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# Disappearing Formal Methods

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## Overview

- Assurance for Safety, Security, and other critical properties
  - Process- vs. product-based assurance
- Formal methods
- Problems with current methods
- Two big ideas
- From refutation to verification
- Disappearing formal methods

## Evidence For Safety, Security, And Other Critical Properties

- How is it done for traditional systems?
  - E.g., an airplane wing
- How is it done for software?
  - Or **software-intensive** systems
  - E.g., a flight-control system

## Safety Cases for Traditional Systems

- Mostly done by mathematical modeling and analysis
  - Build mathematical models of the design, its environment, and requirements
  - Use calculation to establish that the design in the context of the environment satisfies the requirements
  - Only useful when mechanized

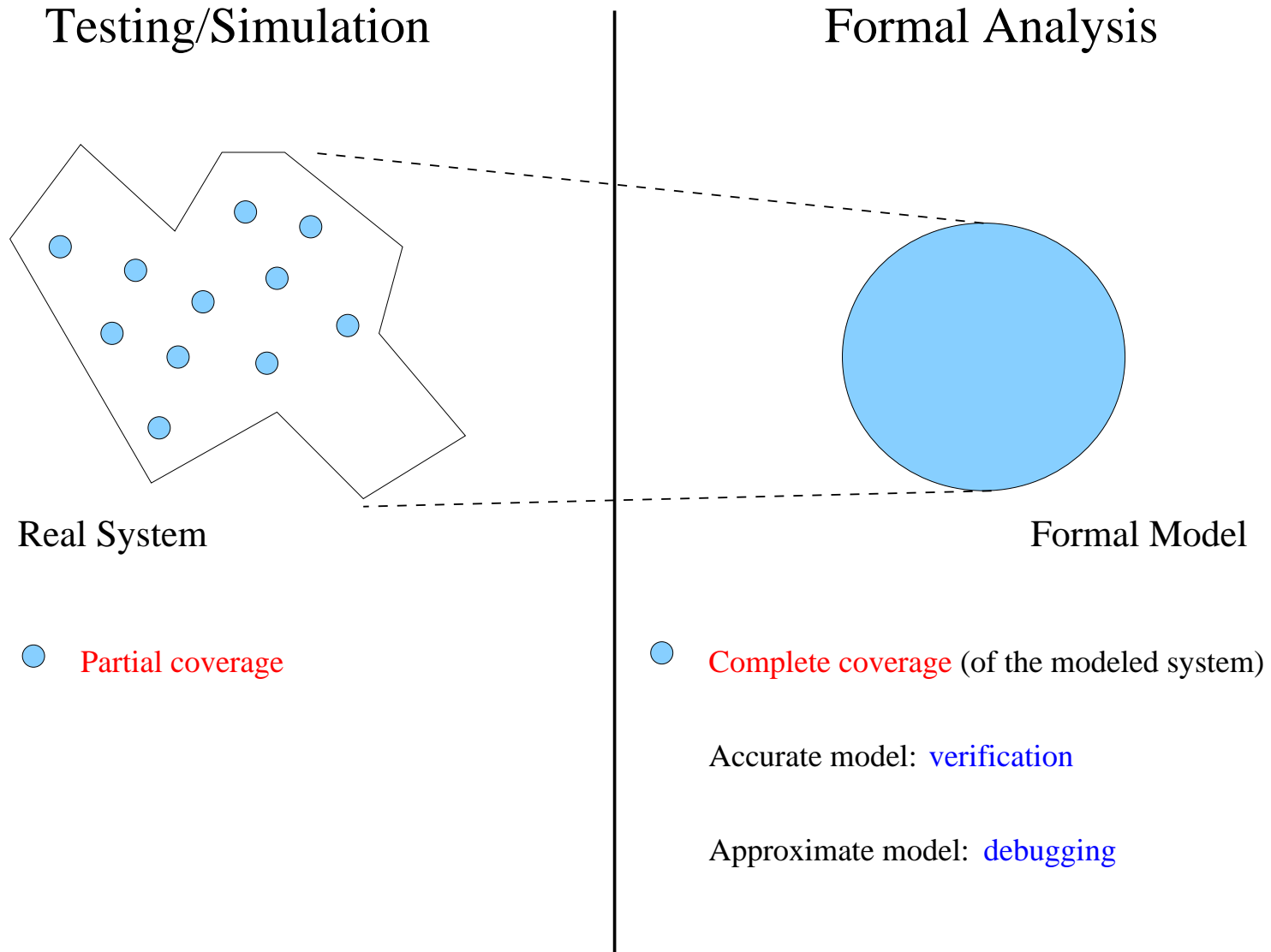
E.g., finite elements analysis

- The modeling is validated by tests
  - Limited testing is sound because we are dealing with continuous systems
- This is product-based certification
  - It concerns properties of (mathematical models of) the product

## Safety Cases for Software Systems

- Mostly done by controlling, monitoring, and documenting the process used to create the software
  - Different industries have different recommended processes (e.g., DO-178B for avionics)
- This is **process-based** certification
  - Provides no direct evidence about the product
    - “We cannot show how well we’ve done, so we’ll show how hard we tried”
- NB. Testing is product-based, but cannot provide evidence beyond  $10^{-4}$  **because we are dealing with discrete systems**
  - Complete testing is infeasible: 114,000 years test for  $10^{-9}$
  - And extrapolation from incomplete tests is unjustified

