Automated Theory Substitution

Toward Proof-Driven Software Development

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TOC

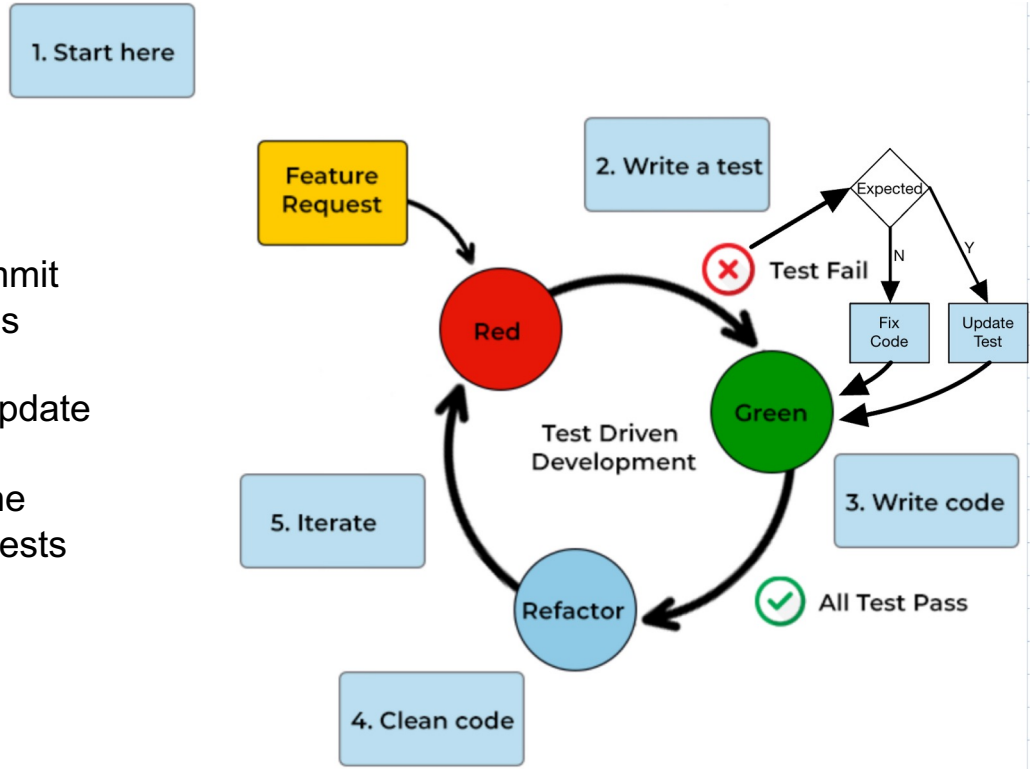
AMETHYST Introduction

- Formal verification of real world software has shown increasing success
  - seL4
  - CompCert
- But still no widespread adoption
  - Expert knowledge required to write and maintain formal proofs
  - Experience with interactive theorem provers (ITP)
- AutoMatEd THeorY SubsTitution (AMETHYST)
  - Artificially intelligent proof assistant for the Isabelle ITP
  - User interaction designed for non-experts
    - Software engineers
    - Integrated into VSCode
  - Move maintenance from the expert into the CI/CD cycle

