

# **An Uncertain Graph-based Approach for Cyber-security Risk Assessment**

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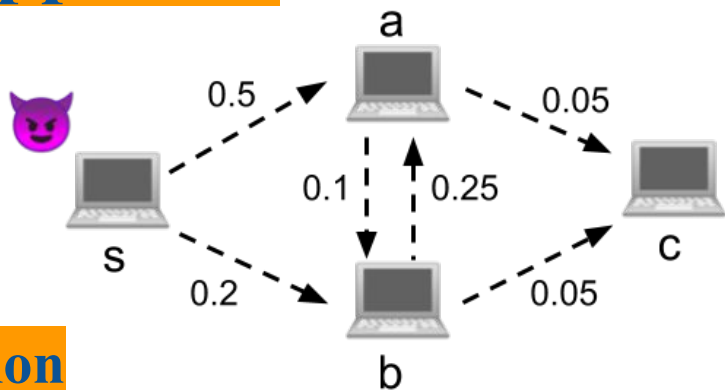
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# I. Background and Motivation

- Understanding the impacts of cyber-attacks allow business to compare the effectiveness of different defense solutions.
- Assessing the risk of a cyber-attacker who *gets access to the network, moves laterally, compromises critical assets, and causes damages* is challenging due to *uncertainty about the system's vulnerabilities and the attacker's ability to find and exploit them*.
- Quantification of losses to the network due to cyber-attacks must explicitly account for such uncertainty.

## II. Modeling Approach

Figure 1:



### (a) Attack propagation

Model the network as an *uncertain graph*  $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{p})$  [1]

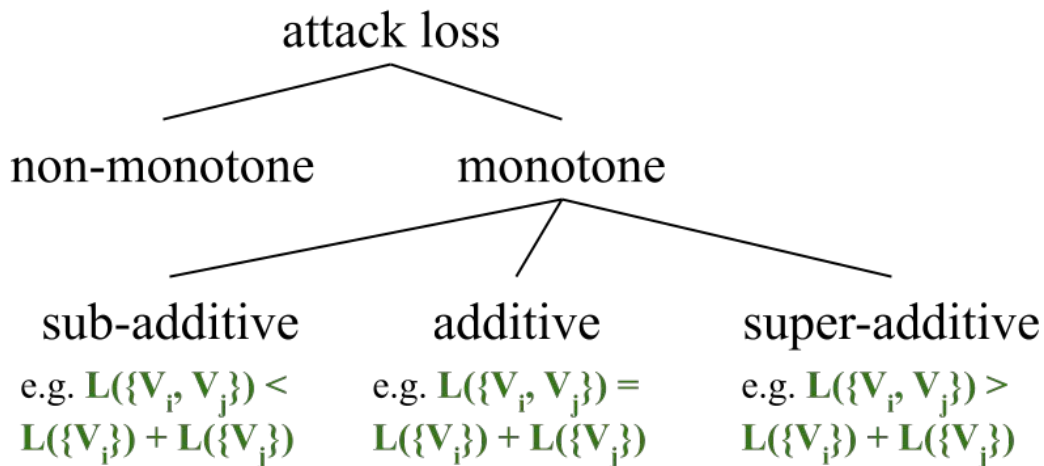
- $\mathbf{V} = \{V_1, V_2, \dots, V_n\}$ : hosts in the network
- $\mathbf{E} = \{E_1, E_2, \dots, E_m\}$ : links between hosts that allow attacks
- $\mathbf{p} = (p_1, p_2, \dots, p_m)$  where  $p_i$  is the probability that  $E_i$  exists

Let  $\mathbf{s} \in \mathbf{V}$  be the starting point of the attack (Figure 1).

[1] Nguyen, H. H., Palani, K., and Nicol, D. M. *An approach to incorporating uncertainty in network security analysis*, HoTSoS (2017).