# Formal Methods In Pictures



## Product-Based Certification For Software

- Build mathematical models of a design, its environment, and requirements
  - The applied math of Computer Science is formal logic
  - So models are formal descriptions in some logical system:
- Use calculation to establish that the design in the context of the environment satisfies the requirements
  - Calculation in formal logic is done by theorem proving or model checking
    - assumptions + design + environment ⊢ requirements*
  - Formal calculations can cover all modeled behaviors, even if numerous or infinite (the power of symbolic reasoning)
- Only useful when mechanized
  - So need automated theorem proving or model checking



## Formal Methods for Product-Based Assurance and Certification

- Want highly accurate formal models, so that calculations support strong claims—i.e., verification
- Then, using formal calculations, some activities that are traditionally performed by reviews
  - Processes that depend on human judgment and consensus can be replaced or supplemented by analyses
  - Processes that can be repeated and checked by others, and potentially so by machine

### Language from DO-178B/ED-12B

- That is, formal methods help us move from process-based to product-based assurance

## However...

- Formal calculations
  - Are undecidable in general
  - And even decidable problems have much greater computational complexity than mechanizations of continuous mathematics
- So full automation is impossible in general
- Must rely on heuristics (guesses) which will sometimes fail
  - Heuristic theorem proving
- Or rely on human guidance
  - Interactive theorem proving
- Or trade off accuracy or completeness of the model for tractability and automation of calculation
  - Model checking

## The Difficulty With Theorem Proving Is...

- Theorem proving can handle accurate models, but requires interactive human guidance
  - Focuses on proof, and idiosyncrasies of the prover, not on the design
  - Difficult to interpret failure (bug, or bad proof?)

“Interactive theorem proving is a waste of human talent”

- Also, must strengthen invariants to make them inductive
- And it's all or nothing
- Payoff is definitive assurance... with caveats
  - May also find subtle bugs