Empower the novice, Unburden the expert

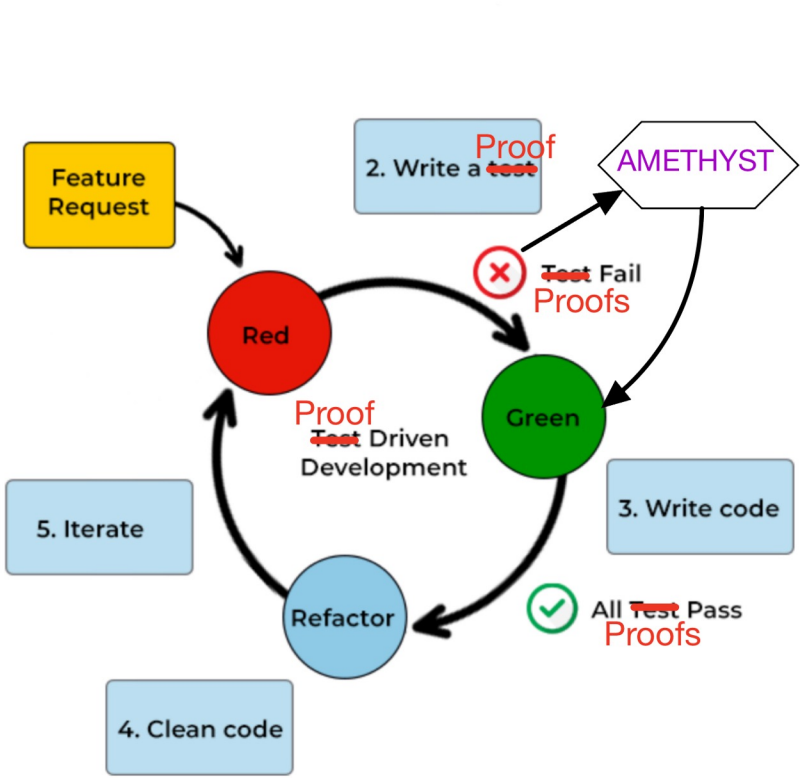
Test-Driven Development

- Tests may be
  - Happy path scenarios
  - Edge case scenarios
  - Regression/unit
- Often run automatically before commit
- Failed tests can go in two directions
  - New bug in code - fix it
  - Expected change in code - update test
- Pro: Tests are written in code so the developer can maintain code and tests
- Con: Can never explicitly cover all cases



Proof-Driven Development

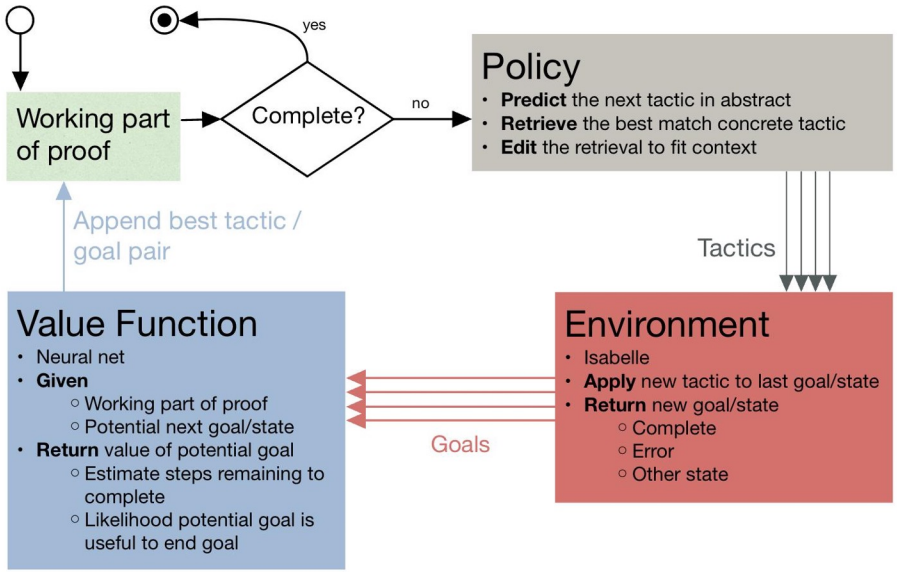
- Proofs implicitly cover all cases
- Should run automatically before commit
- It is easier to break a test than to break a proof
- Fixing a proof is less straightforward - may be a combination of code and proof edits
- Code is easy, proofs are hard
- Amethyst can help with the proof



AMETHYST AI

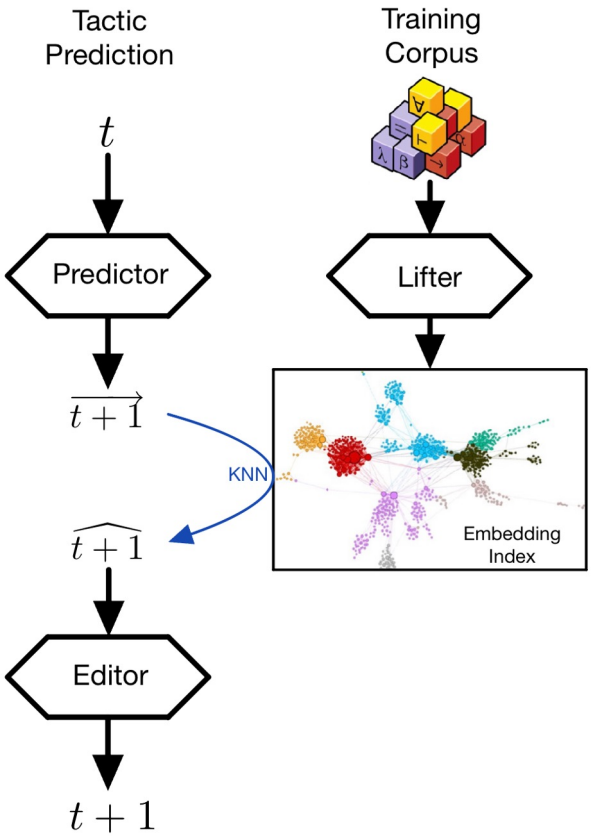
Amethyst AI

- Two machine learning models
  - **Policy** and **Value Function**
  - Pretrained offline on seL4 and Isabelle proofs
- Isabelle/HOL proof assistant
- Coordinated by a tree search agent
  - Finishes a partial proof
  - Updates models in online learning fashion



