Tutorial: How to Cook a Static Analyzer

or, The Surprising Effectiveness of Substructural Proof Theory

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People

This is about work by the

SpaceInvader Team (Ldn): Cristiano Calcagno, Dino Distefano, Peter O'Hearn, Hongseok Yang

benefitting from lots of collaboration with our
SLAyer Colleagues (MSR): Josh Berdine, Byron Cook

A Substructural Logic

$$A \;\not\vdash\; A * A$$

$$10 \mapsto 3 \;\not\vdash\; 10 \mapsto 3 * 10 \mapsto 3$$

$$A * B \;\not\vdash\; A$$

$$10 \mapsto 3 * 42 \mapsto 5 \;\not\vdash\; 10 \mapsto 3$$

Program Verification Extremes

- **Extreme 1: Interactive Proof.** HOL, Coq, Isabelle... More human effort, more expressive.
- **Extreme 2: Static Analysis.** Less expressive, more automatic.
- And there is population in between (SPARK-Ada, Spec-#...).

Program Verification Extremes

- **Extreme 1: Interactive Proof.** HOL, Coq, Isabelle... More human effort, more expressive.
  - *Several embeddings of separation logic in proof assistants.*
- **Extreme 2: Static Analysis.** Less expressive, more automatic.
  - *Several prototype program analysis tools: SpaceInvader, SLAyer, THOR, Xisa...*
- And there is population in between (SPARK-Ada, Spec-#...).
  - *Smallfoot, SmallfootRG, jStar*

Verification by Static Analysis, Current Context

- 2000's: impressive practical advances in automatic program verification E.g.
  - SLAM: Protocol properties of procedure calls in device drivers, e.g. any call to `ReleaseSpinLock` is preceded by a call to `AquireSpinLock`
  - ASTRÉE: no run-time errors in Airbus code

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- The Missing Link
  - ASTRÉE assumes: no dynamic pointer allocation
  - SLAM assumes: memory safety
  - Wither automatic heap verification? (for substantial programs)

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  - ASTRÉE: no run-time errors in Airbus code
- The Missing Link
  - ASTRÉE assumes: no dynamic pointer allocation
  - SLAM assumes: memory safety
  - Wither automatic heap verification? (for substantial programs)
- Many important programs make serious use of heap: Linux, Apache, TCP/IP, IOS... but heap verification is hard.

Part I

Symbolic Execution and Verification

Symbolic Heaps (say less, to do more..)

A special form[1]

$$(B_1 \wedge \cdots \wedge B_n) \wedge (H_1 * \cdots * H_m)$$

where

$$\begin{array}{lcl} H & ::= & E \mapsto \rho \mid \mathsf{tree}(E) \mid \mathsf{lseg}(E, E) \\ B & ::= & E{=}E \mid E{\neq}E \end{array}$$

$$E ::= x \mid \mathsf{nil}$$

$$\rho ::= f_1 : E_1, \ldots, f_n : E_n$$

$$B ::= E{=}E \mid E{\neq}E$$

[1] assertional if-then-else as well

Verification = Symbolic Execution + Entailment Checking

- Inductive Definitions unrolled only on demand (on heap access) during execution.
- Rolled up only after execution, during entailment checking

- The tree definition

$$\mathsf{tree}(E) \iff \textit{if } E{=}\mathsf{nil} \textit{ then } \mathsf{emp}$$
$$\textit{else } \exists x, y.\, (E{\mapsto}l : x, r : y) * \mathsf{tree}(x) * \mathsf{tree}(y)$$

Copytree Verification

Just inside the if (where $p \neq \texttt{nil}$)...

$\{p \neq \texttt{nil} \wedge \texttt{tree}(p)\}$

$$\mathsf{tree}(E) \iff \textit{if } E = \mathsf{nil} \textit{ then } \mathsf{emp}$$
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Copytree Verification

Just inside the if (where $p \neq \texttt{nil}$)...

$\{p \neq \texttt{nil} \land \texttt{tree}(p)\}$ unroll it...