## Inductive Invariants

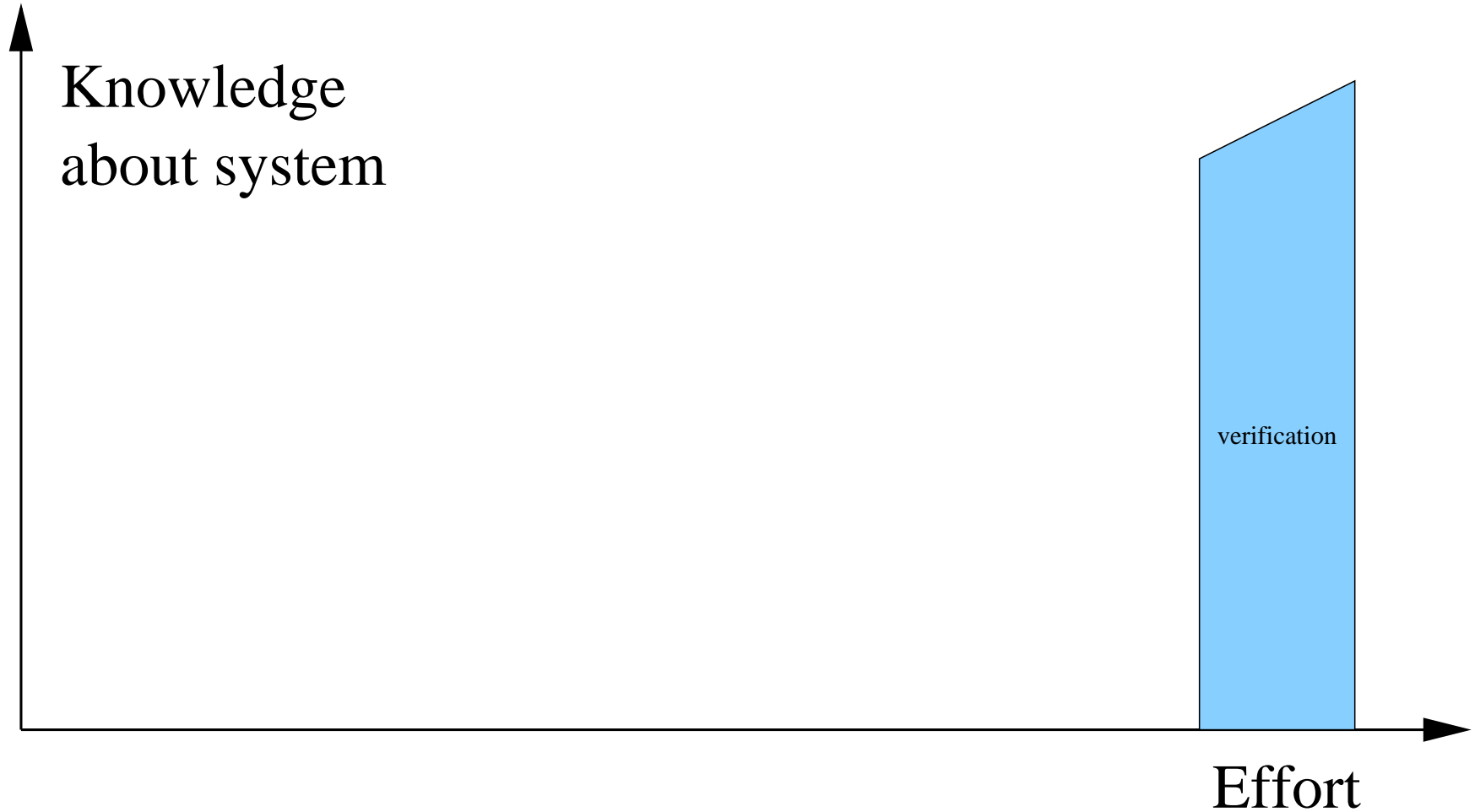


- To establish an invariant or safety property (one true of all reachable states) by theorem proving, we invent another property that implies the one of interest and that is **inductive**
  - Includes all the initial states
  - Is closed on the transitions

The reachable states are the smallest set that is inductive

- Trouble is, **naturally stated invariants are seldom inductive**
  - The second condition is violated
- Postulate a new invariant that excludes the states (so far discovered) that take you outside the desired invariant
- Iterate until success or exasperation
- Bounded retransmission protocol required **57** such iterations

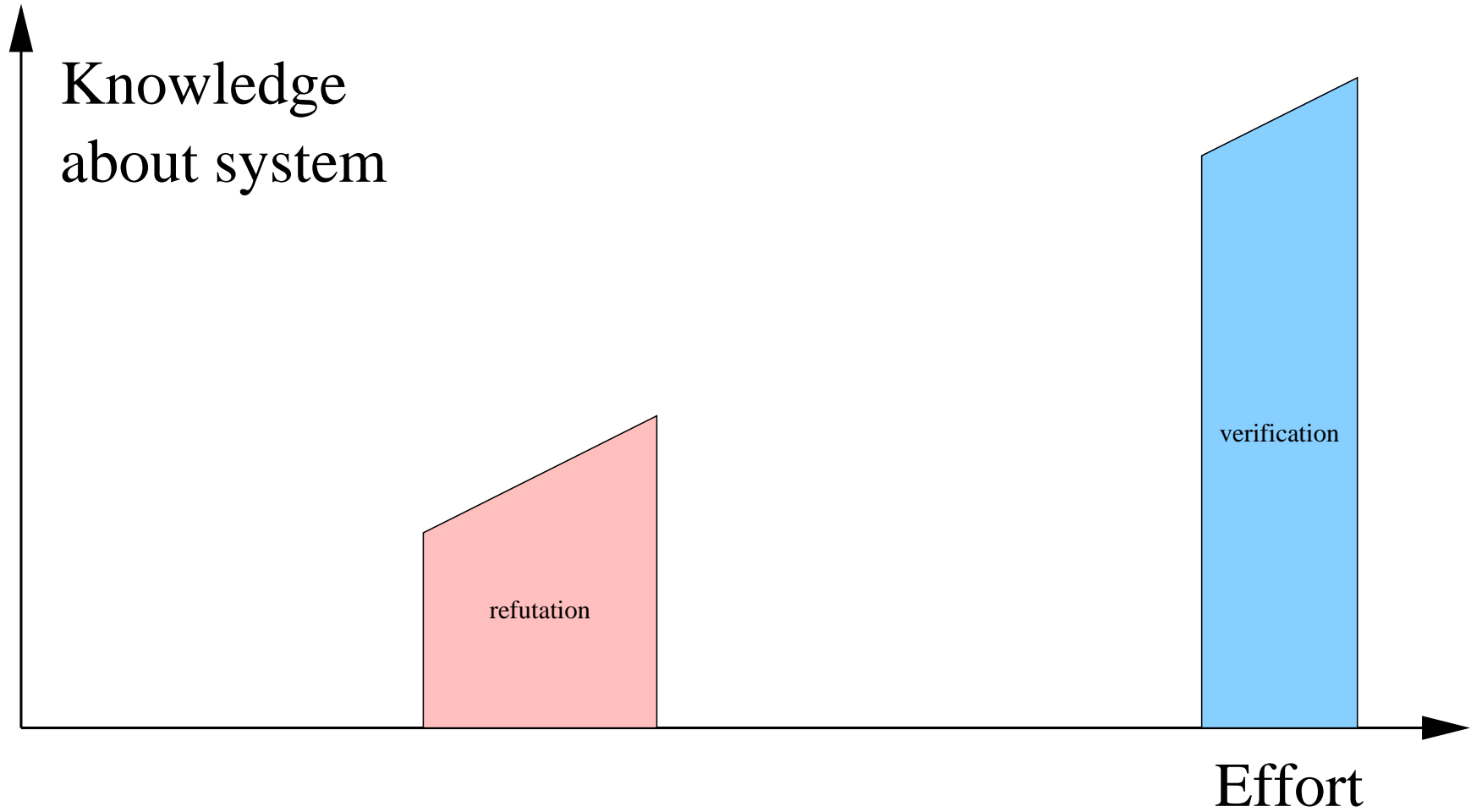
# The Wall of Formal Verification



## The Difficulty with Model Checking Is...

- The models (and properties) have to be simplified to make them tractable to fully automated analysis
- But simplified models may not be fully accurate with respect to the property of interest
  - And that's why they cannot be used for verification
- However, this approach works for refutation (finding bugs)
  - Experience indicates we learn more (find more bugs) by exploring **all** behaviors of a simplified model than by probing just some of the behaviors of the real thing (as with testing or simulation)
- But when to stop?
  - Lack of refutation is not the same as verification

