Security Reasoning via Substructural Dependency Tracking

Hemant Gouni (with Frank Pfenning & Jonathan Aldrich)

April 16, 2026

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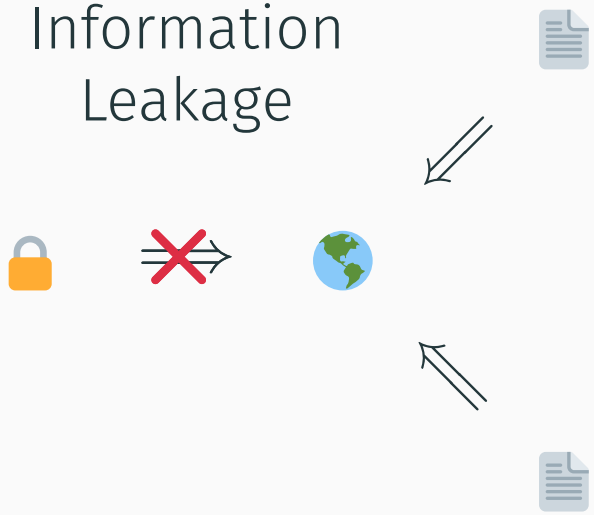
Static Dependency Tracking



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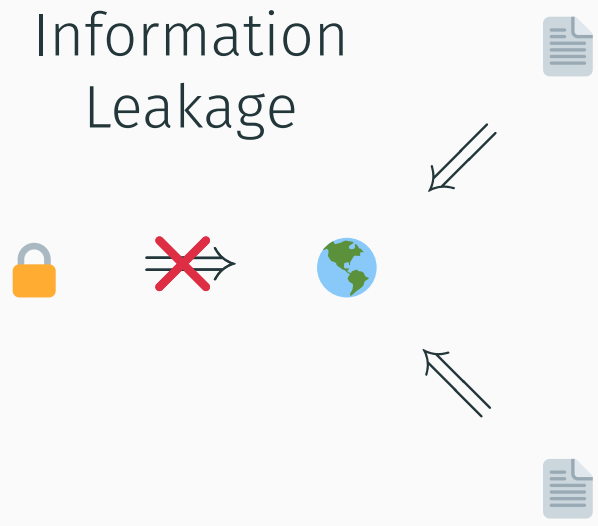
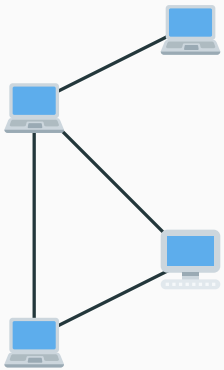
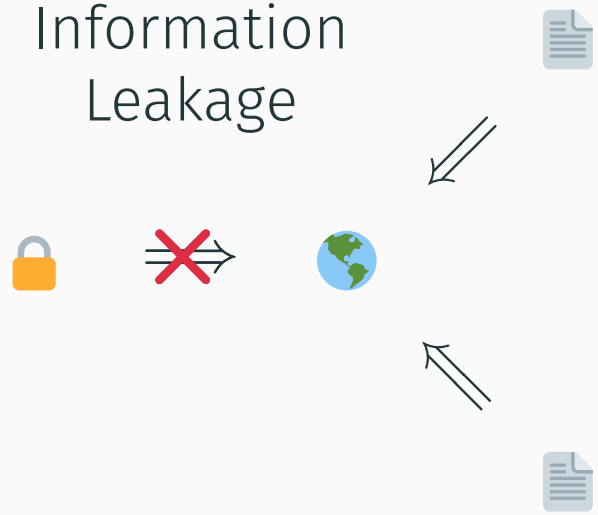