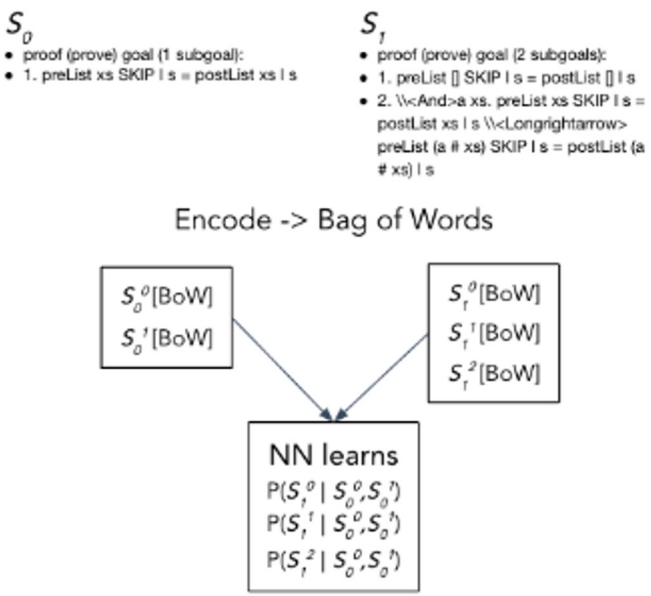
Policy

- Lifter
  - Pretrained to embed lexically similar tactics near each other
  - Embeddings are indexed for NN lookup
- Predictor
  - Given text of a tactic, predict embedding of next tactic
  - Lookup NN in index
- Editor
  - GPT edits the retrieval to fit context of current proof
  - This is remedial RAG



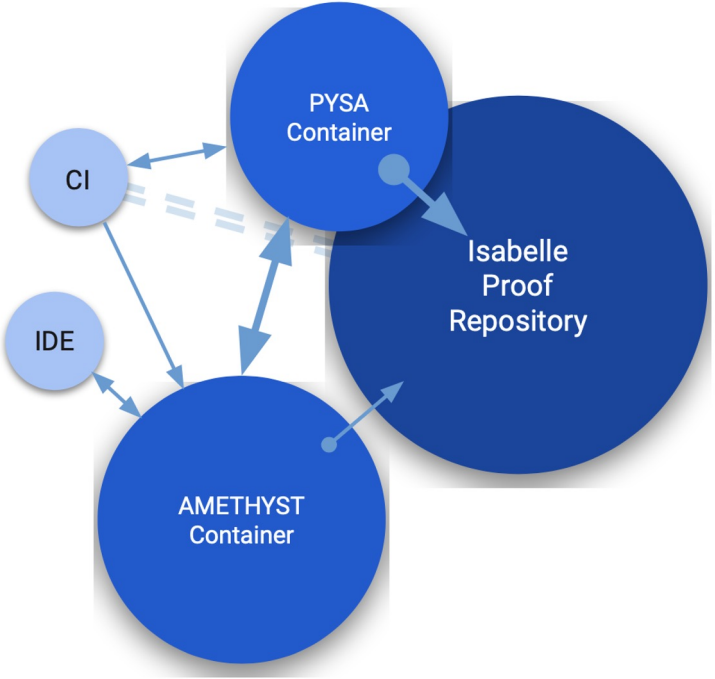
Value Function

- Proof: series of interleaved states and tactics
  - A state s is composed of one or more sub goals
  - Tactic t_n operates on s_n to produce s_{n+1}
- Measure: given states 0-n, how useful is s_{n+1} to completing the proof?
- Train a neural network to predict if the sub goals of s_{n+1} truly follow sub goals of s_n
  - Positive example - select s_n s_{n+1} from contiguous proofs
  - Negative example - select s_n from one proof, select s_{n+1} from random other proof