# Refutation and Verification



## Formal Methods in Current Practice

- *Model checking saved the reputation of formal methods*  
(Daniel Jackson)
- Formal methods have achieved a modest degree of acceptance in some areas
  - E.g., hardware, protocols
- But mainly for purposes of **refutation**
  - That is, looking for errors
  - E.g., debugging, testing
- **Verification** is much less practiced
  - That is, showing the absence of errors



## Summarizing

- Refut'n can be cost-effective, but doesn't get you to verif'n
  - Interaction concerns the model, the technology is automated, it resembles familiar activities
  - It is acceptable to practitioners

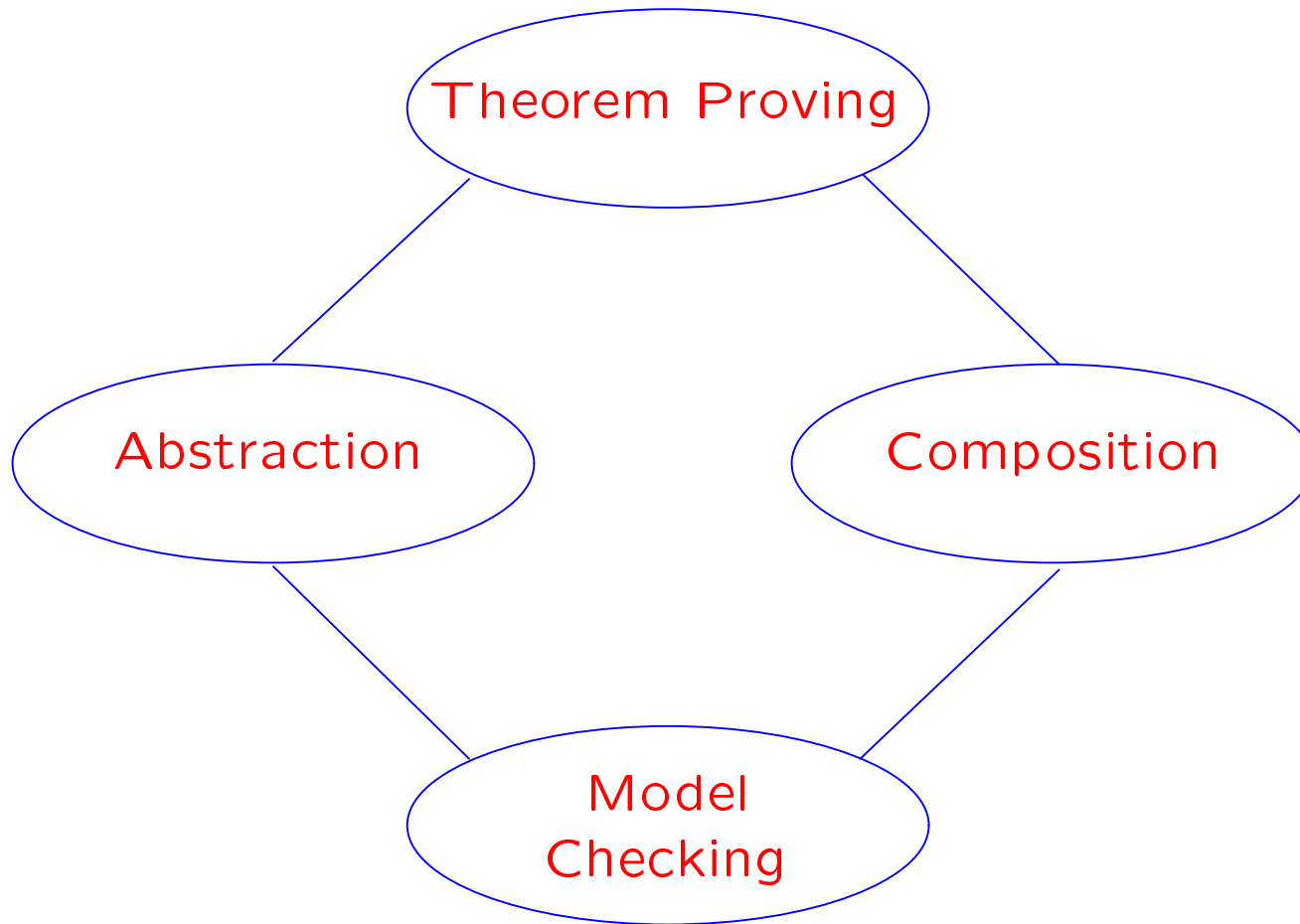
Challenge: why cannot the technology of refutation (particularly model checking) be used for verification?