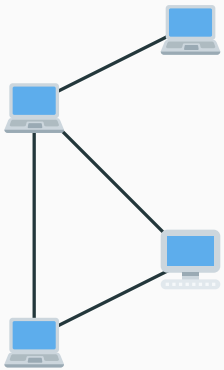
Concurrent
+
Distributed
Systems

Static Dependency Tracking

Information Leakage



Concurrent + Distributed Systems



Build Systems 🛠️

Static Dependency Tracking

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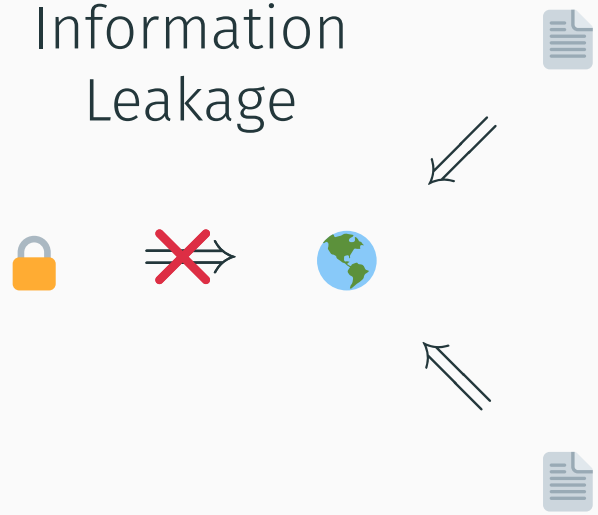
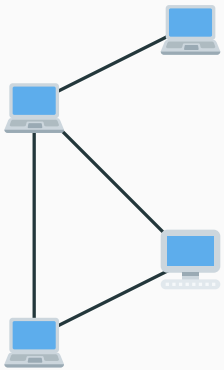


Build Systems 🛠️

Reactive Programming ⚛️



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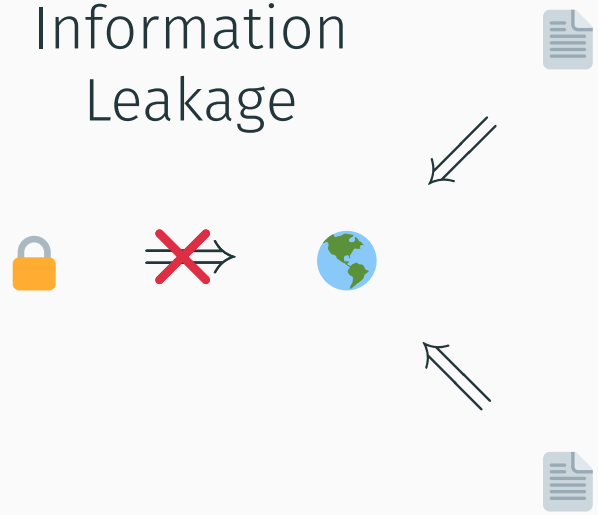

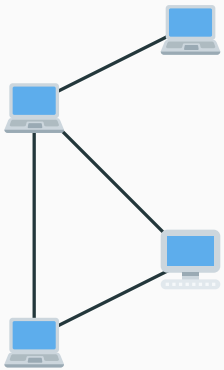

Concurrent
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Reactive Programming ⚛️

Program Slicing 🔪

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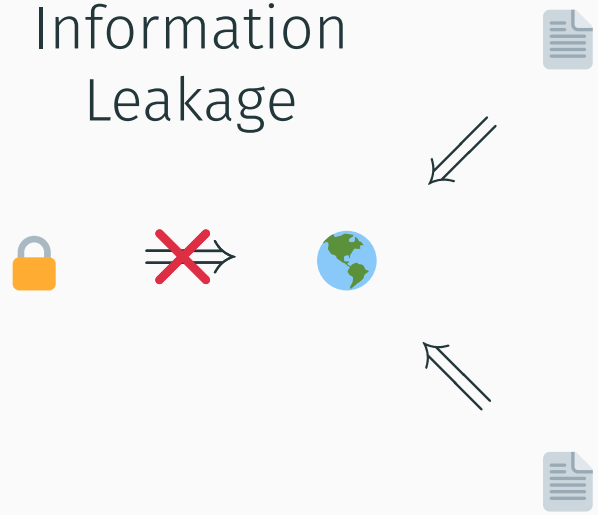

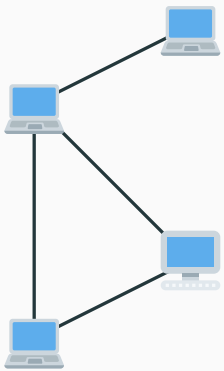

Concurrent
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Distributed Systems

Build Systems 🛠️

Reactive Programming ⚛️

Program Slicing 🔪

Dependent how often?