PYSA

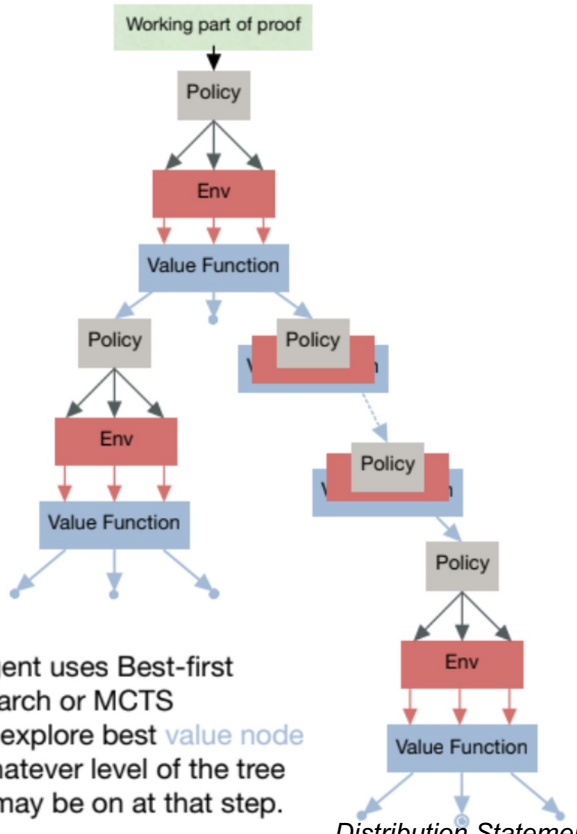
- Provide Python API for Isabelle actions
  - Inspired by Portal to Isabelle (PISA)
  - Parse and navigate theory files
  - Build heap images
  - Gather curriculum data from git history
- Get proof state from Isabelle
  - Lesson plans for AMETHYST training
  - Feedback for AMETHYST evaluation
  - Proof code testing for CI



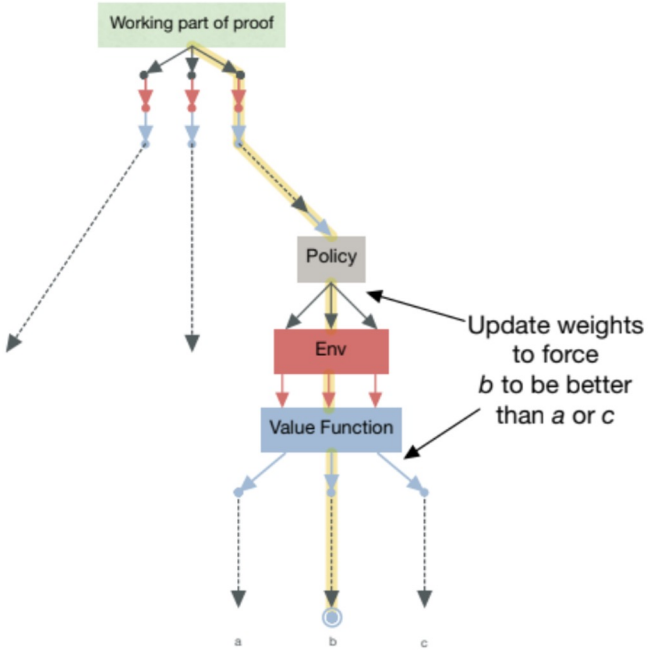
Tree Search Agent

Agent "owns" all modules

- Receives partial working proof
- Policy predicts possible next tactics
- Execute tactics via PYSA
- Value function scores valid PYSA returns
- Iterate until complete proof or max depth
- Take user feedback and back propagate up the tree from best leaf updating models.

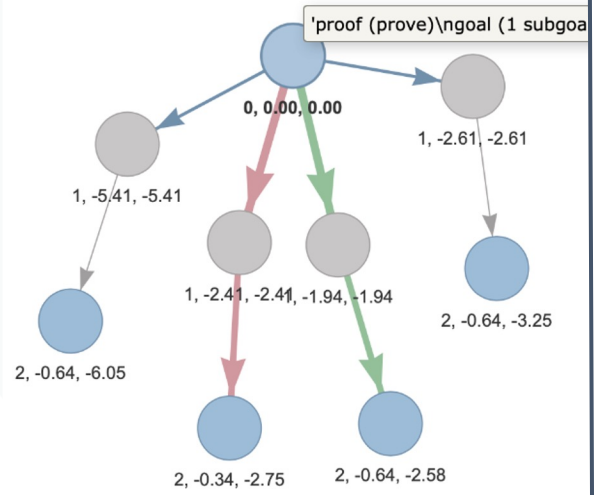


Agent uses Best-first search or MCTS to explore best value node whatever level of the tree it may be on at that step.

