cfg Val := [a-z]+ | "'" [a-z0-9]* "'" | [0-9]+;
```

SQL Query



```
val q := concat("SELECT msg FROM messages WHERE topicid=", v, "");
```

```
assert v in [0-9]+;
```

```
assert q in SqlSmall;
```

SQLI attack conditions



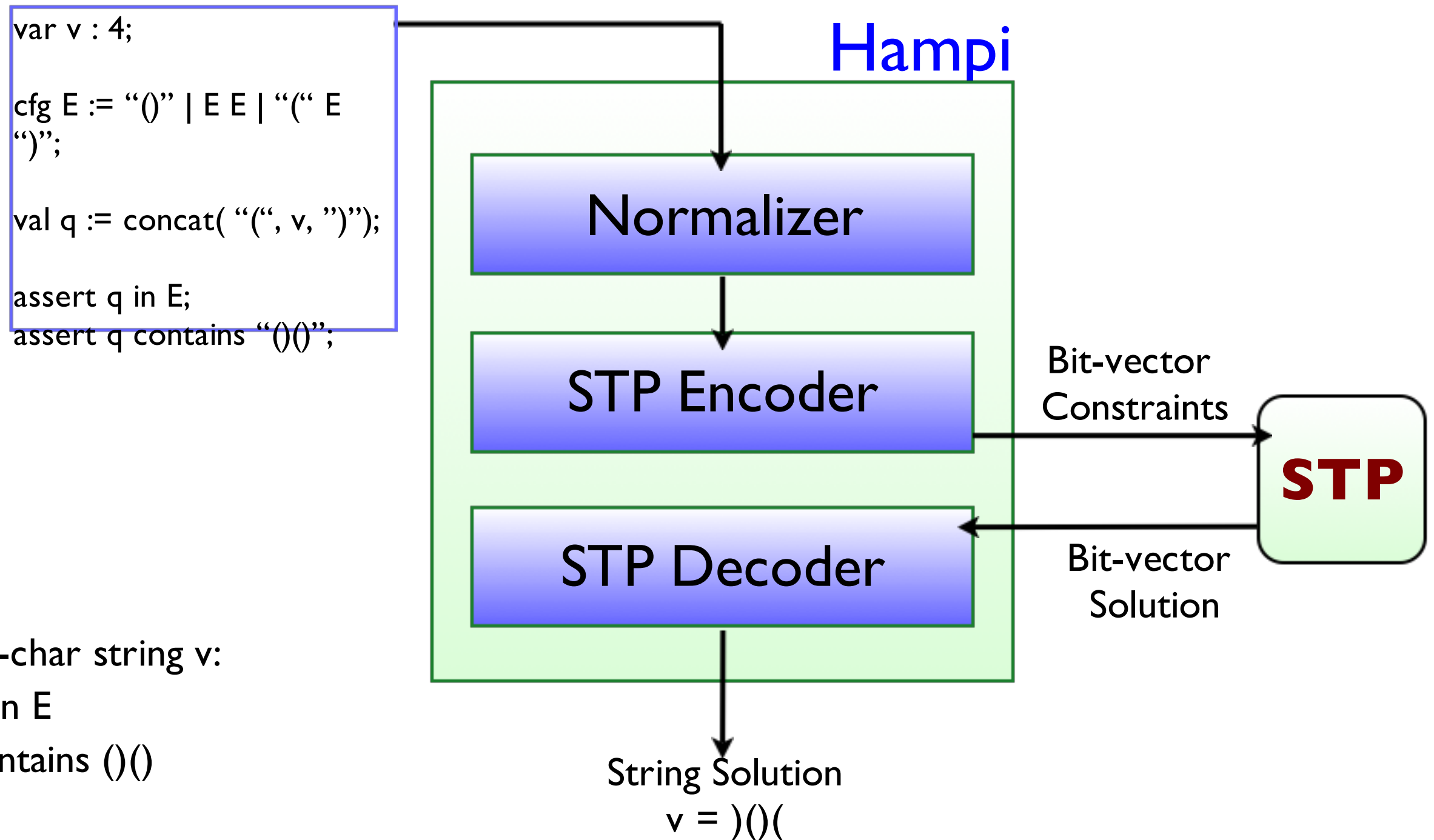
```
assert q contains "OR '1'='1'";
```

“q is a valid SQL query”

“q contains an attack vector”

# How Hampi Works

## Bird's Eye View: Strings into Bit-vectors



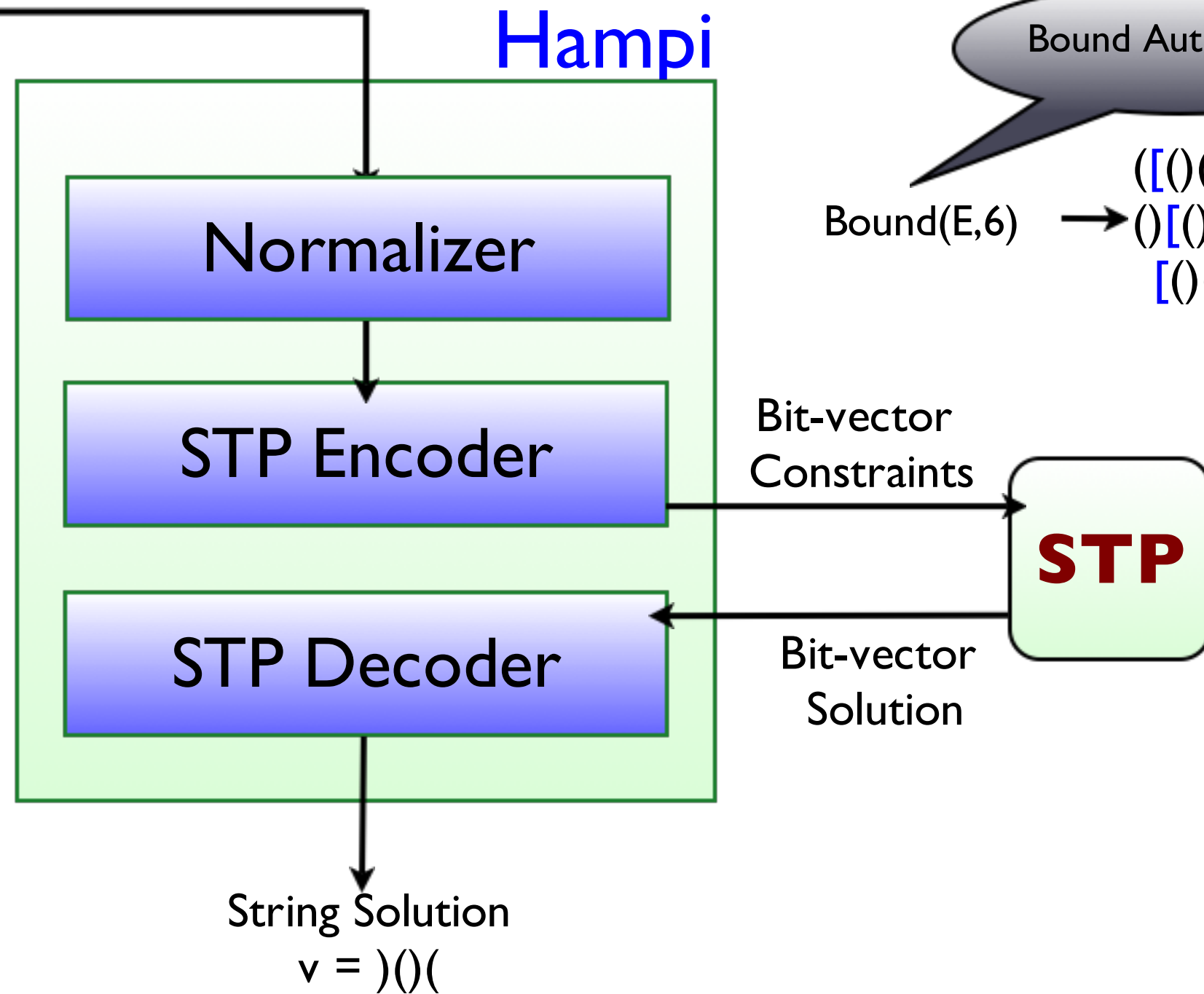
Find a 4-char string v:

- (v) is in E
- (v) contains ()()

# How Hampi Works