Static Dependency Tracking

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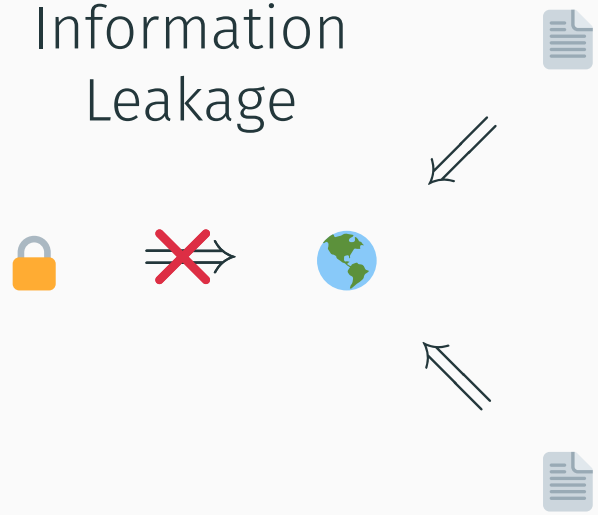

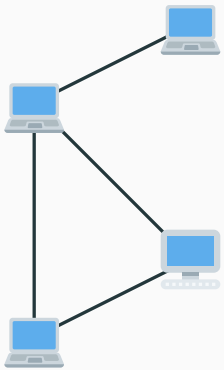

Concurrent
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Build Systems 🛠️

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Dependent how often?

Dependency guaranteed?

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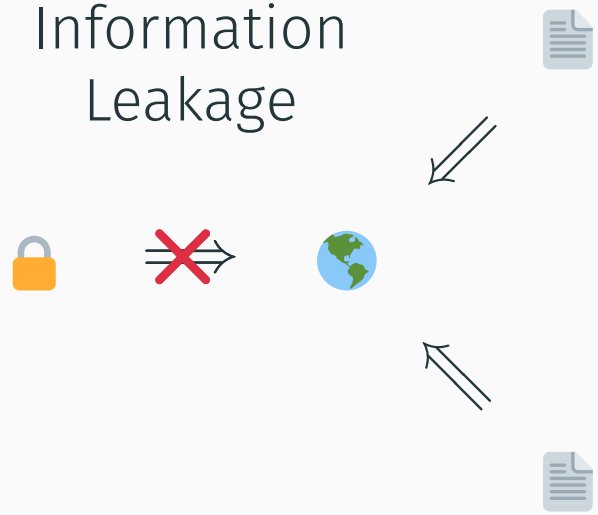

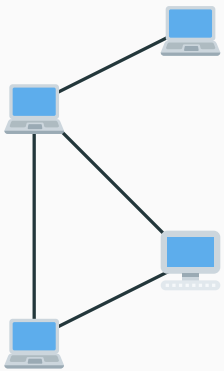

Concurrent
+
Distributed Systems

Build Systems 🛠️

Reactive Programming ⚛️

Program Slicing 🔪

Dependent how often?

Dependency guaranteed?

What dependency order?

Reinventing Our Approach 👨‍🍳

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⇓

Substructural

Dependency Tracking

Affine Types in Rust

```
let x: String = format!("hello");
```

Affine Types in Rust

```
let x: String = format!("hello");

func1(x);
```

Affine Types in Rust

```
let x: String = format!("hello");

func1(x);
func2(x);
```

Affine Types in Rust

```
let x: String = format!("hello");

func1(x);  ⟵ x moved here
func2(x);  ⟵ x used here after move ⚠
```

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let x: String = format!("hello");

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Data in Rust is *ephemeral*: it can only be consumed a **single** time!

Affine Types in Rust

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let x: String = format!("hello");

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Data in Rust is *ephemeral*: it can only be consumed a **single** time! Ephemerality leads to resource reasoning.

Bounded Affine Types

Generalize from **single** to **finitely-bounded** consumption.

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```
val f : int {⛽ ⛽ ⛽} -> int
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```
let f x = x * x
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val f : int {⛽ ⛽ ⛽} -> int

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let f x = x * x * x
let f x = x * x * x * x  ⟵ out of fuel for x ⚠
```

Bounded Affine Types

Generalize from **single** to **finitely-bounded** consumption.

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val f : int {🔥 🔥 🔥} -> int
```

```
let f x = x * x
let f x = x * x * x
let f x = x * x * x * x  ⟵ out of fuel for x ⚠
```

Observe that 🔥 on the **input** dictates the structure of `f`.

Our Approach: Reverse! ↩️

Use output—rather than input—restrictions to create resources.