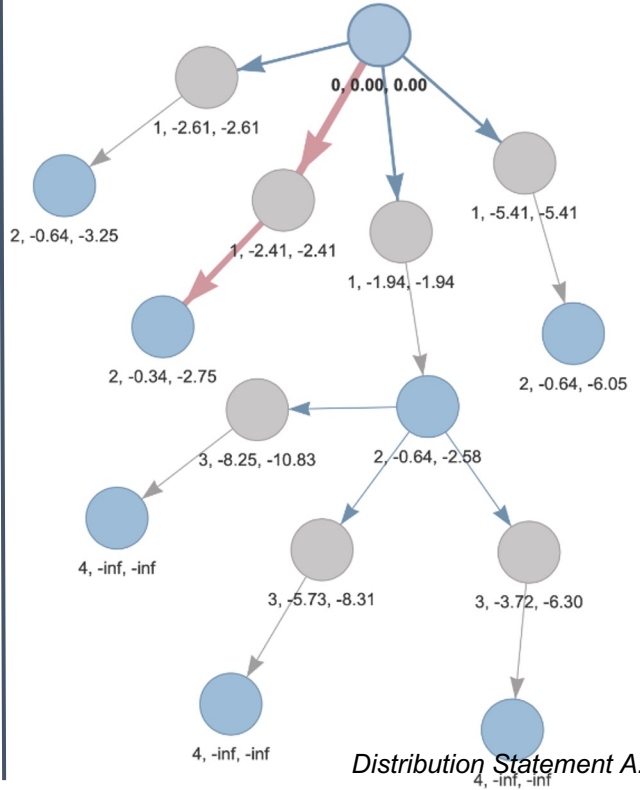


Tree Search Teacher-Forced Learning

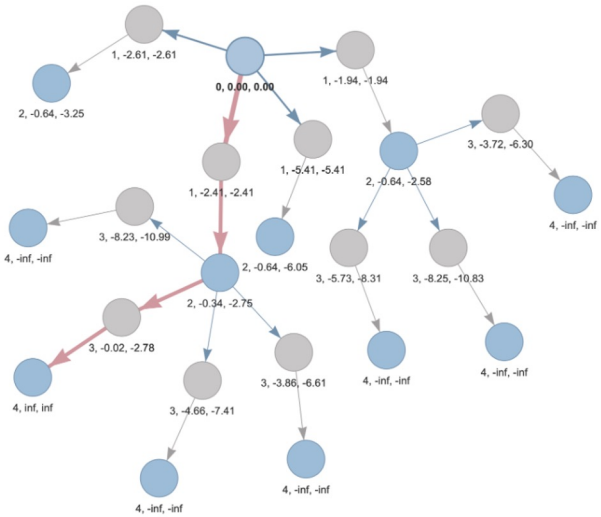
Step 1



Step 2



Step 3



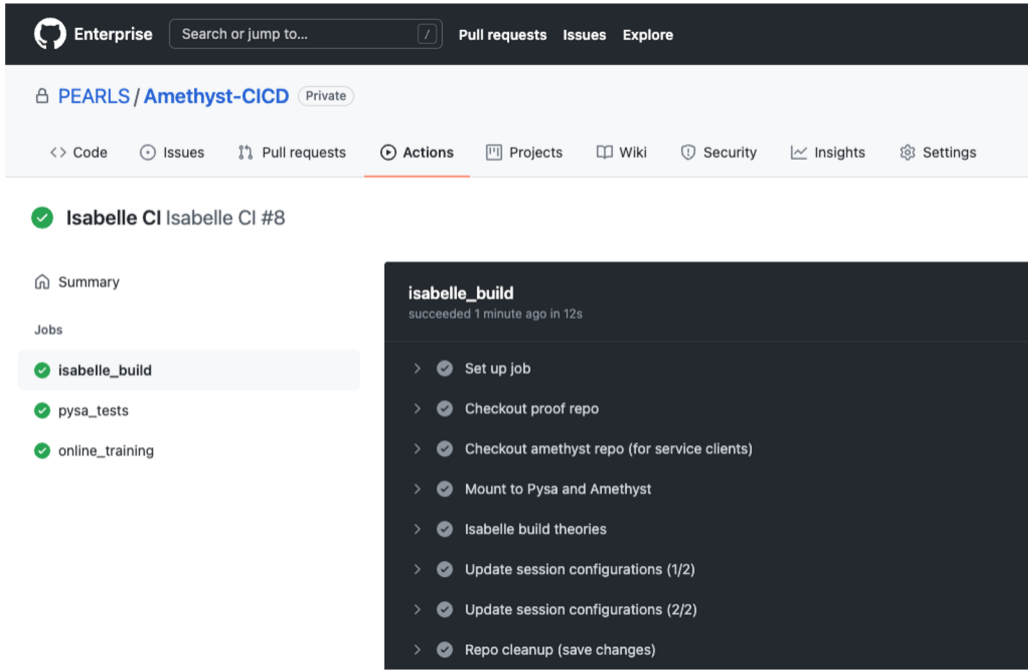
- Blue node - proof state
- Grey node - generated tactic
- Green edge - best path
- Red Edge - teacher's correct path