## Unroll Bounded CFGs into Regular Exp.

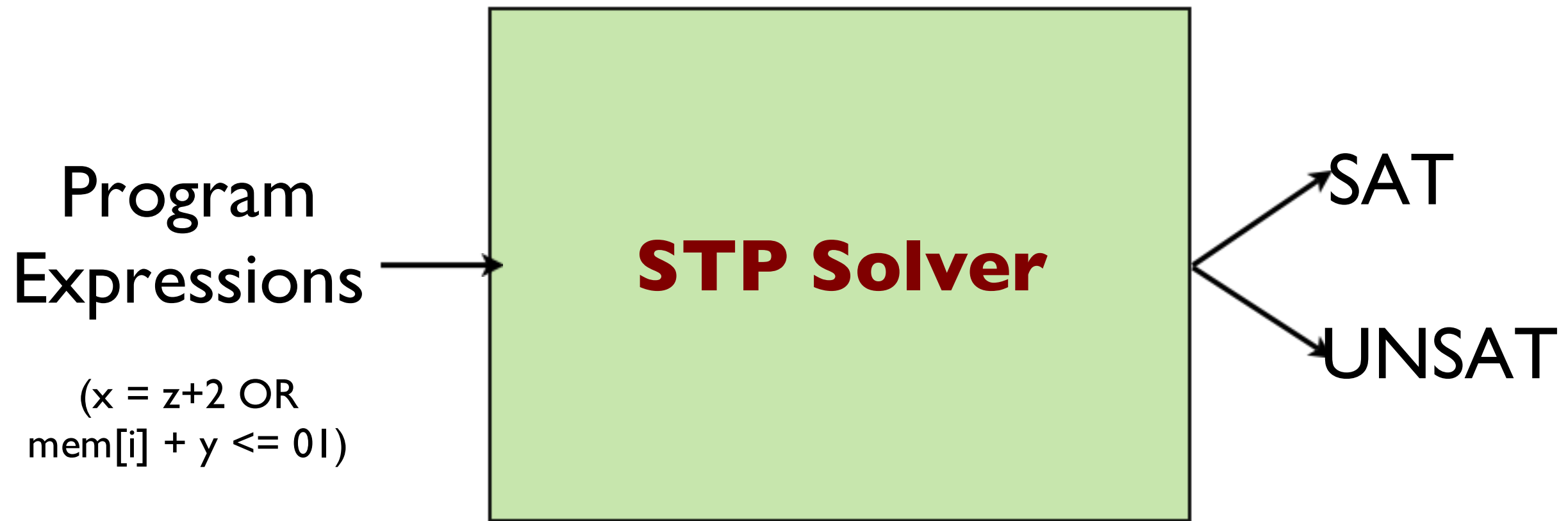
```
var v : 4;  
cfg E := “()” | E E | “(“ E  
“)””;  
val q := concat(“(“, v, ””);  
assert q in E;  
assert q contains “()”;
```



Bound Auto-derived

$$\text{Bound}(E,6) \rightarrow \begin{aligned} &([()() + (())] + \\ &() [()() + (())] + \\ &[()() + (())] () \end{aligned}$$

# STP Bit-vector & Array Solver



- Bit-vector or machine arithmetic
- Arrays for memory
- C/C++/Java expressions
- NP-complete

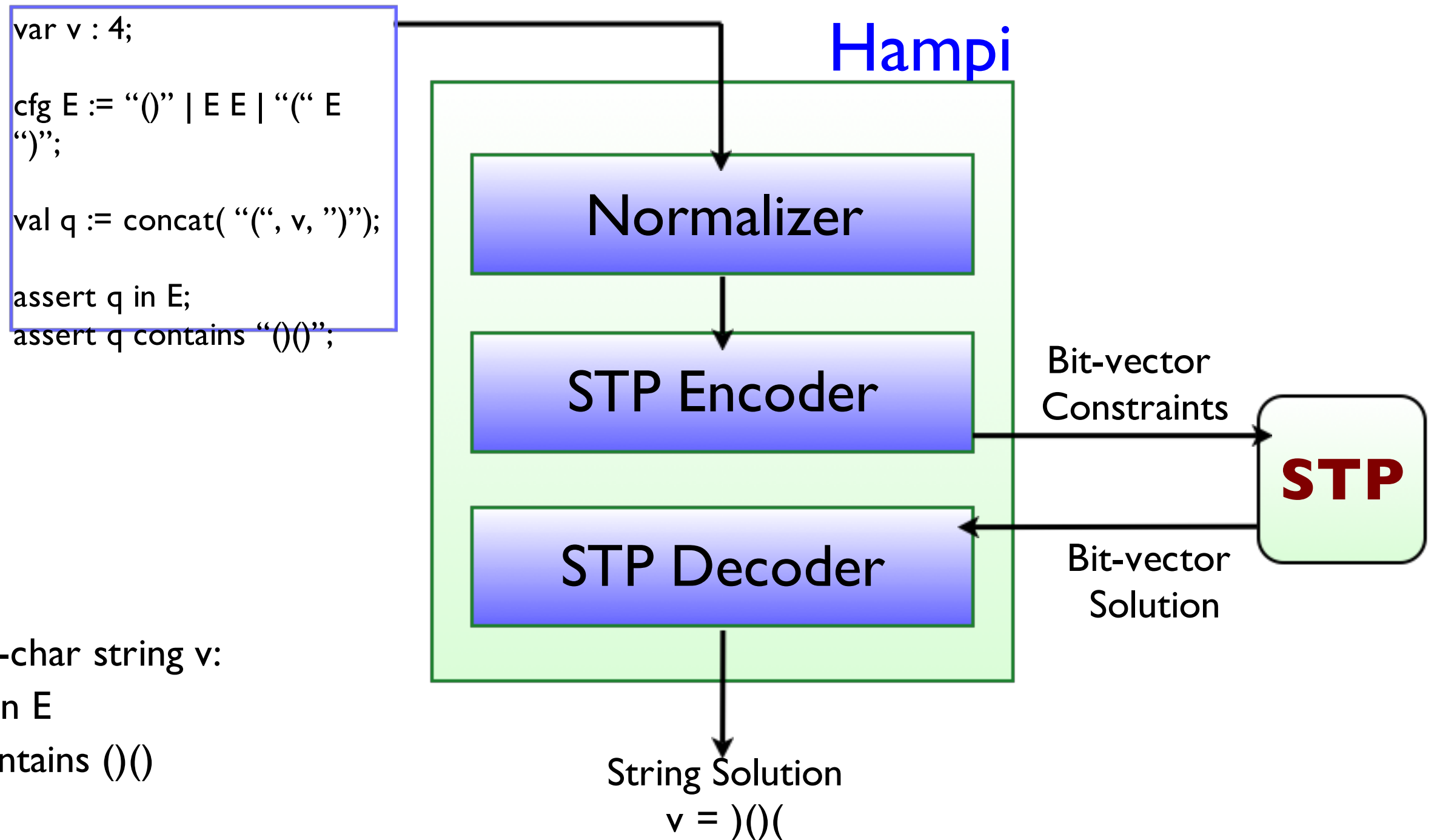
# Impact of STP: Notable Projects

- Played an important role in the development of symbolic testing techniques
- 100+ reliability and security projects

Category	Research Project	Project Leader/Institution
Formal Methods	ACL2 Theorem Prover + STP Verification-aware Design Checker Java PathFinder Model Checker	Eric Smith & David Dill/ <a href="#">Stanford</a> Jacob Chang & David Dill/ <a href="#">Stanford</a> Mehlitz & Pasareanu/ <a href="#">NASA</a>