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Use output—rather than input—restrictions to create resources.

```
val g : [🪙] int -> [🪙 🪙 🪙] int
```

```
let g x = x * x
let g x = x * x * x
let g x = x * x * x * x  ⟵ 🪙 🪙 🪙 🪙 ⚠️
```

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Use output—rather than input—restrictions to create resources.

```
val g : [🪙] int -> [🪙 🪙 🪙] int
```

```
let g x = !x * !x
let g x = !x * !x * !x
let g x = !x * !x * !x * !x  ⟵ 🪙 🪙 🪙 🪙 ⚠
```

The resource count increases when `x` is *run*, not merely *used*.

Our Approach: Reverse! ↩️

Use output—rather than input—restrictions to create resources.

```
val g : [🪙] int -> [🪙 🪙 🪙] int
        suspended
let g x = !x * !x
let g x = !x * !x * !x
let g x = !x * !x * !x * !x  ⟵ 🪙 🪙 🪙 🪙 ⚠
```

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          resuming running
```

The resource count increases when `x` is *run*, not merely *used*.

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Use output—rather than input—restrictions to create resources.

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val g : [🪙] int -> [🪙 🪙 🪙] int

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```

The resource count increases when `x` is *run*, not merely *used*.

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Use output—rather than input—restrictions to create resources.

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                      ⇒ 🪙
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Our Approach: Reverse! ↩️

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let g x = let y be !x in y * y * y * y
                ⇒ 🪙          : int
```

The resource count increases when `x` is *run*, not merely *used*.

Aside: Why the focus on computations?

```
val f : int {🍻 🍻 🍻} -> int

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```

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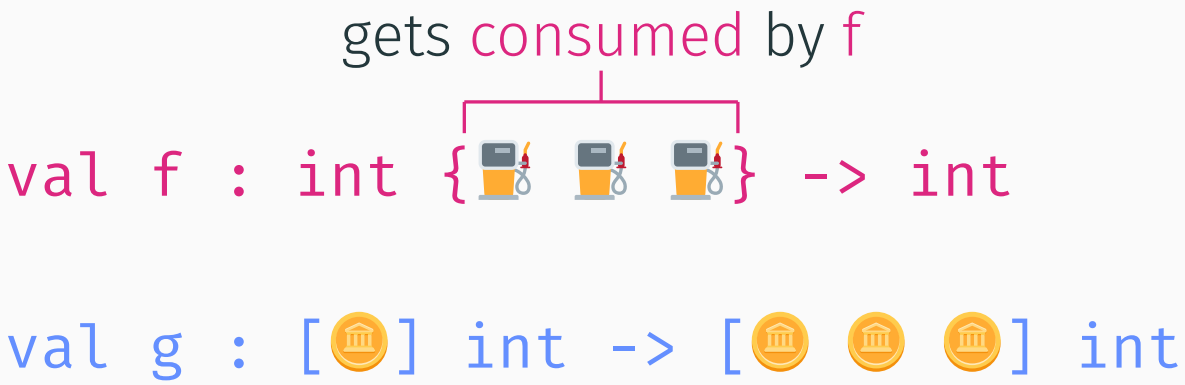

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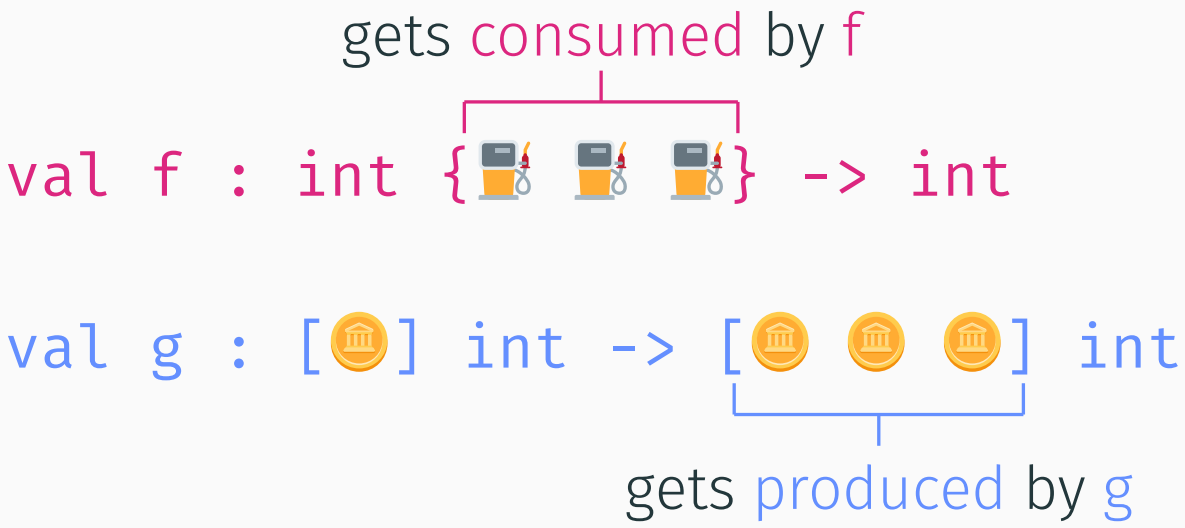

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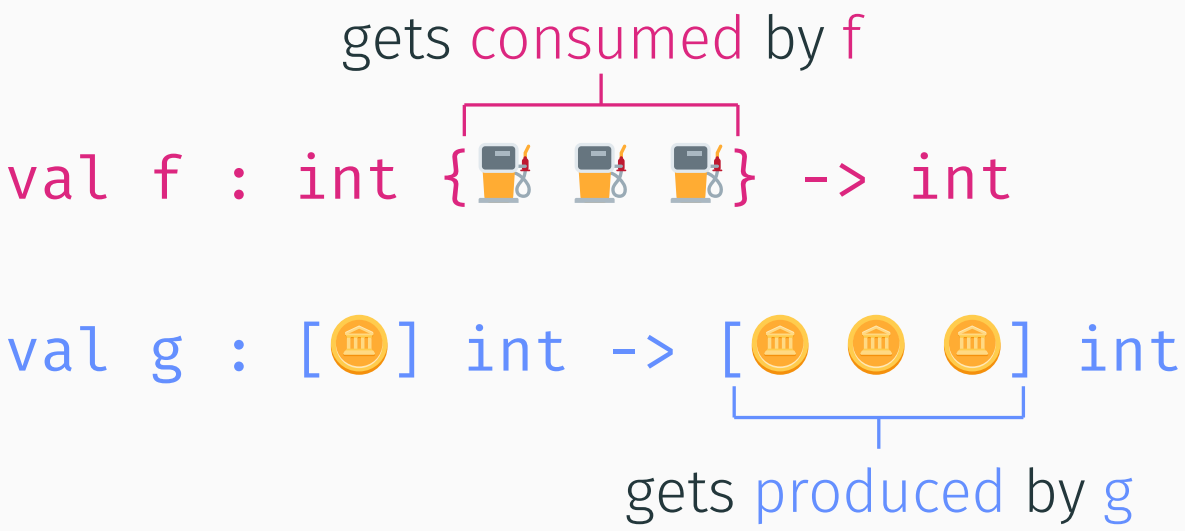

Consumption vs Production

Conventional resources regard consumption via variable use.

Our resources regard production via running computations.

Examples

Example: Quantity-Sensitive Leakage