- Search terminates when PYSA returns zero subgoals.
- Value function assigns score of infinity.
- Teacher path is highest scoring

AMETHYST UX

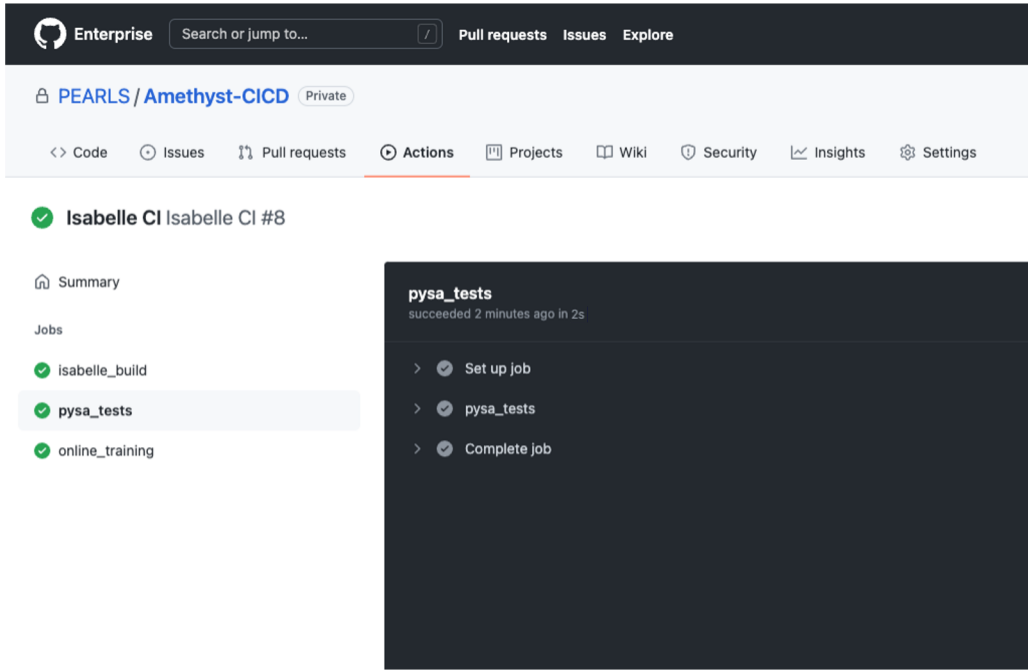
CI with GitHub Actions

- Builds modified theories, reports meaningful deltas



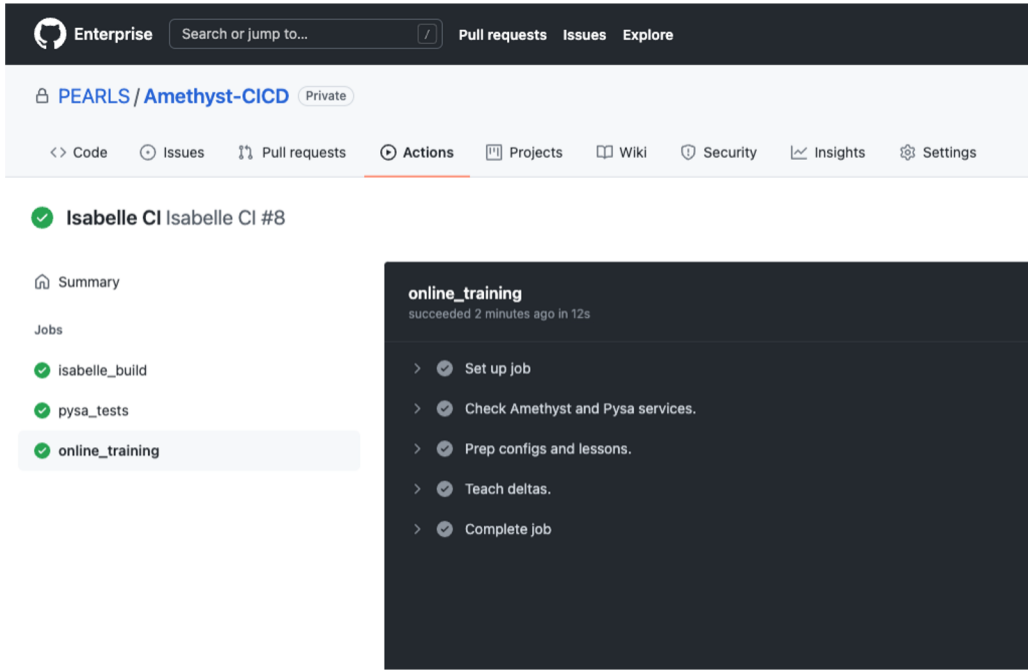
CI with GitHub Actions

- Builds modified theories, reports meaningful deltas
- Tests all proofs, checking PYSA capabilities.



CI with GitHub Actions

- Builds modified theories, reports meaningful deltas
- Tests all proofs, checking Pysa capabilities.
- Online training: teaches model new proofs automatically



VSCode Extension

- Indexer
  - Search tool based on KNN in an embedding space
  - Helps find similar lemmas or tactics
- Interactive Proof Search
  - Visualize AI driven proof search
  - See which branches are promising
    - Proof states ranked by a score