Program Analysis	BitBlaze & WebBlaze BAP	Dawn Song et al./ <a href="#">Berkeley</a> David Brumley/ <a href="#">CMU</a>
Automatic Testing Security	Klee, EXE SmartFuzz Kudzu S2E & Cloud9	Engler & Cadar/ <a href="#">Stanford</a> Molnar & Wagner/ <a href="#">Berkeley</a> Saxena & Song/ <a href="#">Berkeley</a> Bucur & Candea/ <a href="#">EPFL</a>
Hardware Bounded Model-checking (BMC)	Blue-spec BMC BMC	Katelman & Dave/ <a href="#">MIT</a> Haimed/ <a href="#">NVIDIA</a>

# How Hampi Works

## Bird's Eye View: Strings into Bit-vectors

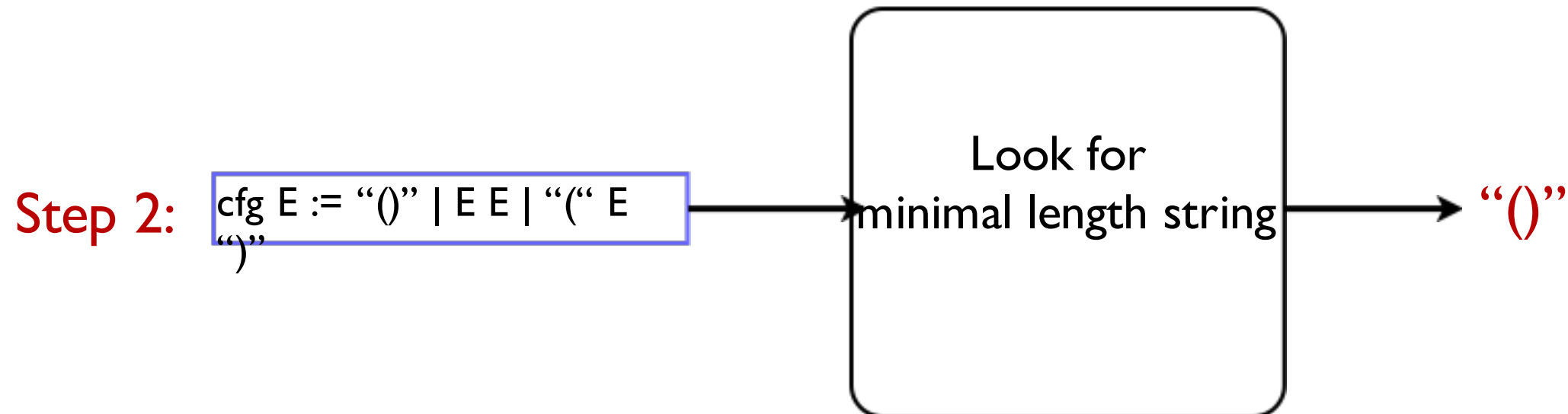
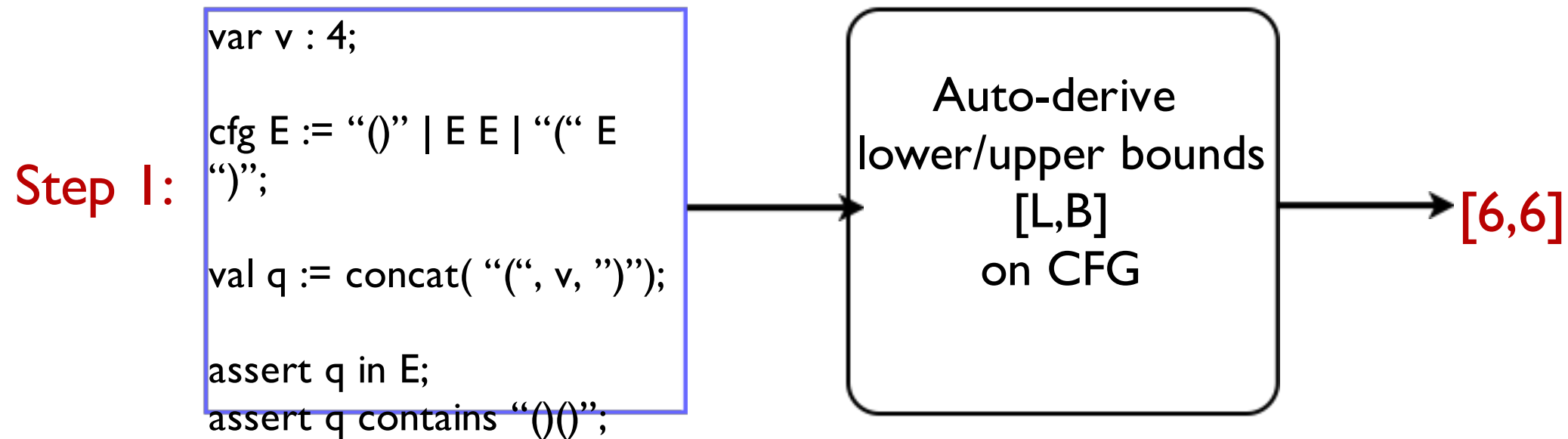


Find a 4-char string v:

- (v) is in E
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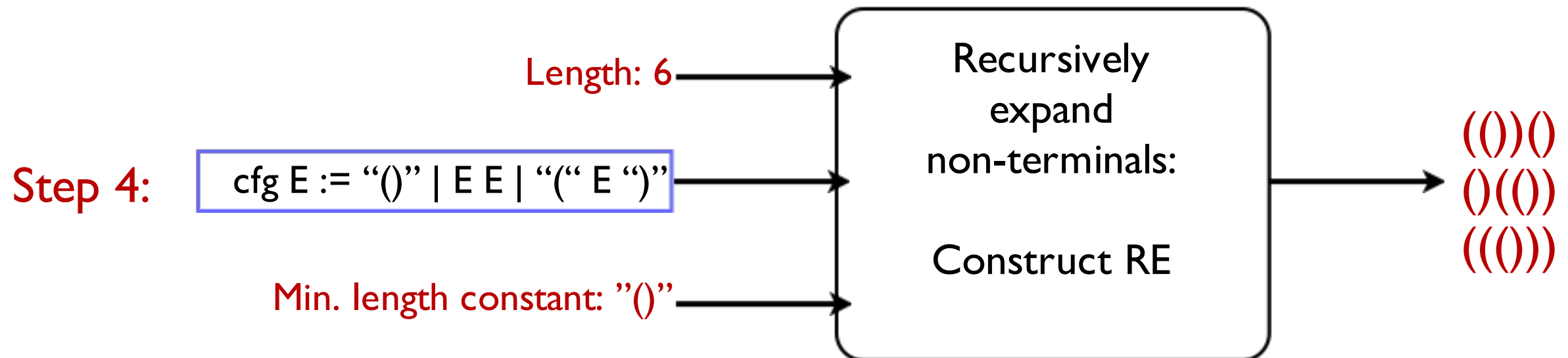
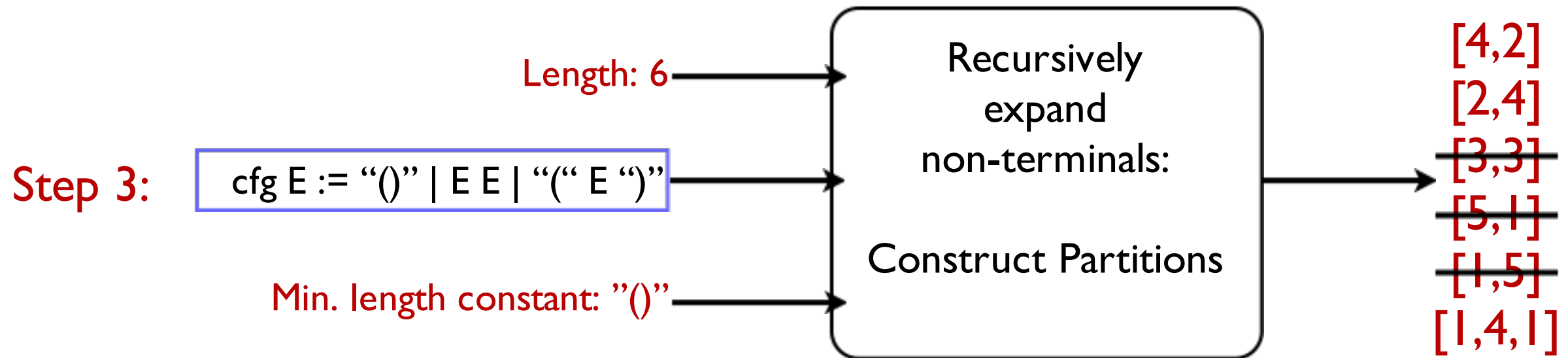