```
module PasswordChecker : sig


end
```

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```
module PasswordChecker : sig
      affine resource 🔐

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.....

```
open PasswordChecker as pc
```

Example: Quantity-Sensitive Leakage

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let _ : [pc.🔑] bool = pc.check "faxe"
```

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Example: Capabilities

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```

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module Authorize : sig
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      val authenticate : string ->
            unit | [💎] unit
end
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```
let secured : [💎] int -> ...
```

Example: Capabilities


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      val authenticate : string ->
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end
```

```
let secured : [💎] int -> ...

match authenticate "argaven" with
| Left _ -> ...
| Right (tok : [💎] unit) -> secured (4 <~ tok)
```

Example: Protocols, or `seccomp`

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```
module DropProto : sig




end
```

Example: Protocols, or `seccomp`


```
module DropProto : sig
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Example: Protocols, or `seccomp`


```
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     (immobile) resource drp, hi




end
```

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      val drop : [drp] unit
      val hi : [hi] unit
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let _ : [hi drp lo] … = !hi; !lo; !drop; !lo
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```

Versus Conventional Resource Reasoning

The *production* perspective naturally characterizes a different range of resources than the *consumption* one:

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`drp` `seccomp`-style sandboxing

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- 🔐 Quantity-Sensitive Leakage
- 💎 Authorization via Capabilities

`drp` `seccomp`-style sandboxing

More examples in the paper!