  - Direct agent to explore certain paths

Indexer

Equivalence_Relations.thy

```
le_lang_equiv: "(\<simeq>) \<le> (\<simeq>\<^sub>L)"
begin

lemma eq_implies_ltl_equiv:
  "\<phi> \<simeq> \<psi> \<Longrightarrow> w \<Turnstile>\<^sub>n \<phi> = w \<Turnstile>\<^sub>n \<psi>"
  using le_lang_equiv ltl_lang_equiv_def by blast

lemma const_implies_eq:
  "\<phi> \<simeq>\<^sub>C \<psi> \<Longrightarrow> \<phi> \<simeq> \<psi>"
  using ge_const_equiv by blast

lemma eq_implies_lang: "\<phi> \<simeq> \<psi> \<Longrightarrow> \<phi> \<simeq>\<^sub>L \<psi>"
using le_lang_equiv
by blast

lemma eq_refl[simp]:
  "\<phi> \<simeq> \<phi>"
  by (meson eq_equivp equivp_reflp)

lemma eq_sym[sym]:
  "\<phi> \<simeq> \<psi> \<Longrightarrow
  by (meson eq_equivp equivp_symp)
```

- Pysa Indexer
- Pysa Tree Visualizer
- Go to Definition
- Peek
- Change All Occurrences
- Refactor...
- Cut
- Copy
- Paste
- Command Palette...