$\{p \mapsto [l : x, r : y] * \texttt{tree}(x) * \texttt{tree}(y)\}$

$i := p \to l\ ;\ j := p \to r;$

$\{p \mapsto [l : i, r : j] * \texttt{tree}(i) * \texttt{tree}(j)\}$

$\mathsf{tree}(E) \iff$ *if* $E = \mathsf{nil}$ *then* emp

else $\exists x, y.\ (E \mapsto l : x, r : y) * \mathsf{tree}(x) * \mathsf{tree}(y)$

Copytree Verification

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$\{p \mapsto [l : x, r : y] * \texttt{tree}(x) * \texttt{tree}(y)\}$

$i := p \to l \; ; \; j := p \to r;$

$\{p \mapsto [l : i, r : j] * \texttt{tree}(i) * \texttt{tree}(j)\}$

$\texttt{tree\_copy}(ii \; ; \; i) \; ; \; \texttt{tree\_copy}(jj \; ; \; j)$

$s := \texttt{new}() \; ; \; s \to l := ii \; ; \; s \to r := jj;$

$\text{tree}(E) \iff$ *if* $E = \text{nil}$ *then* emp

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$\texttt{tree\_copy}(ii\ ;\ i)\ ;\ \texttt{tree\_copy}(jj\ ;\ j)$

$s := \texttt{new}()\ ;\ s \rightarrow l := ii\ ;\ s \rightarrow r := jj;$

$\{p \mapsto [l : i, r : j] * \texttt{tree}(i) * \texttt{tree}(j) * s \mapsto [l : ii, r : jj] * \texttt{tree}(ii) * \texttt{tree}(jj)\}$

$$\text{tree}(E) \iff \textit{if } E = \text{nil } \textit{then } \text{emp}$$
$$\textit{else } \exists x, y.\ (E \mapsto l : x, r : y) * \text{tree}(x) * \text{tree}(y)$$

Copytree Verification

We are left with an entailment

$$p \mapsto [l : i, r : j] * \texttt{tree}(i) * \texttt{tree}(j) * s \mapsto [l : ii, r : jj] * \texttt{tree}(ii) * \texttt{tree}(jj)$$

$$\vdash \qquad \texttt{tree}(p) * \texttt{tree}(s)$$

$$\text{tree}(E) \iff \textit{if } E = \text{nil } \textit{then } \text{emp}$$
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Copytree Verification

We are left with an entailment

$$p \mapsto [l : i, r : j] * \texttt{tree}(i) * \texttt{tree}(j) * s \mapsto [l : ii, r : jj] * \texttt{tree}(ii) * \texttt{tree}(jj)$$

$$\vdash \qquad \texttt{tree}(p) * \texttt{tree}(s)$$

let me roll it...

$$\text{tree}(E) \iff \textit{if } E = \text{nil } \textit{then } \text{emp}$$
$$\textit{else } \exists x, y.\, (E \mapsto l : x, r : y) * \text{tree}(x) * \text{tree}(y)$$

Flawed Copytree Failed Verification

When we mistakenly point back into the source tree

we are left with an entailment

$$p \mapsto [l : i, r : j] * \mathtt{tree}(i) * \mathtt{tree}(j) * s \mapsto [l : i, r : j] * \mathtt{tree}(ii) * \mathtt{tree}(jj)$$

$$\vdash \qquad \mathtt{tree}(p) * \mathtt{tree}(s)$$

that we can't roll up...

Part II

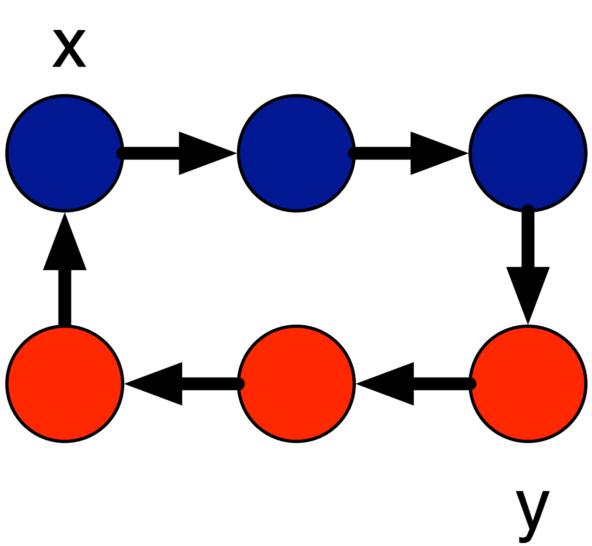
Proving Entailments

Induction and Linked Lists

List segments   ($\mathsf{list}(E)$ is shorthand for $\mathsf{lseg}(E, \mathsf{nil})$)