🛒 3-in-1: {Quantity, Capability, Protocol} Safety

Substructurality via Inequality

- Weakening: [...] $\longmapsto$ [... 💎]

Substructurality via Inequality

- ~~Weakening~~: [...]  [... 💎]

Substructurality via Inequality

- Weakening: $[\ldots] \sqsubseteq [\ldots$ 💎$]$

Substructurality via Inequality

- ~~Weakening~~: $[\ldots] \not\subseteq [\ldots \text{💎}]$

Substructurality via Inequality

- ~~Weakening~~: `[...]` $\not\sqsubseteq$ `[... 💎]` $\Longrightarrow$ `strict`

Substructurality via Inequality

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- Contraction: [🔐 🔐] $\sqsubseteq$ [🔐]

Substructurality via Inequality

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- Exchange: `[hi drp]` $\sqsubseteq$ `[drp hi]`

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Substructurality via Inequality

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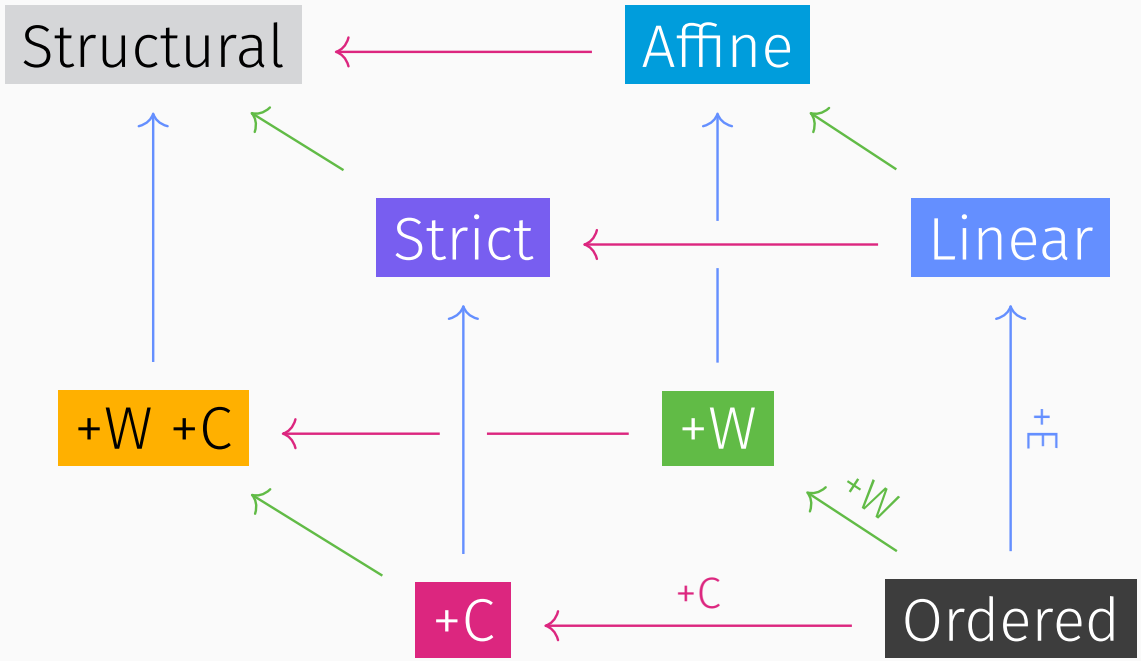

Soundness Theorem

If e `: [a1 a2 ...] A` then `!`$e \longmapsto v$ producing resources `[b1 b2 ...]` and `[b1 b2 ...]` $\sqsubseteq$ `[a1 a2 ...]`.