## (b) Attack impact

- Cyber-attacks may induce losses of various kinds including *direct financial losses* (e.g. system downtime) and *indirect losses* (e.g. loss of reputation).
- Define the *attack loss function*  $\mathbf{L}: \mathbf{V} \rightarrow \mathbf{R}_{\geq 0}$  and consider  $\mathbf{L}$  as *a function of the set of hosts in  $\mathbf{V}$  that can be reached from  $\mathbf{s}$* .
- Several types of  $\mathbf{L}$ : for some  $\mathbf{V}_i, \mathbf{V}_j \in \mathbf{V}$



# III. Cyber-security Risk Assessment

## (a) Risk triplet [2]

- **Realization**: let  $\mathcal{X} = \{0, 1\}^m$  and  $\mathbf{X} = (X_1, X_2, \dots, X_m)$  the multivariate random variable where  $X_i \sim \text{Bernoulli}(p_i)$  for  $i = 1, 2, \dots, m$  and  $X_i = 1$  implies  $E_i$  exists. An element  $\mathbf{x} = (x_1, x_2, \dots, x_m)$  in  $\mathcal{X}$  is a realization of  $\mathbf{X}$ .
- **Probability**: given  $\mathbf{x}$ , assume  $X_i$ 's are *mutually independent*  
$$\Pr(\mathbf{X} = \mathbf{x}) = \prod_i (x_i p_i + (1-x_i) (1-p_i))$$
(otherwise see technique in [1] for modeling edge correlations)
- **Impact**: given  $\mathbf{x}$ , define the directed graph  $\mathbf{G}(\mathbf{x}) = (\mathbf{V}, \mathbf{E}(\mathbf{x}))$  where  $\mathbf{E}(\mathbf{x}) = \{E_i \in \mathbf{E}: x_i = 1\}$ . Let  $\mathbf{V}_s(\mathbf{x}) \subseteq \mathbf{V}$  containing all nodes in  $\mathbf{G}(\mathbf{x})$  that can be reached from  $s$ . The impact is simply defined as  $\mathbf{L}(\mathbf{x}) \equiv \mathbf{L}(\mathbf{V}_s(\mathbf{x}))$ .

## (b) Risk measure as security metric

- Expected loss (EL):  $E(L(X)) = \sum_{x \in X} L(x) \Pr(X = x)$

Example: assume additive loss with  $L(\{s\}) = 0$ ,  $L(\{a\}) = 1$ ,  $L(\{b\}) = 2$ , and  $L(\{c\}) = 3$ . The model in Figure 1 generates  $2^6 = 64$  realizations with  $L(X) \in \{0, 1, \dots, 6\}$  (Figure 2) and  $E(L(X)) = 1.119$ .

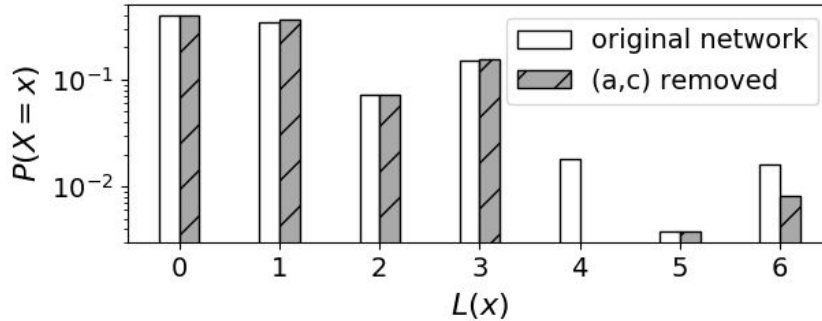
- Loss tail probability (LTP): let  $T$  be a selected threshold and  $\mathcal{T} = \{x \in X : L(x) > T\}$ . The LTP is defined as

$$\Pr(L(X) > T) = \sum_{x \in X} 1_{\{x \in \mathcal{T}\}} \Pr(X = x)$$

Example (cont.): using  $T = 3$ , we have  $\Pr(L(X) > 3) = 0.038$ .

Suppose the network access control is hardened and link  $(a,c)$  is removed as a result. The new EL remains relatively the same at **1.041** but the new LTP drops by 3x times to **0.012**.

Figure 2:



## IV. Future Work

- Study different forms of the attack loss function.
- Study *computational techniques* for estimating the expected loss and loss tail probability (both are #P-complete [3]).
- Extend the model to capture *attacker's behaviors* and *interactions between the attacker and the defender*.
- Use the model to compute *premium for cyber-insurance*.

# Thank you!

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