$$\mathsf{lseg}(E, F) \iff \texttt{if } E = F \texttt{ then } \mathsf{emp}$$
$$\texttt{else } \exists y. E \mapsto tl : y * \mathsf{lseg}(y, F)$$

$$\mathsf{lseg}(x, y) * \mathsf{lseg}(y, x)$$



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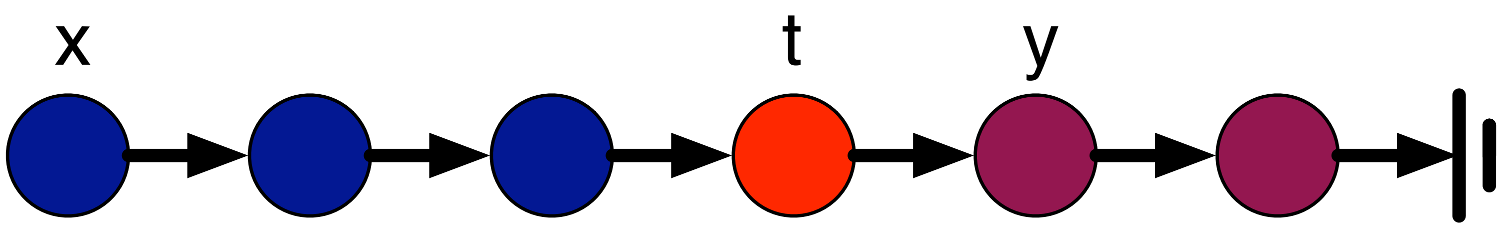
$$\mathsf{lseg}(x, t) * t \mapsto [tl : y] * \mathsf{list}(y)$$

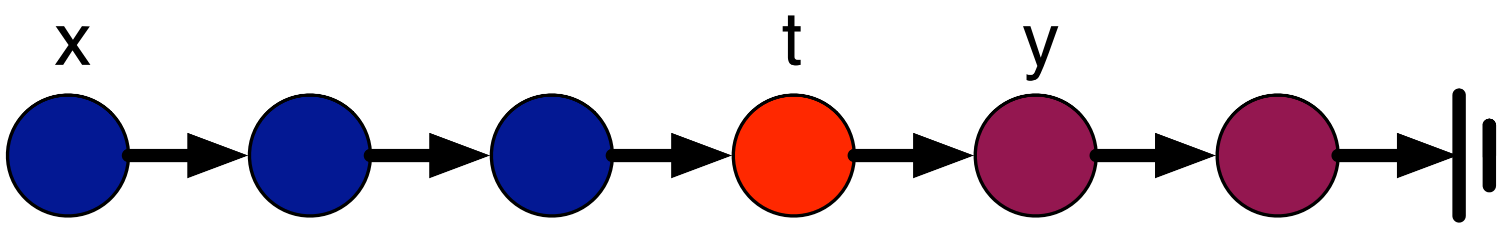


Induction and Linked Lists

List segments (list(E) is shorthand for lseg(E, nil))

$$\text{lseg}(E,F) \iff \texttt{if } E = F \texttt{ then emp}$$
$$\texttt{else } \exists y. E \mapsto tl : y * \text{lseg}(y, F)$$

Entailment $\text{lseg}(x,t) * t \mapsto [tl : y] * \text{list}(y) \vdash \text{list}(x)$