# How Hampi Works

## Unroll Bounded CFGs into Regular Exp.



# How Hampi Works

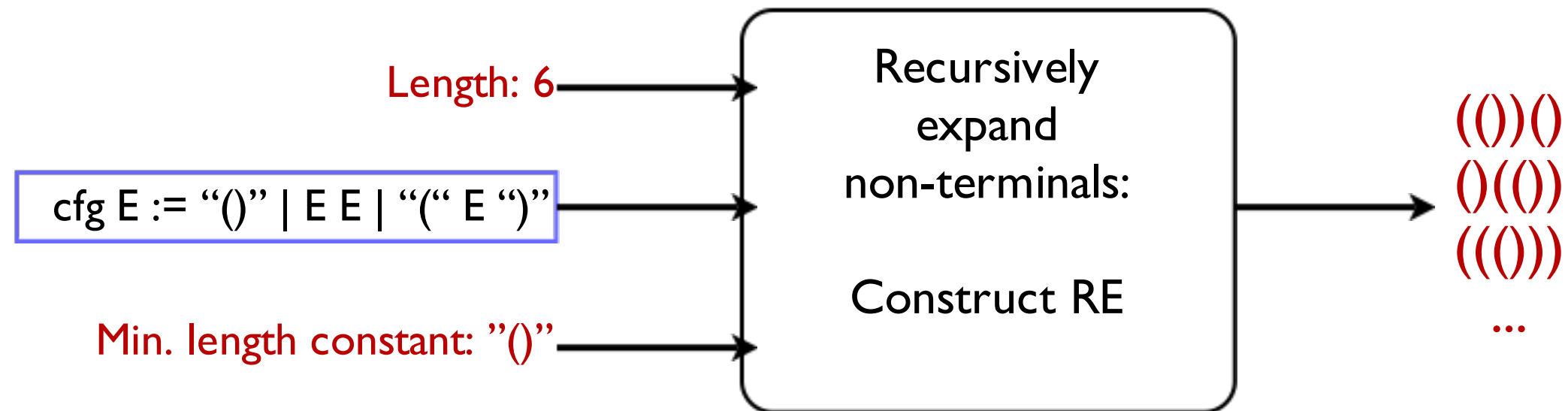
## Unroll Bounded CFGs into Regular Exp.





# Unroll Bounded CFGs into Regular Exp.

## Managing Exponential Blow-up



- Memoize common sub-expressions
- Lots of redundant sub-expression in commonly occurring regular expressions
- Works well in practice

# How Hampi Works

## Converting Regular Exp. into Bit-vectors

Encode regular expressions recursively

- Alphabet  $\{ (, ) \} \rightarrow 0, 1$
- constant  $\rightarrow$  bit-vector constant
- union  $+$   $\rightarrow$  disjunction  $\vee$
- concatenation  $\rightarrow$  conjunction  $\wedge$
- Kleene star  $*$   $\rightarrow$  conjunction  $\wedge$
- Membership, equality  $\rightarrow$  equality

$( \vee ) \in ( ) [ ( ) ( ) + ( ( ) ) ] + [ ( ) ( ) + ( ( ) ) ] ( ) + ( [ ( ) ( ) + ( ( ) ) ] )$

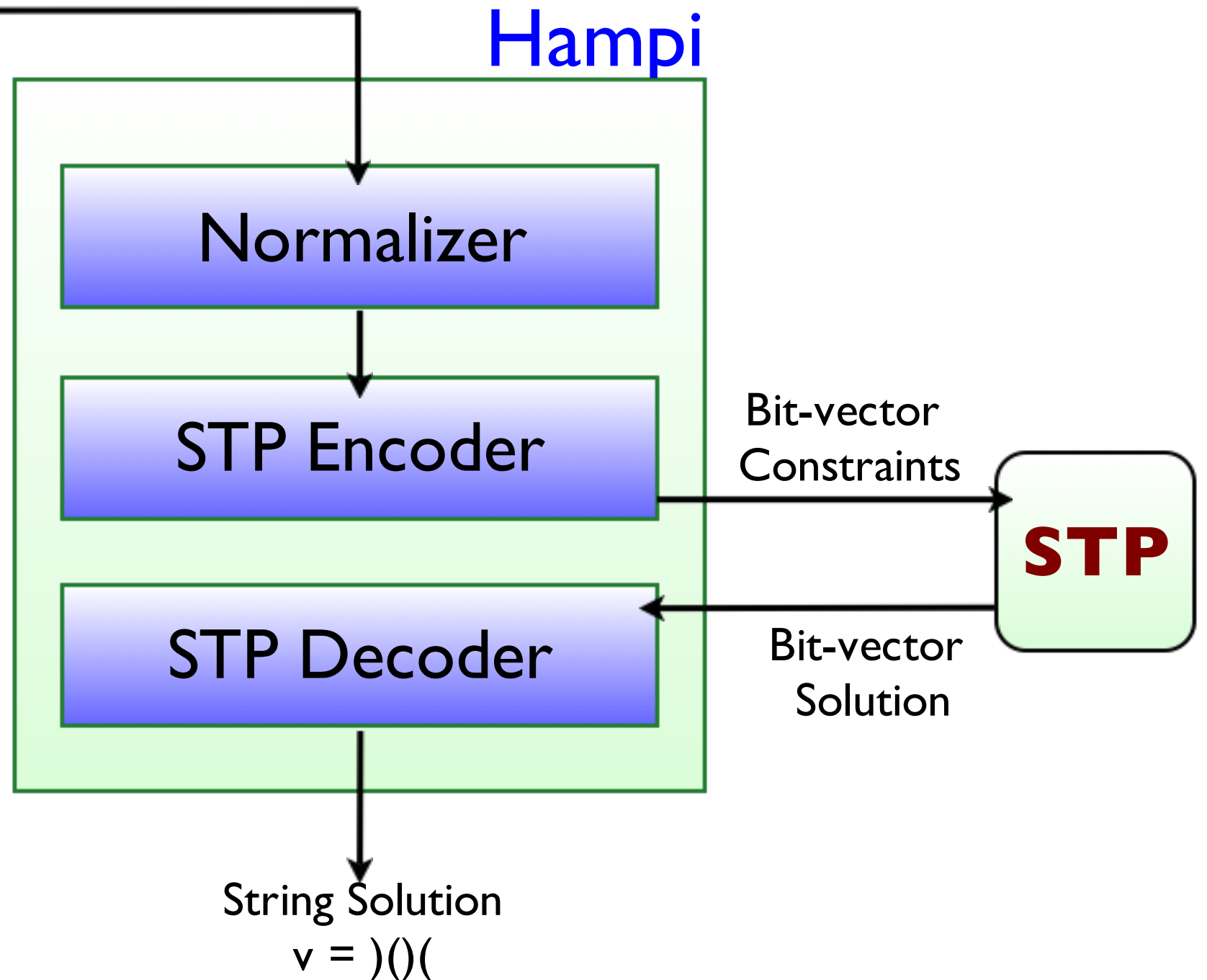