Indexer

[Extension Development Host] Pysa Indexer — LTL

Equivalence_Relations.thy

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520     le_lang_equiv: "(\<sim>) \<le> (\<sim>\<^sub>L)"
521   begin
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545
546   lemma eq_trans[trans]:
```

Pysa Indexer

Pysa Tree Visualizer

Search

lemma eq_implies_lang: "\<phi> \<sim> \<psi> \<Longrigh Search

Results

lemma eq_implies_lang:
"\<phi> \<sim> \<psi> \<Longrightarrow> \<phi> \<sim>\<^sub>L \<psi>"

lemma const_implies_eq:
"\<phi> \<sim>\<^sub>C \<psi> \<Longrightarrow> \<phi> \<sim> \<psi>"

lemma af_lang_congruent:
"\<phi> \<sim>\<^sub>L \<psi> \<Longrightarrow> af \<phi> w \<sim>\<^sub>L af \<psi> w"

lemma ltl_prop_equiv_implies_ltl_lang_equiv:
"\<phi> \<sim>\<^sub>P \<psi> \<Longrightarrow> \<phi> \<sim>\<^sub>L \<psi>"

lemma af_const_congruent:
"\<phi> \<sim>\<^sub>C \<psi> \
\<phi> w \<sim>\<^sub>C af \<psi

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VSCode Extension

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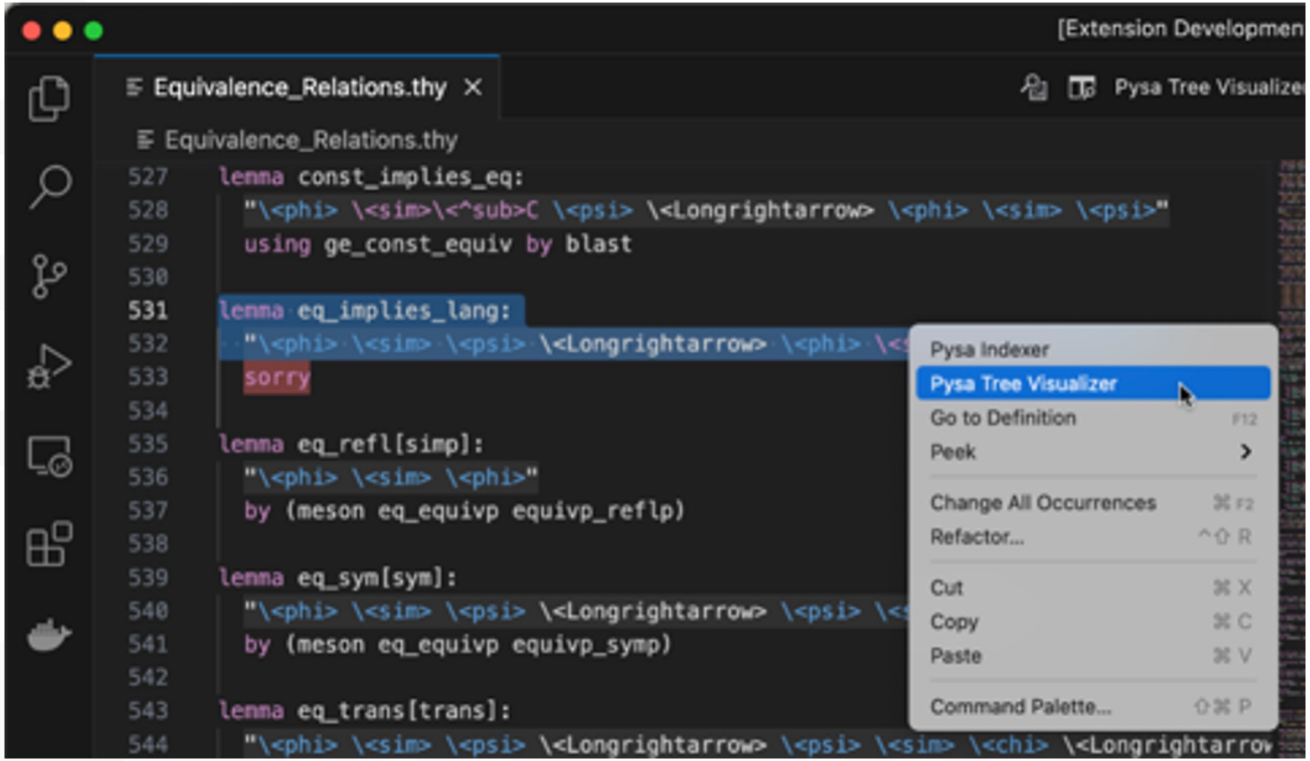
Proof Search Visualizer



Proof Search Visualizer

[Extension Development Host] Pysa Tree Visualizer — LTL

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Pysa Tree Visualizer

Lemma	lemma eq_implies_lang: "\<phi> \<sim> \<psi> \<Longrightarrow> \<phi> \<s
Tactics	
Session	LTL
Theory	/workspace/LTL/Equivalence_Relations.thy

Start | Full Proof | Step | Trace | Refresh

Proof Search Visualizer

[Extension Development Host] Pysa Tree Visualizer — LTL

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Tactics:

Session: LTL

Theory: /workspace/LTL/Equivalence_Relations.thy

Start | Full Proof | Step | Trace | Refresh

```
proof (prove)
goal (1 subgoal):
 1. ϕ ~ ψ ⟹ ϕ ~⇩L ψ
```

Proof Search Visualizer

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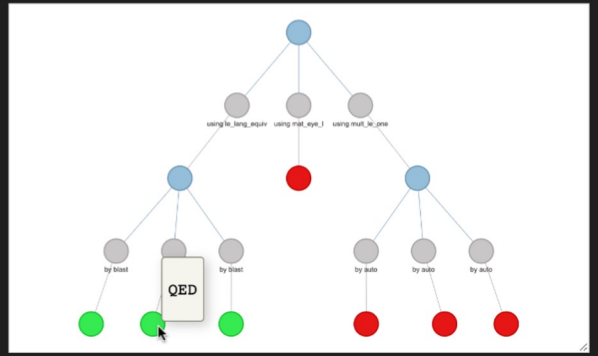

Proof Search Visualizer

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Tactics:

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Proof Search Visualizer

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