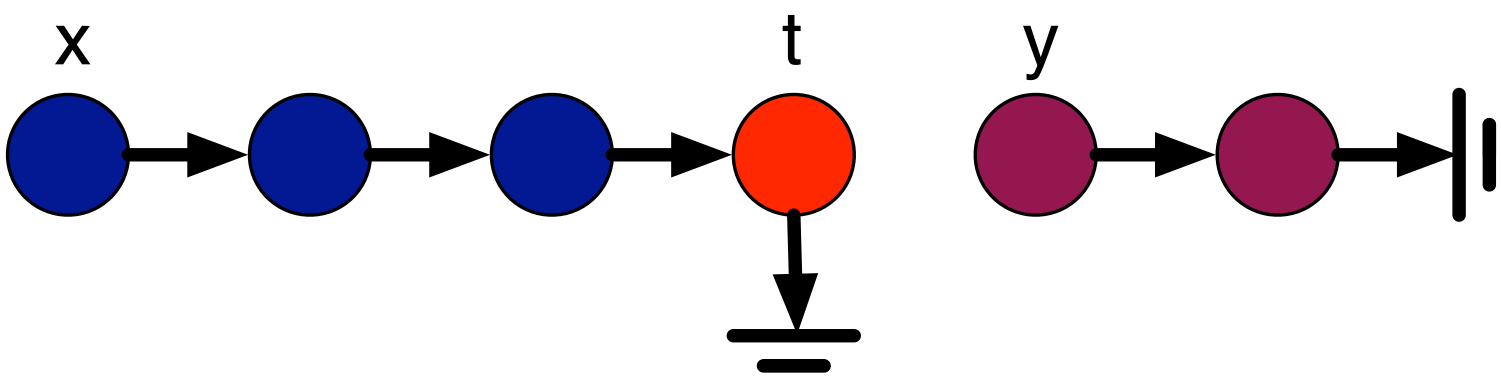


Induction and Linked Lists

List segments ($\mathsf{list}(E)$ is shorthand for $\mathsf{lseg}(E, \mathsf{nil})$)

$$\mathsf{lseg}(E, F) \iff \texttt{if } E = F \texttt{ then } \mathsf{emp}$$
$$\texttt{else } \exists y. E \mapsto tl : y * \mathsf{lseg}(y, F)$$

Non-Entailment $\mathsf{lseg}(x, t) * t \mapsto \texttt{nil} * \mathsf{list}(y) \not\vdash \mathsf{list}(x)$



Solution (Berdine and Calcagno)

- A proof theory oriented around Abstraction and Subtraction.

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$$\mathsf{lseg}(x, t) * \mathsf{list}(t) \vdash \mathsf{list}(x)$$

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$$\mathsf{lseg}(x, t) * \mathsf{list}(t) \ \vdash \ \mathsf{list}(x)$$

- Subtraction Rule

$$\frac{Q_1 \vdash Q_2}{Q_1 * S \vdash Q_2 * S}$$

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- A proof theory oriented around Abstraction and Subtraction.
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- Subtraction Rule

$$\frac{Q_1 \vdash Q_2}{Q_1 * S \vdash Q_2 * S}$$

- Try to reduce an entailment to the axiom

$$\frac{}{B \wedge \mathsf{emp} \vdash \mathsf{true} \wedge \mathsf{emp}}$$

Works great!

$$\mathsf{lseg}(x, t) * t \mapsto [tl : y] * \mathsf{list}(y) \;\vdash\; \mathsf{list}(x) \qquad\qquad \text{Abstract (Roll)}$$

Works great!

$$\mathsf{lseg}(x, t) * \mathsf{list}(t) \vdash \mathsf{list}(x) \qquad \text{Abstract (Inductive)}$$

$$\mathsf{lseg}(x, t) * t \mapsto [tl : y] * \mathsf{list}(y) \vdash \mathsf{list}(x) \qquad \text{Abstract (Roll)}$$

Works great!

$\mathsf{list}(x) \;\vdash\; \mathsf{list}(x)$	Subtract
$\mathsf{lseg}(x, t) * \mathsf{list}(t) \;\vdash\; \mathsf{list}(x)$	Abstract (Inductive)
$\mathsf{lseg}(x, t) * t \mapsto [tl : y] * \mathsf{list}(y) \;\vdash\; \mathsf{list}(x)$	Abstract (Roll)

Works great!

$\ddot{\smile}$

$\mathsf{emp} \vdash \mathsf{emp}$	Axiom!
$\mathsf{list}(x) \vdash \mathsf{list}(x)$	Subtract
$\mathsf{lseg}(x, t) * \mathsf{list}(t) \vdash \mathsf{list}(x)$	Abstract (Inductive)
$\mathsf{lseg}(x, t) * t \mapsto [tl : y] * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Abstract (Roll)

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$\mathsf{lseg}(x, t) * t \mapsto \mathtt{nil} * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Abstract (Inductive)

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$\mathsf{emp} \vdash \mathsf{emp}$	Axiom!
$\mathsf{list}(x) \vdash \mathsf{list}(x)$	Subtract
$\mathsf{lseg}(x, t) * \mathsf{list}(t) \vdash \mathsf{list}(x)$	Abstract (Inductive)
$\mathsf{lseg}(x, t) * t \mapsto [tl : y] * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Abstract (Roll)