Formula  $\Phi_1$   $\vee$  Formula  $\Phi_2$   $\vee$  Formula  $\Phi_3$

$B[0]=0 \wedge B[1]=1 \wedge \{ B[2]=0 \wedge B[3]=1 \wedge B[4]=0 \wedge B[5]=1 \vee \dots$

# How Hampi Works

## Decoder converts Bit-vectors to Strings

```
var v : 4;  
cfg E := “()” | E E | “(“ E  
“)””;  
val q := concat(“(“ , v, “)””);  
assert q in E;  
assert q contains “()”;
```

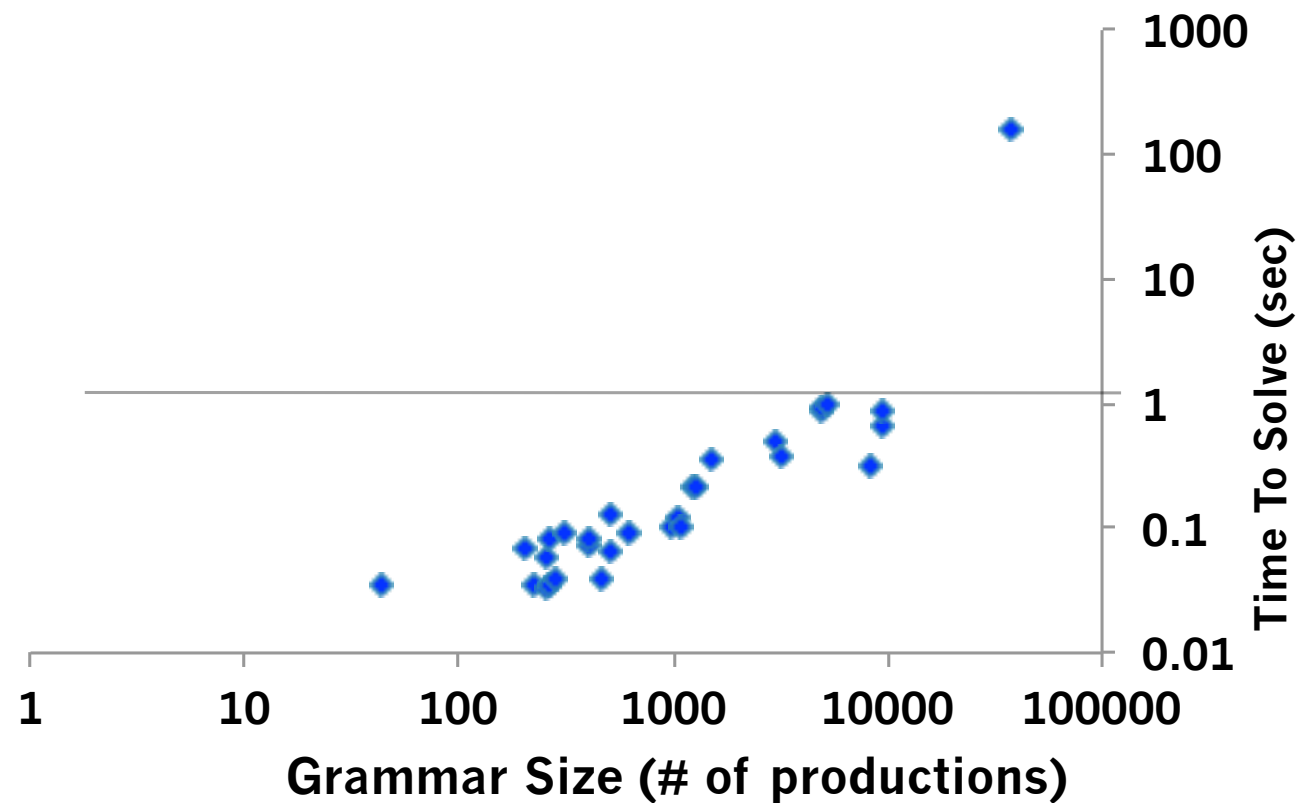


Find a 4-char string v:

- (v) is in E
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# HAMPI: Result I

## Static SQL Injection Analysis



- 1367 string constraints from Wasserman & Su [PLDI'07]
- Hampi scales to **large grammars**
- Hampi solved 99.7% of constraints in  $< 1$  sec
- All solvable constraints had short solutions

# HAMPI: Result 2

## Security Testing and XSS

- Attackers inject client-side script into web pages
- Somehow circumvent same-origin policy in websites