Soundness Theorem

well-typed under
expected resources

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Soundness Theorem

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evaluates to a value

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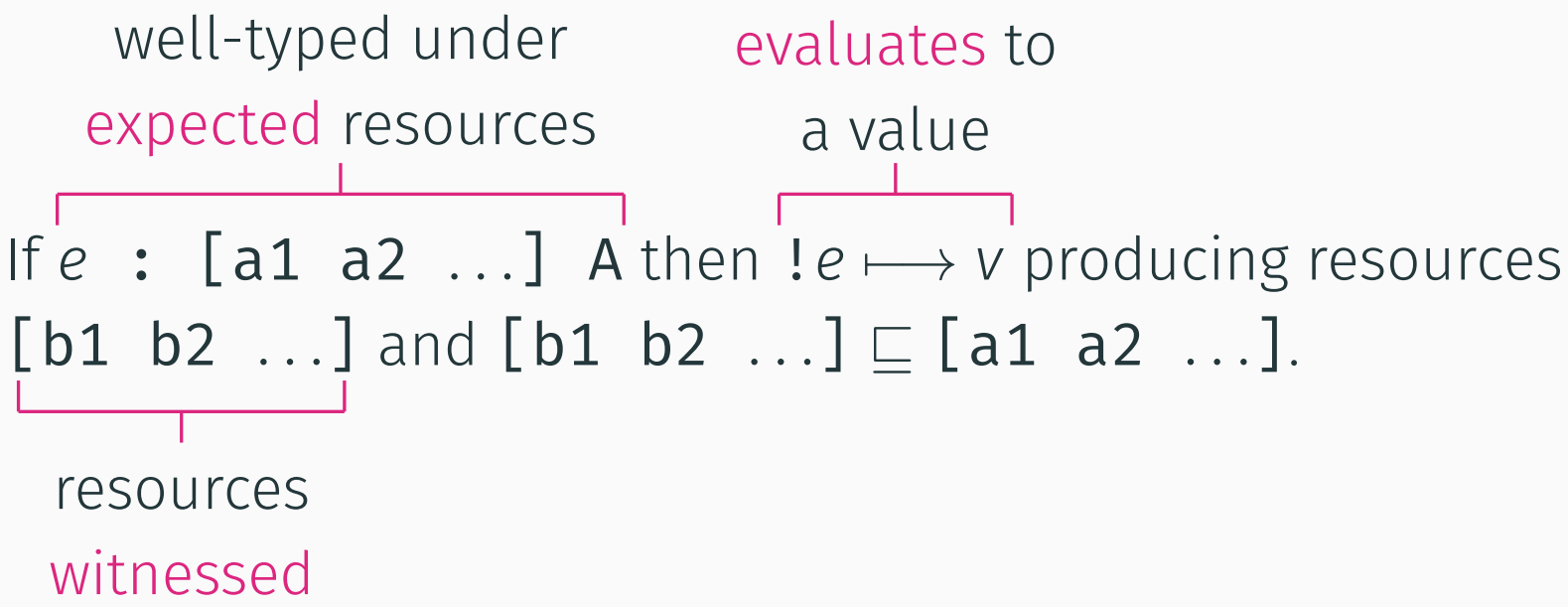
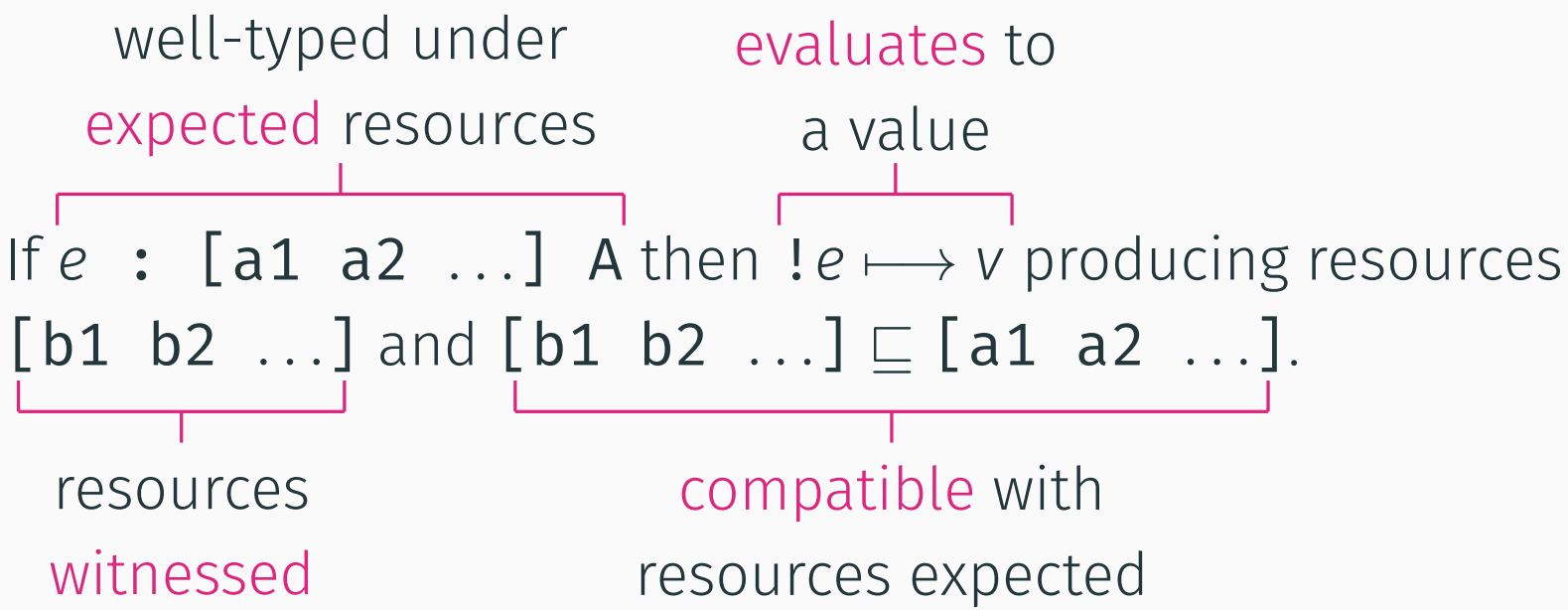
Soundness Theorem