$\ddot\frown$

$\mathsf{list}(y) \vdash \mathsf{emp}$	Junk: Not Axiom!
$\mathsf{list}(x) * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Subtract
$\mathsf{lseg}(x, t) * t \mapsto \mathtt{nil} * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Abstract (Inductive)

List of abstraction rules for lseg

Rolling

$$\mathsf{emp} \quad \longrightarrow \quad \mathsf{lseg}(E, E)$$

$$E_1 \neq E_3 \wedge E_1 \mapsto [tl : E_2, \rho] * \mathsf{lseg}(E_2, E_3) \quad \longrightarrow \quad \mathsf{lseg}(E_1, E_3)$$

Induction Avoidance

$$\mathsf{lseg}(E_1, E_2) * \mathsf{lseg}(E_2, \mathsf{nil}) \quad \longrightarrow \quad \mathsf{lseg}(E_1, \mathsf{nil})$$

$$\mathsf{lseg}(E_1, E_2) * E_2 \mapsto [t : \mathsf{nil}] \quad \longrightarrow \quad \mathsf{lseg}(E_1, \mathsf{nil})$$

$$\mathsf{lseg}(E_1, E_2) * \mathsf{lseg}(E_2, E_3) * E_3 \mapsto [\rho] \quad \longrightarrow \quad \mathsf{lseg}(E_1, E_3) * E_3 \mapsto [\rho]$$

$$E_3 \neq E_4 \wedge \mathsf{lseg}(E_1, E_2) * \mathsf{lseg}(E_2, E_3) * \mathsf{lseg}(E_3, E_4)$$

$$\longrightarrow \quad \mathsf{lseg}(E_1, E_3) * \mathsf{lseg}(E_3, E_4)$$

Proof Procedure for $Q_1 \vdash Q_2$, Normalization Phase

- Substitute out all equalities

$$\frac{Q_1[E/x] \vdash Q_2[E/x]}{x = E \land Q_1 \vdash Q_2}$$

- Generate disequalities. E.g., using

$$x \mapsto [\rho] * y \mapsto [\rho'] \;\rightarrow\; x \neq y$$

- Remove empty lists and trees: $\mathsf{lseg}(x, x)$, $\mathsf{tree}(\mathtt{nil})$
- Check antecedent for inconsistency, if so, return "valid". Inconsistencies:
  $x \mapsto [\rho] * x \mapsto [\rho'] \qquad \mathtt{nil} \mapsto - \qquad x \neq x \qquad \cdots$
- Check pure consequences (easy inequational logic), if failed then "invalid"

Proof Procedure for $Q_1 \vdash Q_2$, Abstract/Subtract Phase

Trying to prove $B_1 \wedge H_1 \vdash H_2$

- For each spatial predicate in H_2, try to apply abstraction rules to match it with things in H_1.
- Then, apply subtraction rule.

$$\frac{Q_1 \vdash Q_2}{Q_1 * S \vdash Q_2 * S}$$

- If you are left with

$$B \wedge \mathsf{emp} \vdash \mathsf{true} \wedge \mathsf{emp}$$

report "valid", else "invalid"

Completeness

- Proof procedure is complete (and quadratic) when we know all the listsegs are nonempty (when $x \neq y$ is there for each $\mathsf{lseg}(x, y)$).

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- Complete procedure for general case, using excluded middle

$$\frac{x = y \wedge Q_1 \vdash Q_2 \qquad\qquad x{\neq}y \wedge Q_1 \vdash Q_2}{Q_1 \vdash Q_2}$$

Completeness

- Proof procedure is complete (and quadratic) when we know all the listsegs are nonempty (when $x \neq y$ is there for each $\mathsf{lseg}(x, y)$).
- Complete procedure for general case, using excluded middle

$$\frac{x = y \wedge Q_1 \vdash Q_2 \qquad x \neq y \wedge Q_1 \vdash Q_2}{Q_1 \vdash Q_2}$$

- The resulting proof procedure is exponential. Never implemented.