- echo “Thank you \$my\_poster for using the message board”;
- Unsanitized \$my\_poster
- Can be JavaScript
- Execution can be bad

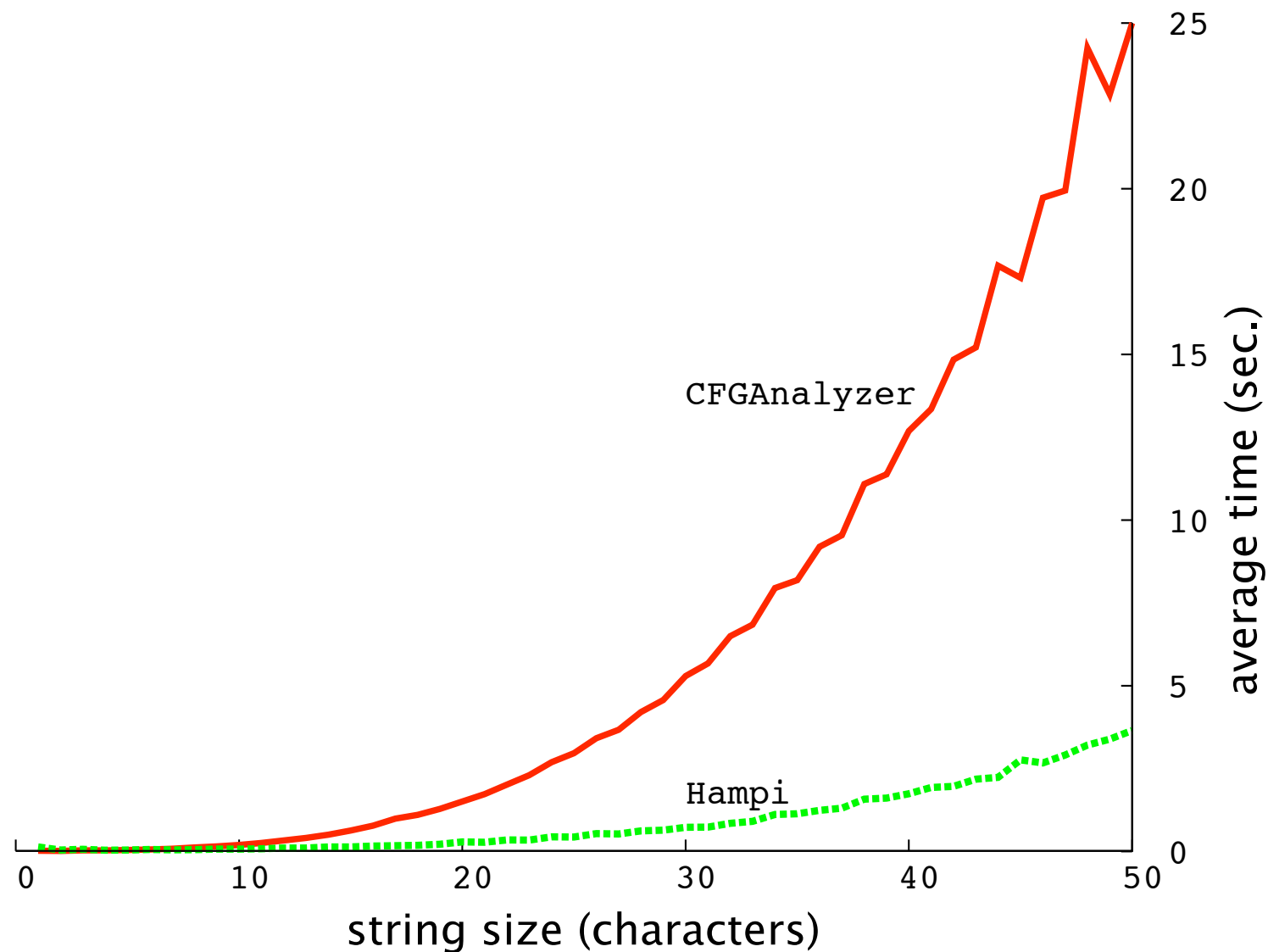
# HAMPI: Result 2

## Security Testing

- Hampi used to build Ardilla security tester [Kiezun et al., ICSE'09]
  - 60 new vulnerabilities on 5 PHP applications (300+ kLOC)
    - 23 SQL injection
    - 37 cross-site scripting (XSS)
- 5 added to  
US National Vulnerability DB
- 46% of constraints solved in < 1 second per constraint
  - 100% of constraints solved in < 10 seconds per constraint

# HAMPI: Result 3

## Comparison with Competing Tools

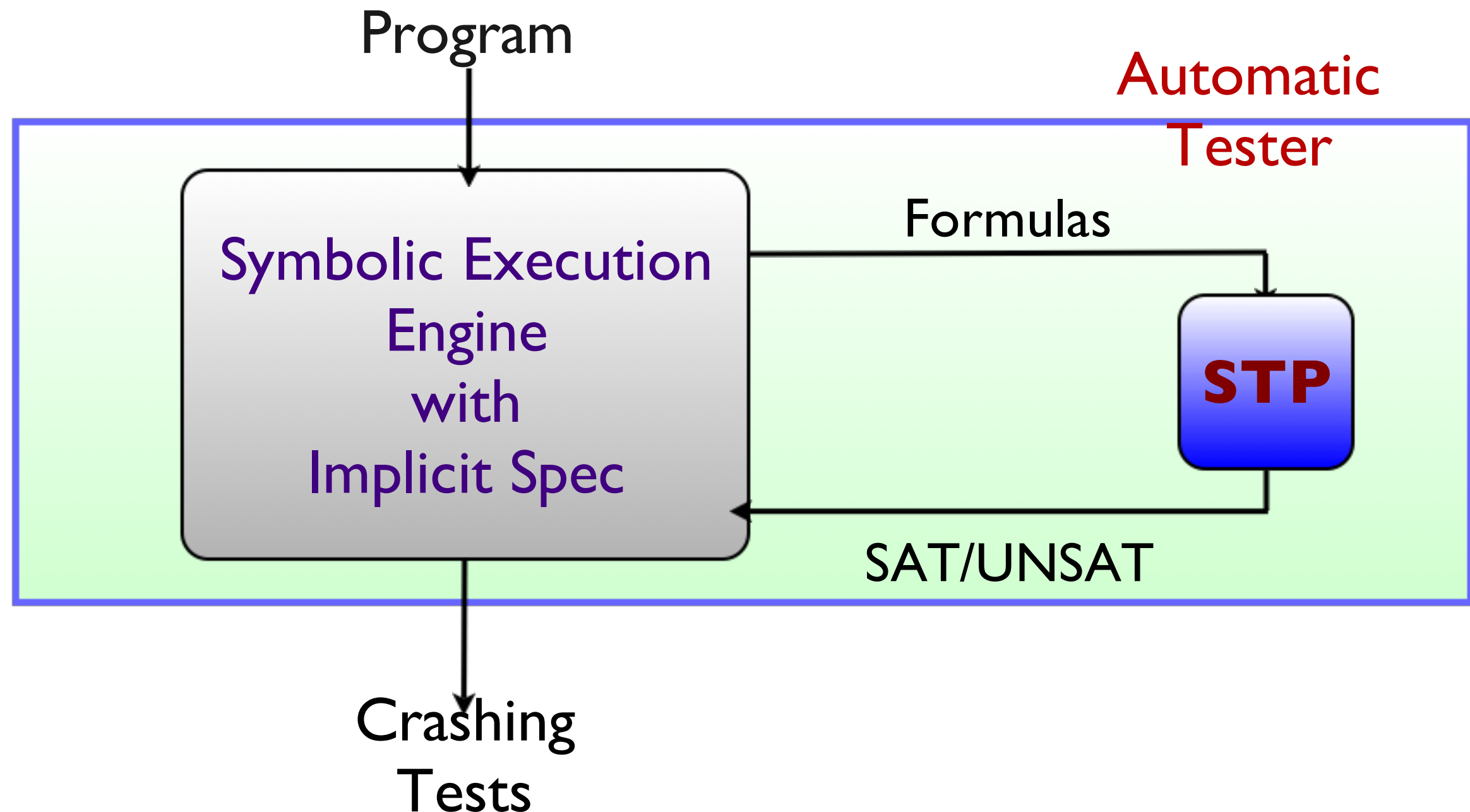


- [HAMPI vs. CFGAnalyzer \(U. Munich\)](#): HAMPI ~7x faster for strings of size 50+
- HAMPI vs. Rex (Microsoft Research): HAMPI ~100x faster for strings of size 100+
- HAMPI vs. DPRLE (U. Virginia): HAMPI ~1000x faster for strings of size 100+