- Verif'n has high potential payoff, but few interm'd benefits
  - Interaction concerns the proof and the prover, technology is not automated, intimidating
  - It is not acceptable to practitioners

Challenge: why cannot theorem proving be made automatic?

- Overall challenges: why cannot model checking and theorem proving work together? And why cannot we move smoothly from refutation to verification?

**Abstraction is a Bridge  
Between Deductive and Algorithmic Methods  
And Between Refutation and Verification**



## Using Model Checking For Verification

- Model checking requires simple models (e.g., finite state)
- But can be used to verify properties of a complex model if it has a simple **property-preserving abstraction**
- Trouble is, it usually requires theorem proving to justify the abstraction
  - **45** of the 57 invariants required for BRP
- **First Big Idea**: use theorem proving to **calculate** the abstraction

## Making Theorem Proving More Automatic

- The general theorem proving problem is undecidable
  - So full automation requires heuristics
  - Which will sometimes fail
- Classical verification poses correctness as a single “big theorem”
  - So failure to prove it (when true) is catastrophic
- **Second Big Idea:** “failure-tolerant” theorem proving
  - Prove lots of small theorems instead of one big one
  - In a context where some failures can be tolerated
- **Aha!** Automated abstraction provides this context

## Abstraction

- Given a transition system  $G$  on  $S$  and property  $P$ , a property-preserving abstraction yields a transition system  $\hat{G}$  on  $\hat{S}$  and property  $\hat{P}$  such that

$$\hat{G} \models \hat{P} \Rightarrow G \models P$$

- Strongly property preserving abstraction:

$$\hat{G} \models \hat{P} \Leftrightarrow G \models P$$

- A good abstraction typically (for universal properties) introduces nondeterminism while preserving the property
- Remaining problem: Construction of reasonably precise  $\hat{G}$  and  $\hat{P}$  given  $G$  and  $P$

## Data Abstraction [Cousot & Cousot]

- Replace concrete variable  $x$  over datatype  $C$  by an abstract variable  $x'$  over datatype  $A$  through a mapping  $h : [C \rightarrow A]$ .
- Examples: Parity,  $\text{mod } N$ , zero-nonzero, intervals, cardinalities,  $\{0, 1, \text{many}\}$ ,  $\{\text{empty}, \text{nonempty}\}$
- Given  $f : [C \rightarrow C]$ , construct  $\hat{f} : [A \rightarrow \text{set}[A]]$ :  
(observe how data abstraction introduces nondeterminism)

$$b \in \hat{f}(a) \Leftrightarrow \exists x : a = h(x) \wedge b = h(f(x))$$

$$b \notin \hat{f}(a) \Leftrightarrow \vdash \forall x : a = h(x) \Rightarrow b \neq h(f(x))$$

- **Theorem-proving failure affects accuracy, not soundness**
- Mechanized in Bandera (Corbett, Dwyer and Hatcliff, KSU)

## Predicate Abstraction [Graf-Saïdi]

- Abstracts out **relations** between variables, e.g.,  $x < y$ ,  
 $x + y = z$
- Variables ranging over infinite datatypes can be replaced by Boolean variables representing the predicates on those variables
- Predicates can be extracted from guards, assignments, and the property of interest
- Guessing predicates is easier than invariant strengthening (and is also more general [Rusu & Singerman, TACAS 99])
- Mechanized in PVS (SRI)

## Construction of Predicate Abstractions

- Given  $\phi : [S \rightarrow \hat{S}]$  induced by the abstracted predicates, construct  $\hat{G}$  by

$$\hat{G}(\hat{s}_1, \hat{s}_2) \Leftrightarrow \exists s_1, s_2 : \hat{s}_1 = \phi(s_1) \wedge \hat{s}_2 = \phi(s_2) \wedge G(s_1, s_2)$$

$$\neg \hat{G}(\hat{s}_1, \hat{s}_2) \Leftrightarrow \vdash \forall s_1, s_2 : \hat{s}_1 \neq \phi(s_1) \vee \hat{s}_2 \neq \phi(s_2) \vee \neg G(s_1, s_2)$$

- **Theorem-proving failure affects accuracy, not soundness**
- There is another method (exponentially more efficient) [Saïdi & Shankar, CAV 99]
- More powerful than data abstraction, but construction is more complex