Part III

Automatically Inferring Frame Axioms

A Small Spec, and a Small Proof

- Spec
  $[\mathtt{tree}(p)]$ $\mathtt{DispTree}(p)$ $[\mathtt{emp}]$

- Proof of body of recursive procedure

  $[\mathtt{tree}(i) * \mathtt{tree}(j)]$
  $\mathtt{DispTree}(i);$
  $[\mathtt{emp} * \mathtt{tree}(j)]$
  $\mathtt{DispTree}(j);$
  $[\mathtt{emp}]$

$$\frac{\{P\}C\{Q\}}{\{P * R\}C\{Q * R\}} \text{ Frame Rule}$$

A Small Spec, and a Small Proof

- Spec
  $[\texttt{tree}(p)]\ \texttt{DispTree}(p)\ [\texttt{emp}]$

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  $[\texttt{tree}(i) * \texttt{tree}(j)]$
  $\texttt{DispTree}(i);$
  $[\texttt{emp} * \texttt{tree}(j)]$
  $\texttt{DispTree}(j);$
  $[\texttt{emp}]$

To automate
we must infer frames
during ``execution”

$$\frac{\{P\}C\{Q\}}{\{P * R\}C\{Q * R\}}\ \text{Frame Rule}$$

Frame Inference: An Extension to the Entailment Question

$$A \vdash B$$

Frame Inference: An Extension to the Entailment Question

$$A \vdash B * ?$$

Frame Inference: An Extension to the Entailment Question

$$\texttt{tree}(i) * \texttt{tree}(j) \ \vdash \ \texttt{tree}(i) * ?$$

Frame Inference: An Extension to the Entailment Question

$$\texttt{tree}(i) * \texttt{tree}(j) \;\vdash\; \texttt{tree}(i) * \texttt{tree}(j)$$

Frame Inference: An Extension to the Entailment Question

$$x \neq \texttt{nil} \wedge \mathsf{list}(x) \;\vdash\; \exists x'.\, x \mapsto x' * ?$$

Frame Inference: An Extension to the Entailment Question

$$x \neq \mathtt{nil} \wedge \mathsf{list}(x) \ \vdash \ \exists x'.\, x \mapsto x' * \mathsf{list}(x')$$

Frame Inference: An Extension to the Entailment Question

$$A \vdash B * \textcolor{red}{?}$$

How to infer a frame

Convert a failed derivation

$\mathsf{list}(y) \vdash \mathsf{emp}$	Junk: Not Axiom!
$\mathsf{list}(x) * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Subtract
$\mathsf{lseg}(x, t) * t \mapsto \mathtt{nil} * \mathsf{list}(y) \vdash \mathsf{list}(x)$	Abstract (Inductive)

into a successful one

$\mathsf{emp} \vdash \mathsf{emp}$	Axiom
$\mathsf{list}(y) \vdash \mathsf{list}(y)$	Subtract
$\mathsf{list}(x) * \mathsf{list}(y) \vdash \mathsf{list}(x) * \mathsf{list}(y)$	Subtract
$\mathsf{lseg}(x, t) * t \mapsto \mathtt{nil} * \mathsf{list}(y) \vdash \mathsf{list}(x) * \mathsf{list}(y)$	Abstract (Inductive)

How to infer a frame, more generally

- Problem: $A \vdash B * ?$
- Apply abstraction and subtraction to shrink your goal: if you get to $F \vdash \texttt{emp}$ then F is your frame axiom.

$$
\begin{array}{cc}
F \vdash \texttt{emp} & \Uparrow \\
\vdots & \Uparrow \\
A \vdash B & \Uparrow
\end{array}
$$

- Sometimes you need to deal with multiple leaves at top (case analysis)

Part IV

Abstract Interpretation

Cooking a Program Analyzer

1. Just write an interpreter. (Well, an *abstract* interpreter.)
2. Symbolically execute statements using in-place reasoning (all true Hoare triples).
3. Interpret while loops by using abstractin rules like

$$\mathsf{ls}(x, t') * \mathsf{list}(t') \vdash \mathsf{list}(x)$$

   to automatically find loop invariants. This uses the rule of consequence on the right to find the invariant for the while rule

$$\frac{\{P\}C\{Q\} \quad Q \vdash Q'}{\{P\}C\{Q'\}} \qquad \frac{\{I \wedge B\}C\{I\}}{\{I\}\texttt{while}\ B\ \texttt{do}\ \{I \wedge \neg B\}}$$

4. A terminating run of the interpreter will give us a **proof** of assertions at all program points.

Example

```
{emp}
x=nil;
while (_ ){
        new(y);
        y ->tl = x;
        x=y;
}
```