Soundness Theorem



Soundness Theorem $\Rightarrow$ Capability Safety

A token used as an identifier for an object such that ***possession of the token confers access rights for the object.*** *A capability can be thought of as a ticket.*

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Proof Sketch

`[b1 b2 ...]` $\sqsubseteq$ `[💎 a1 a2 ...]` `[...]` $\not\sqsubseteq$ `[... 💎]`

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Proof Sketch

`[b1 b2 ...]` $\sqsubseteq$ `[💎 a1 a2 ...]` by soundness

`[...]` $\not\sqsubseteq$ `[... 💎]`

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Proof Sketch

`[b1 b2 ...]` $\sqsubseteq$ `[💎 a1 a2 ...]` — by soundness

`[...]` $\not\sqsubseteq$ `[... 💎]` — by strictness

Soundness Theorem ⇒ Quantity Safety

If e `: [a1 a2 ...] A` with n 🔐 where 🔐 **`affine`** then
`!`$e \longmapsto v$ produces `[b1 b2 ...]` with k 🔐 where $k \leq n$.

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Proof Sketch

`[b1 b2 ...]` $\sqsubseteq$ `[a1 a2 ...]` $\qquad$ [🔑 🔑] $\not\sqsubseteq$ [🔑]

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Proof Sketch

$$\texttt{[b1 b2 ...]} \sqsubseteq \texttt{[a1 a2 ...]} \qquad [🔑\ 🔑] \not\sqsubseteq [🔑]$$

by soundness

Soundness Theorem ⇒ Quantity Safety

If e `: [a1 a2 ...] A` with n 🔑 where 🔑 `affine` then
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Proof Sketch

$$\texttt{[b1 b2 ...]} \sqsubseteq \texttt{[a1 a2 ...]}$$

by soundness

$$[\text{🔑}\ \text{🔑}] \not\sqsubseteq [\text{🔑}]$$

by affinity

Soundness Theorem ⇒ Quantity Safety

If e `: [a1 a2 ...] A` with n 🔑 where 🔑 `linear` then
`!`$e \longmapsto v$ produces `[b1 b2 ...]` with k 🔑 where $k = n$.

Proof Sketch

$$\underbrace{\texttt{[b1 b2 ...]} \sqsubseteq \texttt{[a1 a2 ...]}}_{\text{by soundness}} \qquad \underbrace{[🔑\ 🔑] \not\sqsubseteq [🔑]}_{\text{by affinity}}$$

Soundness Theorem ⇒ Quantity Safety

If e `: [a1 a2 ...] A` with n 🔑 where 🔑 `linear` then
`!`$e \longmapsto v$ produces `[b1 b2 ...]` with k 🔑 where $k = n$.

Proof Sketch

`[b1 b2 ...]` $\sqsubseteq$ `[a1 a2 ...]` by soundness

`[`🔑 🔑`]` $\not\sqsubseteq$ `[`🔑`]` by affinity

`[...]` $\not\sqsubseteq$ `[...` 🔑`]` by strictness

Soundness Theorem $\Rightarrow$ Protocol Safety

If e `: [a1 a2 a3 ...] A` where `a1 a2` **`ordered`** then $!e \longmapsto v$ produces `[b1 b2 b3 ...]`.

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If e `: [a1 a2 a3 ...] A` where `a1 a2 ordered` then $!e \longmapsto v$ produces `[a1 a2 b3 ...]`.

Proof Sketch: Analogous from soundness + weakening, contraction, exchange

More in the paper!

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- ‼️ More examples!

Takeaway: Our view on resources newly unifies a set of old tools under a single language

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