## Automated Abstraction

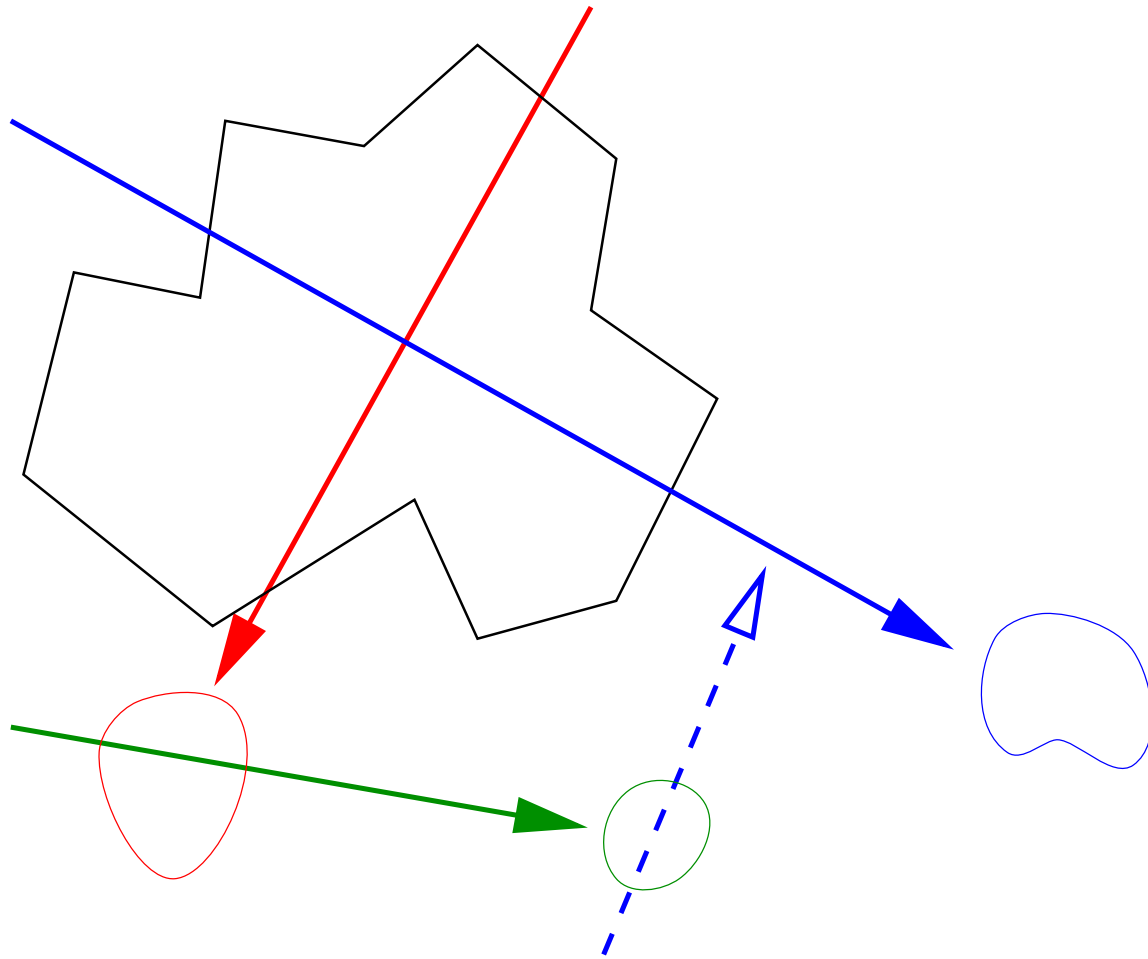
- Can often construct a simplified model that is faithful to the original (for a given property of interest)
  - The reduced model can be analyzed by model checking
  - And failure to detect bugs **does** certify their absence
- These reduced models can be constructed automatically by mechanized **data** or **predicate abstraction**
  - The construction is done by trying to prove lots of little theorems
    - ★ If a proof fails, the abstracted model will be more conservative, but often still good enough
- **But still the construction often requires auxiliary invariants**

## The Bridge Goes In Both Directions



- Model checkers often calculate the reachable stateset
  - Which is the **strongest invariant**And then throw it away
- The concretization of the reachable states of an abstraction is an invariant of the concrete system
  - And often a strong one
- **So modify a model checker to return the reachable states as a formula that a theorem prover can manipulate**
- Has been done (by Sergey Berezin) for CMU SMV and is used in InVeSt [Bensalem, Lakhnech & Owre, CAV 99]

# Integrated, Iterated Analysis



## Even More Integrated, Iterated Analysis!



- (Approximations to) fixpoints of weakest preconditions or strongest postconditions also generate invariants and can strengthen those extracted from an abstraction
  - Mechanized by theorem proving
  - (Strongest postconditions are equivalent to symbolic simulation, which is independently useful)
- Counterexamples from failed model check help distinguish bugs from weak abstractions, and also help refine the abstraction
  - Suggest additional properties (invariants) that will help the theorem prover construct a tighter model
  - Suggest additional predicates on which to abstract

## Truly Integrated, Iterated Analysis!

- Recast the goal as one of calculating and accumulating properties about a design (symbolic analysis)
- Rather than just verifying or refuting a specific property
- Properties convey information and insight, and provide leverage to construct new abstractions
  - And hence more properties
- Requires restructuring of verification tools
  - So that many work together
  - And so that they return symbolic values and properties rather than just yes/no results of verifications
- This is what **SAL** is about: **Symbolic Analysis Laboratory**