Calculated Loop Invariant

$\vee$

$\vee$

Example

```
{emp}
x=nil;
while (_ ){      x = nil ∧ emp
        new(y);
        y ->tl = x;
        x=y;
}
```

Calculated Loop Invariant

$$x = \mathtt{nil} \land \mathsf{emp}$$

$\lor$

$\lor$

Example

$\{\texttt{emp}\}$
$x{=}\texttt{nil};$
$while\ (\_\ )\{ \qquad x \mapsto \texttt{nil}$
$\qquad \texttt{new}(y);$
$\qquad y \texttt{->} tl = x;$
$\qquad x{=}y;$
$\}$

Calculated Loop Invariant

$$x = \texttt{nil} \wedge \mathsf{emp}$$
$$\vee \quad x \mapsto \texttt{nil}$$
$$\vee$$

Example

$\{\texttt{emp}\}$

x=$\texttt{nil}$;

while (_){ $\quad x \mapsto x' * x' \mapsto \texttt{nil}$

$\quad\quad \texttt{new}(y);$

$\quad\quad y$ -> $tl = x;$

$\quad\quad x=y;$

}

Calculated Loop Invariant

$$x = \texttt{nil} \wedge \texttt{emp}$$

$$\vee \quad x \mapsto \texttt{nil}$$

$$\vee$$

Example

$\{\texttt{emp}\}$

```
x=nil;
while (_ ){          ls(x, nil)
    new(y);
    y->tl = x;
    x=y;
}
```

Calculated Loop Invariant

$$
\begin{aligned}
& x = \texttt{nil} \wedge \mathsf{emp} \\
\vee \quad & x \mapsto \texttt{nil} \\
\vee \quad & \mathsf{ls}(x, \texttt{nil})
\end{aligned}
$$

Example

$\{\texttt{emp}\}$

x=$\texttt{nil}$;

while (_){ $\quad x \mapsto x' * \mathsf{ls}(x', \texttt{nil})$

  $\texttt{new}(y)$;

  y ->$tl = x$;

  x=y;

}

Calculated Loop Invariant

$$x = \texttt{nil} \land \mathsf{emp}$$

$$\lor \quad x \mapsto \texttt{nil}$$

$$\lor \quad \mathsf{ls}(x, \texttt{nil})$$

Example

$\{\texttt{emp}\}$

$x{=}\texttt{nil};$

while (_){ $\qquad \mathsf{ls}(x, \texttt{nil})$

$\quad \texttt{new}(y);$

$\quad y \texttt{->} tl = x;$

$\quad x{=}y;$

}

Calculated Loop Invariant

$$x = \texttt{nil} \wedge \mathsf{emp}$$

$$\vee \quad x \mapsto \texttt{nil}$$

$$\vee \quad \mathsf{ls}(x, \texttt{nil})$$

Example

$\{\mathtt{emp}\}$

$x{=}\mathtt{nil};$

$while\ (\_\ )\{$ $\qquad \mathsf{ls}(x, \mathtt{nil})$

$\qquad \mathtt{new}(y);$

$\qquad y \texttt{->} tl = x;$

$\qquad x{=}y;$

$\}$

Calculated Loop Invariant

$$x = \mathtt{nil} \wedge \mathsf{emp}$$
$$\vee \quad x \mapsto \mathtt{nil}$$
$$\vee \quad \mathsf{ls}(x, \mathtt{nil})$$

Fixed-point reached!

Perspective: this seemingly exotic logic has pretty effective proof technology

- Can mix with other provers. Apply subtraction rule to simplify formulae as a tactic, or make call-outs other proof procedures as you go.
  - **Parkinson***'s talk later today: call-out to Z3 SMT solver;*
  - **Shao***'s talk later: proof theory inside Coq tactics.*
  - **Tuerk***'s poster and paper in proceedings: HOLfoot.*
  - *And other tools (Tokyo, Sydney, Singapore) use Omega, Isabelle...*
- Frame inference can be used to modularize a program analysis or (I imagine) to increase automation in an interactive proof.
- The induction-avoiding abstraction rules, like

$$\mathsf{lseg}(x, t) * \mathsf{list}(t) \vdash \mathsf{list}(x)$$

can also be used to infer loop invariants (abstract interpretation) by helping us reach fixed-points.

See talks of **Parkinson**, **Gotsman**, **Magill**, **Distefano** *later today for more on analysis.*