# How to Automatically Crash Programs?

## KLEE: Concolic Execution-based Tester

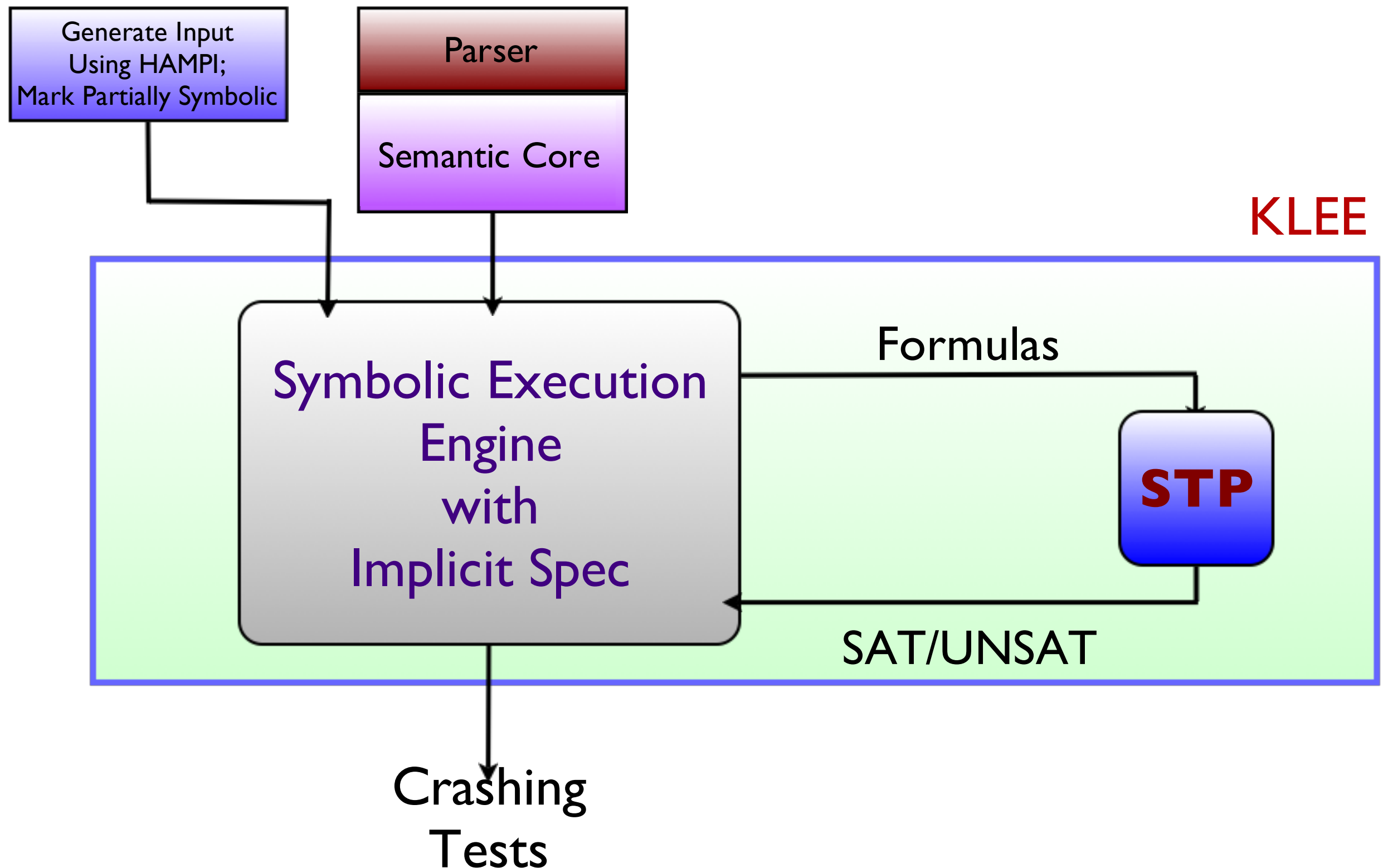
Problem: Automatically generate **crashing tests** given only the code





# HAMPI: Result 4

## Helping KLEE Pierce Parsers



# Impact of Hampi: Notable Projects

Category	Research Project	Project Leader/Institution
Static Analysis	SQL-injection vulnerabilities	Wasserman & Su/UC, Davis
Security Testing	Ardilla for PHP (SQL injections, cross-site scripting)	Kiezun & Ernst/MIT
Concolic Testing	Klee Kudzu NoTamper	Engler & Cadar/Stanford Saxena & Song/Berkeley Bisht & Venkatakrishnan/U Chicago
New Solvers	Kaluza	Saxena & Song/Berkeley

# Rest of the Talk

- HAMPI string solver
  - String equations and membership in regular expressions/CFGs
  - How HAMPI works
  - Experimental results
- Theoretical results
  - Undecidability of forallexists fragment of word equations
  - Conditional decidability results for word equations and length
  - Open theoretical problems
- Z3-str

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# Word Equations, Reg Exp, and Length

## What is known

Result	Person (Year)	Notes
Undecidability of Quantified Word Equations	Quine (1946)	Multiplication reduced to concat
Undecidability of Quantified Word Equations with single alternation	Durnev (1996), G. (2012)	2-counter machines reduced to words with single quantifier alter.