## From Refutation to Verification

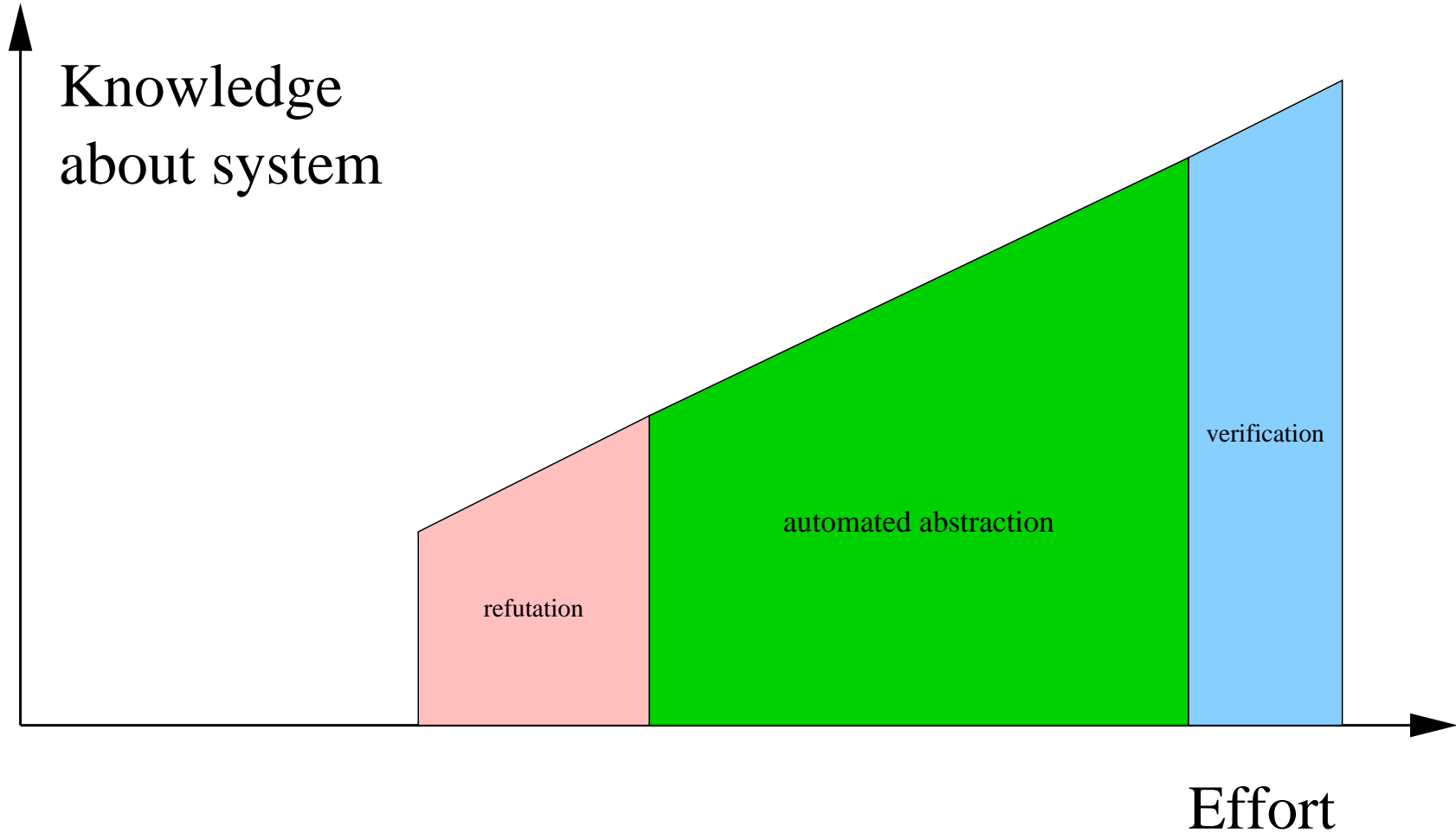
- By allowing unsound abstractions

$$\hat{G} \models \hat{P} \not\Rightarrow G \models P$$

We can do refutation as well as verification

- By selecting abstractions (sound/unsound) and properties (little/big) we can fill in the space between refutation and verification
- Refutation lowers the barrier to entry
- Provides economic incentive: discovery of high value bugs
  - Can estimate the cost of each bug found
  - And can directly compare with other technologies
- Yet allows smooth transition to verification

# From Refutation To Verification



## Filling the Remaining Gap

- Model checking for refutation and (via automated abstraction) for verification imposes a much smaller barrier to adoption than old-style formal verification
- But the barrier is still there
- What about really low cost/low threat kinds of formal analysis?
- **Make the formal methods disappear inside traditional tools and methods**
  - **Invisible** formal methods, or
  - **Ubiquitous** formal methods



## Examples of Disappearing Formal Methods

- Extended static checking (ESC) for Java (Compaq SRC)
- PVS-like type system (predicate subtypes) for any language
  - Traditional type systems have to be trivially decidable
  - But can gain enormous error detection by adding a component that requires theorem proving (lots of small theorems, failure generates a warning)
- Completeness/Consistency checkers for tabular specifications (cf. [Ontario Hydro](#), RSML, SCR)
- Statechart/Stateflow property checkers (cf. OFFIS)
  - Show me a path that activates this state
  - Can this state and that be active simultaneously?
- Test case generators (cf. Verimag/IRISA TGV)

## Tools To Realize These

- Abstraction and model checking
- Automated theorem proving built on powerful decision procedures
  - Combination of: propositional satisfiability, equality over uninterpreted function symbols with (linear) arithmetic, arrays, datatypes
  - Quantifier elimination for decidable fragment of the above

We are making these available as **ICS**

- Also decision procedures for more powerful theories (e.g., Mona for WS1S, available in PVS)
- These can be extended to model checking
  - E.g., Lossy-Channel Systems (LCS)
  - Just as ordinary model checking builds on BDDs and SAT

## What We Are building



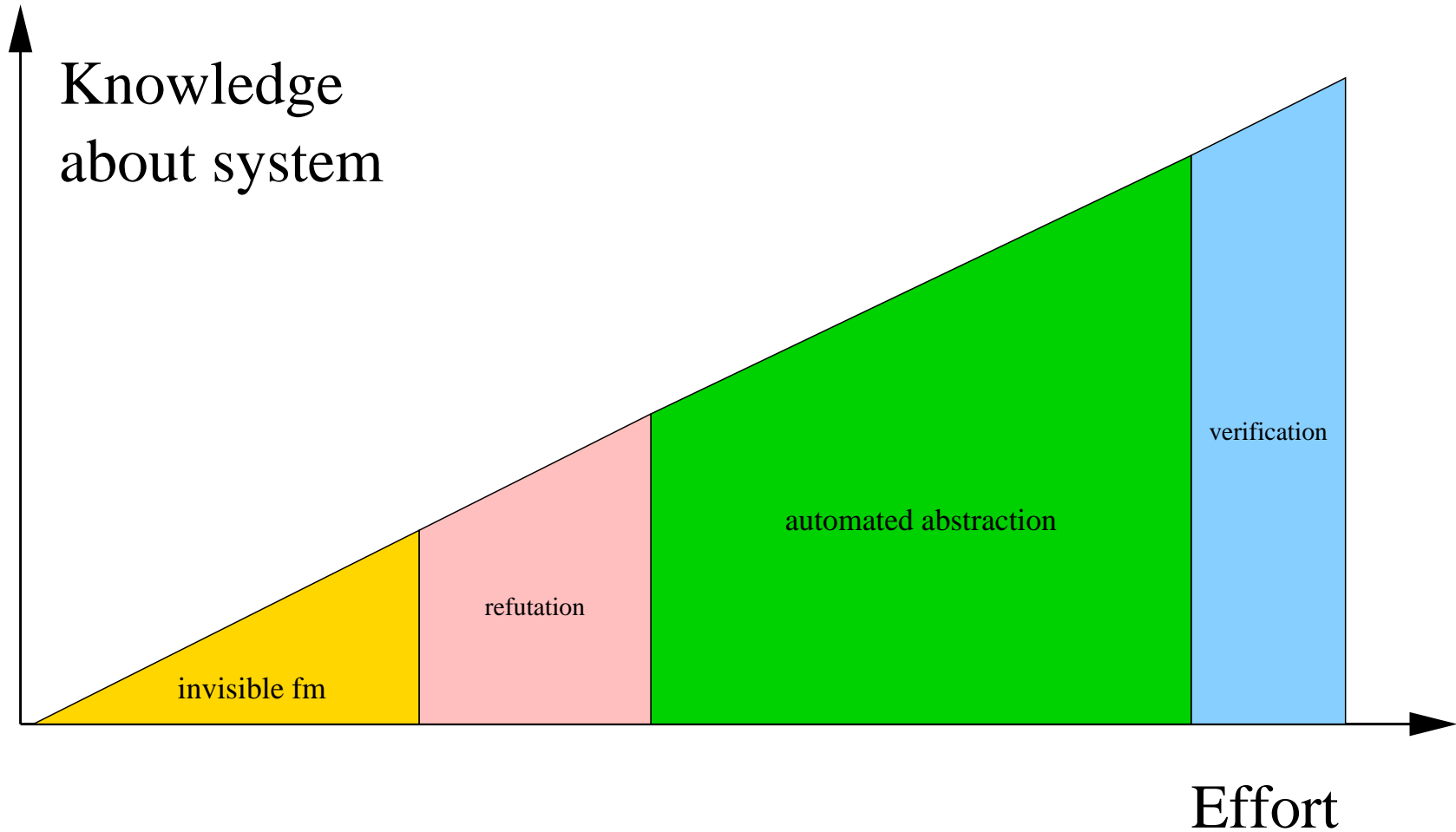
SAL

PVS

ICS

ICS = Integrated Canonizer-Solver (= ICanSolve)

# Disappearing Formal Methods



## Acknowledgments

- N. Shankar, Sam Owre, Harald Rueß, Hassen Saïdi
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## To Learn More

- Check out papers and technical reports at <http://www.csl.sri.com/programs/formalmethods>
- Information about our verification system, PVS, and the system itself are available from <http://pvs.csl.sri.com>
  - Freely available under license to SRI
  - Built in Allegro Lisp for Solaris, or Linux
  - Version 2.3 includes predicate abstraction
- We plan to release SAL and ICS in July 2001