Decidability (PSPACE) of QF Theory of Word Equations	Makanin (1977) Plandowski (1996, 2002/06)	Makanin result very difficult Simplified by Plandowski
Decidability (PSPACE-complete) of QF Theory of Word Equations + RE	Schultz (1992)	RE membership predicate
QF word equations + Length() (?)	Matiyasevich (1971)	Unsolved

# Theory of Word Equations and Length

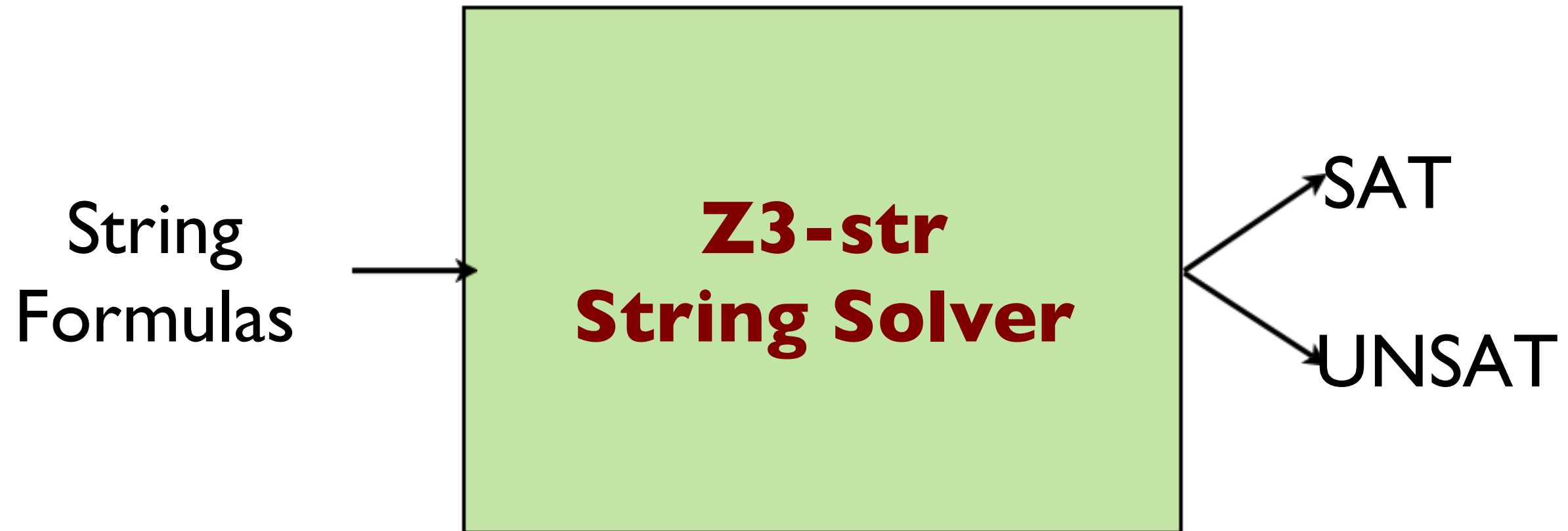
## Our Results (HVC 2012)

<b>Decidability/Undecidability</b>	<b>Result</b>
Undecidability	<ul style="list-style-type: none"> <li>• Theorem: The forall-exists fragment of quantifier-free word equations is undecidable.</li> </ul> <p>Proof Sketch:</p> <ul style="list-style-type: none"> <li>• Reduction from the halting problem for 2-counter machines to SAT problem for forall-exists fragment of word equations</li> <li>• Intuition is to encode computational histories of 2-counter machines into strings</li> </ul>
Conditional decidability	<ul style="list-style-type: none"> <li>• Theorem: The quantifier-free theory of word equations and length is decidable, if word equations can be converted into solved form</li> <li>• Theorem: The quantifier-free theory of word equations with length and regular expressions membership is decidable, if word equations can be converted into solved form</li> </ul>

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- **Z3-str**

# Z3-str String Solver\*



- Quantifier-free theory of word equations and length function
- **Status: unknown**
- Our partial decidability technique
  - Given a word equation partition its solutions space into finite buckets
  - Leverage Z3 for identifying equivalent expressions and length consistency checks
  - Approximate by heuristically solving “overlapping” equations

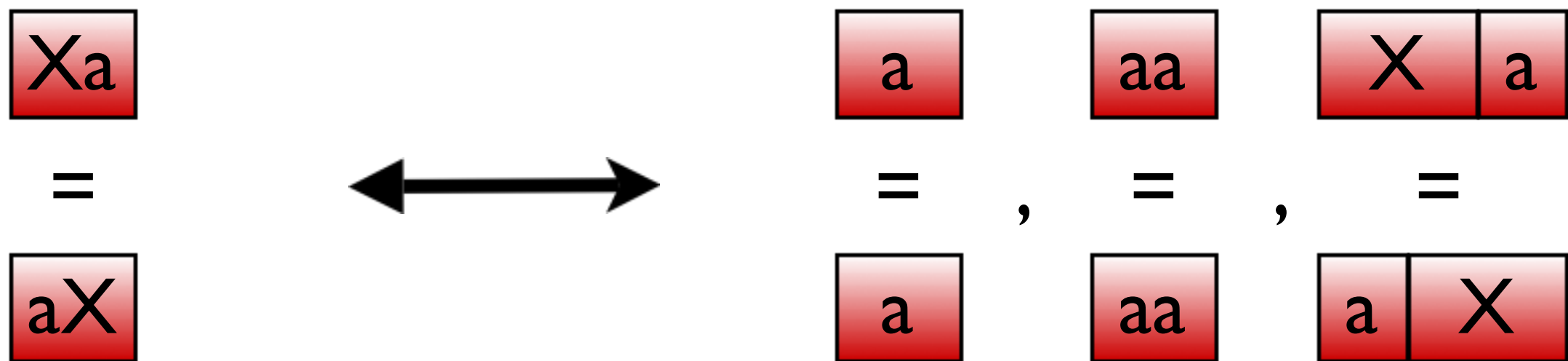
\* ~~Joint work with Xiangyu Zhang and Yunhui Zheng (Purdue University)~~



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# Related Work (Practice)

<b>Tool Name</b>	<b>Project Leader/Institution</b>	<b>Comparison with HAMPI</b>
Rex	Bjorner, Tillman, Vornkov et al. (Microsoft Research, Redmond)	<ul style="list-style-type: none"><li>• HAMPI</li><li>+ Length+Replace(<math>s_1, s_2, s_3</math>)</li><li>- CFG</li><li>• Translation to int. linear arith. (Z3)</li></ul>
Mona	Karlund et al. (U. of Aarhus)	<ul style="list-style-type: none"><li>• Can encode HAMPI &amp; Rex</li><li>• User work</li><li>• Automata-based</li><li>• Non-elementary</li></ul>
DPRLE	Hooimeijer (U. of Virginia)	<ul style="list-style-type: none"><li>• Regular expression constraints</li></ul>

# Some Future Directions

- A Predictive Theory for CDCL SAT solvers and DPLL(T)?
- Attack-resistance programs
  - Can we define a mathematical notion of partial reliability?
- Expanding the scope of testing
  - Automatic counter-example construction for math conjectures
- Open problems regarding theories of strings
  - Is the SAT problem for word equations in NP? Is the quantifier-free theory of word equations and length decidable?
- Richer string solvers
- All-in-one: integrating word equations, length and membership into SMT

# Key Contributions

<https://ece.uwaterloo.ca/~vganesh>

<b>Name</b>	<b>Key Concept</b>	<b>Impact</b>	<b>Pubs</b>
<b>STP</b> Bit-vector & Array Solver <sup>1,2</sup>	Abstraction-refinement for Solving	Concolic Testing	CAV 2007 CCS 2006 TISSEC 2008
<b>HAMPI</b> String Solver <sup>1</sup>	App-driven Bounding for Solving	Analysis of Web Apps	ISSTA 2009 <sup>3</sup> TOSEM 2012 CAV 2011
<b>(Un)Decidability</b> results for Strings	Reduction from two-counter machine halting problem		HVC 2012
<b>Taint-based Fuzzing</b>	Information flow is cheaper than concolic	Scales better than concolic	ICSE 2009
<b>Automatic Input Rectification</b>	Acceptability Envelope: Fix the input, not the program	New way of approaching SE	ICSE 2012

**1. STP won the SMTCOMP 2006 and 2010 competitions for bit-vector solvers**

**2. HAMPI: ACM Best Paper Award 2009**

**3. Google Award 2011**

4. Retargetable Compiler (DATE 1999)

5. Proof-producing decision procedures (TACAS 2003)

6. Error-finding in ARBAC policies (CCS 2011)

7. Programmatic SAT Solvers (SAT 2012)